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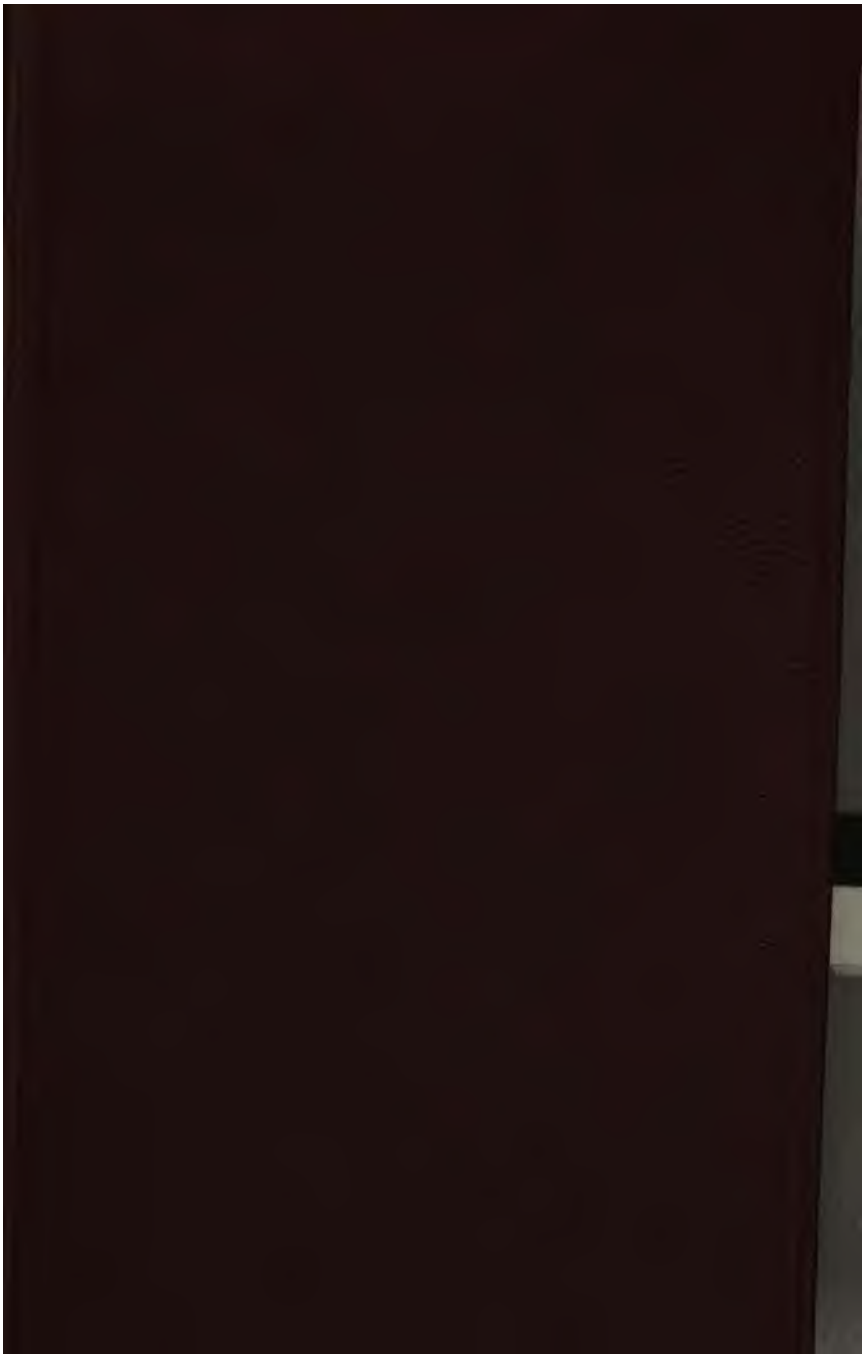

THE

MINERAL SURVEYOR & VALUER'S COMPLETE GUIDE;

COMPRISING A

TREATISE ON IMPROVED MINING SURVEYING,

WITH

NEW TRAVERSE TABLES;

AND DESCRIPTIONS OF IMPROVED INSTRUMENTS; ALSO AN EXPOSITION OF THE CORRECT PRINCIPLES OF LAYING OUT AND VALUING HOME AND FOREIGN IRON AND COAL MINERAL PROPERTIES:

TO WHICH IS APPENDED THE SECOND EDITION, CAREFULLY REVISED, OF

M. THOMAN'S

(*of the Société Crédit Mobilier of Paris*)

TREATISE ON COMPOUND INTEREST AND ANNUITIES, WITH LOGARITHMIC TABLES.

BY

WILLIAM LINTERN,

MINING AND CIVIL ENGINEER.

With Four Plates of Diagrams, Plans, &c.

LONDON:

LOCKWOOD AND CO., 7, STATIONERS' HALL COURT,

1872.

188. g. 28

Cambridge:
PRINTED BY C. J. CLAY, M.A.
AT THE UNIVERSITY PRESS.

PREFACE.

THE TRAVERSE TABLES herewith submitted to the public were computed some years ago by the author, to supply a need he had often felt in his professional duties, when engaged in laying out works for the connection of different and distant operations in collieries and mines: viz. a Set of Tables by the use of which the results of some extensive surveys may be computed, and acted upon, with greater accuracy and confidence than can be sometimes attained by the ordinary methods of plotting.

It occurred to the author that if a survey were made with great care and accuracy, the results should be capable of computation with absolute certainty; and that thus much cost in mining operations would occasionally be saved, by the drivages meeting properly, as intended: for where the cost of driving is large, in hard ground for instance, frequently costing £3, or £4, or more per yard, an error even of 5 yards may thus become an expensive matter; and frequently also there will be no inconsiderable expense in endeavouring to "straighten" a crooked driving, which has resulted from incorrect "points" having been given.

It is not claimed for these Tables that they will supersede good surveying; on the contrary, if the sur-

veying has been incorrectly done the Tables will not, and cannot, correct the error; but where the work of surveying has been done correctly, the employment of these Tables will eliminate the result much more unerringly than plotting; and thus they have often been proved by the author to be very useful and valuable.

Their application to surveys of mineral workings or openings which "pitch" highly must also be a matter of great utility, and they will supply a very ready medium, by which to ascertain the horizontal measure of the different pitches of the lines taken, which often vary very much in the same mine or colliery.

The use of the Magnetic needle is herein strongly defended, as being more generally applicable, as a rule, to mining surveying than any other system that can be suggested. But the author takes the liberty of submitting that the *magnetic meridian* is not a satisfactory "base" of working: and a description will be found herein of an improved circumferentor, by the use of which the True Meridian may be made the regular working "base," instead of the Magnetic Meridian; and by which the error occasioned by the constantly varying declination of the latter may be effectually guarded against.

A description is also given herewith of an improved protractor, by which the angles may be taken off directly from the instrument itself; instead of from points made in the paper either by the "folding arms" or pencil employed with the plainer kinds: this the author thinks must facilitate the making of a good and correct plan, and save the paper from those numerous punctures which are made in using the existing kinds of protractors.

A general description of the operations of careful mining surveying will also be found in the chapters preceding the Tables; and some of the peculiarities, both of home and foreign mining, are there pointed out for the benefit of those who may have to engage therein, and prepare plans, etc. of the same.

A chapter is also given on the Valuation of mining properties, and some examples of the varying class of property connected with mining, which may be also occasionally useful.

The work of surveying etc. enters largely into mining, and the cost of mining is sometimes affected in no slight degree by the manner in which the work of the surveyor is accomplished: and he should therefore omit no opportunity of performing his task in a manner to yield the best possible results. If the accompanying matter and Tables will in any manner facilitate the attainment of these desiderata the author will be pleased, and his anticipations realized.

Possibly some errors may hereafter be discovered in the work, but as every care has been taken to ensure accuracy, it is hoped they will not be found to be numerous.

If any justification is considered necessary for having written this treatise with the Tables for publication, it is submitted in these few words:—the author does not doubt but that others have felt the want that he himself felt, before he set about the computation of these Tables; and that there are probably many who would like to avail themselves of such assistance as they may afford, at a less cost than that of having to construct them for themselves. Of the text it may be said that it seems to be but a natural accompaniment of the Tables.

THOMAN'S TREATISE.—The Revision which the Edition of Thoman's Treatise appended to the present work has undergone is most complete: all the examples have been reworked, and the errors of the former edition have been herein corrected.

The errors which have been expunged from this book are neither few nor unimportant, and in the Note introduced at pages xiv.–xvi. an explanation is given of the manner in which two serious blemishes of the first edition have been met and removed; but throughout the book there were a large number of corrections necessary, either in the stating of the questions, or in the details of working them, and, in some few instances, in the stating of the formulæ on which the computations are based.

From the care with which these corrections have been made, and the proofs of the altered stereotype plates revised, it is hoped and believed that this valuable treatise will now become more generally useful and accessible to those for whom it was originally intended; and that the difficulties which must have accompanied any attempt to utilize its valuable principles from the first issue will in this be found to have been fairly overcome. A careful study of the above-mentioned Note will be of very essential service to a due understanding of the statements of the numerous and valuable examples with which the book abounds.

A few new examples are also given at the end of the Introduction, to facilitate a comparison of these Tables for the computation of money interests by logarithms, with such as those of Smart's which are given in natural numbers.

W. L.

DOWLAIS IRON WORKS,
July, 1872.

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CHAPTER I.

PAPER FOR MAPS.

When a Survey of a property is about to be made for the purpose of obtaining a correct plan, it is important to secure a sheet of good paper mounted on cloth, of about the necessary size, and to keep it in a dry and moderately cool place for some few days before anything is done towards laying down any part of the survey; if this were not attended to, but the paper ordered from the maker and the plotting proceeded with immediately on its arrival, it would soon be discovered that lines which were laid down with care at the first according to scale, shew a different length on the scale being applied to them some little time after, supposing the paper to have remained in a dry room; and it would be a hopeless task to endeavour to get lines laid down subsequently, to "come in" correctly with those laid down at the first. Paper arriving fresh from the mill is sure to be more or less damp, and after it has been exposed a little time to a dry atmosphere the damp evaporates, and, as a natural consequence, the paper shrinks; and until ample time has been allowed for this evaporation, in a medium dry and warm room, the paper should not be used, but should be occasionally unrolled and turned. It will also sometimes occur that paper which has been at hand a considerable time, has been kept in some very dry cupboard, or place in which it may have shrunk below its ordinary dimensions: in this case, if lines were to be laid down to scale immediately on

bringing it out, and the paper then left exposed to ordinary atmospheric influences, it would be found to have expanded, and on applying the scale to the lines previously laid down, it would be found that they now represented a greater length than was originally intended.

Some persons may think that these are unnecessary precautions, and that no appreciable error or difference arises from such causes, but such is far from being the case; when a plan is being made to, say, 2 chains to an inch, and the lines laid down are some of them from 20 to 30 chains each, it would then be no unusual thing to find a difference of 20 links and upwards in the longest lines, and this represents an error, multiplied over two or three such lines, sufficient in some cases to give rise to disagreeable consequences, and to law suits occasionally for infringements of boundaries, the costs of which are sometimes heavy.

This is only one of the many little points of detail which go to make up the difference between a good plan and a bad one,—between an accurate plan and an inaccurate; for with a material so absorbent of moisture as paper is, it is most natural to expect that in laying down long lines to scale, without giving any attention to the normal, or abnormal state of the paper, differences and errors are sure to arise and to accumulate.

Where the plan to be made is a permanent working plan the best paper should be procured, for inferior papers wear badly, and are easily chafed off with the action of indiarubber in the operation of cleansing them from the dust and pencil marks, which are the inevitable accompaniments of a mining plan; but an inordinate use of indiarubber only goes to destroy the face of the paper, and causes it to become the more easily soiled.

The best hot-pressed paper is considered the most durable, and best to work upon; and it should be well mounted either on white or brown cloth, and bound at the edges.

CHAPTER II.

DIVIDING AND LINING THE PAPER.

HAVING given the paper sufficient time to season properly, the next thing that should be done with it is to carefully lay down two series of lines, dividing the paper into squares of, say, 10 chains each way, or 10 acres area each; one set of these lines should represent a meridian (which of the meridians it should be we will hereafter discuss), and the other set should traverse the first at right angles. This operation should be performed with scrupulous care, and to do it properly and accurately a large beam compass is necessary, and a long thin true straight-edge; if one of steel is at hand so much the better,—there are few things more useful and valuable to the surveyor.

If a hand compass (or ordinary dividers) is used to mark off on the two base lines (which a person of experience would at once recognise as the first step) traversing each other, it should be so set that it will accurately divide such a sufficient number of tens, taken together, as shall ensure the accuracy of each division; since it is, in a matter of this kind, unwise to trust to the setting of the compass to one single measure on the scale:—for instance, if a divided beam compass is used the hand compass should be passed over such a number of the multiples of the division determined on for the side of the squares on the paper, as shall ensure its dividing accurately over the length and breadth of the paper.

The lines should then be finely and truly drawn through the points so marked with faint red ink, or some other colour that will be sufficiently distinguishable from the other lines to be drawn upon the plan.

It will be necessary to determine, before the lines are laid down, what direction on the paper the meridional lines must take so as to ensure the plan of the property being got fairly

into the body of the paper, or else the conformation of the outline of the boundary to the edges of the paper will never have a pleasing effect on the eye; this may usually be ascertained from some old plan, or by taking the bearing of some distant object with the theodolite from one of the angles of the boundary, or a combination of two or three such lines across the middle of the property. The correct laying down of these sets of lines is a matter of great importance, as instances are frequently met with where plans of properties of 300 or 400 acres area, or more are made, and the magnetic meridian is represented by a short line in one corner of the paper of about 14 or 16 inches in length; and to plot a series of lines at different points on the paper from this one meridional line is a matter of no little difficulty, and such an operation involves almost certain errors to a greater or less extent; for whether a rolling parallel ruler, or angle and straight edge are used (the latter being decidedly the most accurate) in the operation, errors under such circumstances cannot be wholly avoided; but with these rectangular and parallel lines the protractor can be laid down on the part of the map most convenient for easy plotting, and greater accuracy can be thus ensured. These lines are also useful in another way, for if proper attention has been paid to the preliminary work of the seasoning of the paper; and next, to the marking of the distances, these lines afford a ready gauge at all times, by applying the scale to them, to ascertain whether the Map has been left about in a damp place, and is unduly expanded with moisture; or whether from having been in a very dry and warm place, it has contracted below its normal dimensions.

We have noticed the fact that when a Map has lain open on the table in a warm sunlight for some time the contraction has been sufficiently marked to be distinctly appreciable in a line of 5 inches long; and thus it becomes wise to have regard to influences likely to affect, in a marked manner, the expansion or contraction of the paper.

By having the plan so divided into squares of 10 acres each they also assist the eye in forming a rough estimate of

the quantity of ground exhausted of any particular vein:—supposing the ground to have been worked regular and uniform; they also assist the computation, when a more accurate estimate is to be made.

CHAPTER III.

THE MERIDIONAL LINES.

In respect of the "Meridional" lines it has hitherto been generally, or, so far as we know, universally the practice to represent the *magnetic* meridian as the working base line of the plan, from which, at all subsequent times the workings or additions to the plan have been marked off; and this practice has continued in consequence perhaps of, first, a want of due consideration of the actual difference the magnetic variation really makes from year to year in lines of even moderate length; and, secondly, from an absence of a readily and simply adjustable instrument with which to take the angles with the differences corrected.

The western declination of the magnetic meridian is now decreasing at the rate of about 1° in 8 years, or about 7½ minutes per annum.

If we may suppose the case of a plan made, say, 8 years ago, with a particular instrument to take the angles, and that in the meantime no addition has been made to some particular part of it; but that now, from a resumption of mining operations or some other cause, an addition is to be made to it from surveys now taken; and if we further suppose a portion of the new workings to be a continuation of a straight engine plane, and that the part to be added measures, say, 850 links; and, further, if the length be laid upon the plan according to a magnetic angle taken *now* with the instrument, we shall find that instead of the present angle corresponding with the last taken 8 years ago, and so showing a continuous straight

line, there will be a lateral deviation from a straight line to the extent of 14·83 links to the *right*, looking in the direction in which the extension has been made. But if it is an addition to be made after one year instead of eight years, during which time, at the ordinary rate of decrease, the deviation will have been 7½ minutes; and if the addition to the plan be, say, 630 links, the lateral deviation will amount to 1·36 links for this length; and when we consider that a mining plan is frequently being added to for 16 years or 20 years, and often more, the importance of taking the magnetic variation into consideration cannot well be disputed; for in 20 years the lateral extreme deviation in a line of 20 chains in length, would amount to no less than 87·2 links; and in the simpler case of a 12 *month's* work being added at one survey at the end of the year, and where the aggregate of the lines in one direction has been 9 chains, the accumulating error would amount to 1·96 links, or about 16 inches.

These things fairly considered it will readily be seen what an unsatisfactory base of operations the line of magnetic meridian really is, if treated as an *invariable* line, while it is, as a matter of fact, *constantly variable* and changing.

The plan of taking the angles with the magnetic needle is so general, and answers the purpose of a good mining plan so well, that it is not intended to say anything here in disparagement of its utility and general applicability; indeed we do not conceive that there is, at the present time at least, any other system known, that can be brought into successful competition with it. It is quite right and proper to be able at times to dispense with the use of the needle, when a surveyor is compelled by force of circumstances to fix the instrument in some locality likely to disarrange the polarity of the needle, and to interfere with its proper action, and in such cases it becomes absolutely necessary to dispense with the use of the needle; but we conceive it to be in such cases only that it is found advantageous to dispense with its use, for we hold that with a thoroughly good instrument used and handled with care—the needle being set each time

to zero, and the angle being read on the verniers—the use of the needle affords the most correct results that can be attained in mining surveys, and thus the errors which are *instrumental* can be reduced to a minimum.

But as no instruments hitherto in common use have any adjustable zero to alter with the alteration of the magnetic meridian, either once or twice a year, as may be deemed necessary, it has been usual to set the needle to the common zero of the dial-plate, and to work from this line as though it was *invariable.*

Bearing in mind the foregoing facts, we advance the opinion that the meridional lines to be laid down on the plans should represent the *true* meridian, and not the *magnetic* meridian; and the second series of lines will of course cross these at right angles, and represent the East and West lines upon the plan.

The way in which we propose to provide for the adjustment of the instrument to the True Meridian, and the adoption of that meridian as the working base line instead of the magnetic meridian, will be found described in Chapter VIII. of the present treatise.

CHAPTER IV.

CHAIN, CHAINING, ETC.

THE proper handling of the chain is scarcely of less importance, in mining as in every other surveying, than that of the instrument; and it should not for a moment be supposed that the surveyor can delegate the affairs of the chain altogether to his chainmen.

It is of first importance to have the chain correct in length, and the stretching corrected regularly as found necessary. A correct datum for proving the chain should be at hand, and should frequently be resorted to, to ensure the detection of any extension that may have occurred. It is

not sufficient that this should be done at stated periods, it should rather be done whilst bearing in mind the work the chain has been used in; for it will often occur that the chain will have been strained and stretched more with one or two days' operations in some bad place or ground, than in ordinarily good positions in as many months' use; and hence the chain should be tested whenever there is reason for suspecting it necessary, in fact, it should be done often, so that the surveyor may be assured of its accuracy at all times.

This having been done, it should be constantly kept in mind that no amount of care exercised in the taking of the angles with the instrument will compensate for a want of care in taking the lengths of the lines; for a correct angle and incorrect line, or *vice versa*, cannot possibly go to make up a correct plan; hence, however good an assistant the surveyor may have to take the chain-lengths, he should never wholly lose sight of the work himself, but should always give such an amount of attention to it as would ensure a constant diligence in his assistant, to take off the true lengths of the lines; for, especially in underground work, if the chain is very dirty, the difficulty of recognizing the divisions is greatly increased; and, in the like proportion, the vigilance necessary to obtain the correct lengths. As arrows can seldom or never be used underground, the assistant that draws the chain forward has to supply the substitute, in the manner of marking the chain-ends; and as this man will generally be found to be a man picked up in the colliery or mine for the occasion, and with no particular idea of chaining, or any appreciation of the necessity for nicety in marking the lengths, the greater is the danger that it will be thought by him sufficient if he marks somewhere within three inches of the proper place. We have generally found it necessary to give this leading man a short lesson before commencing operations, with the view to ensure his attention and care: and we have found the most accurate way to mark the ends of the chain to be, to cause the leading man, after he has drawn the chain

forward the whole length, and has been put properly into *line*, to turn around to face the man at the other or "following" end of the chain; and having strained the chain properly into line, to bring the *toe* of his right foot to the handle of the chain,—the handle being held edgewise, and the foot being placed wholly on the ground; if this be carefully done there can be no error so far. Note is then taken of any headings, stalls, faults, threads, soft or inferior coals, commencement of solid rib, or of gob, or of any other thing on the way requiring to be booked down, and the man at the "following" end then walks forward and places the toe of his right foot to the toe of his companion, who then again proceeds forward another length, and the hindmost handle of the chain being brought and held to the toe of the foot the chain is again properly strained, set in line, and again brought to the right toe of the leading man, and so on to the end of the line. Where this is regularly and carefully done, there need be no error of practical moment occurring in the operation of chaining.

Where arrows are and can be used, the number we have found to be best is *eleven;* by using eleven there is always one in the ground, ten being given up to the leading man when the eleventh has been put into the ground, and it thus becomes the first in the next ten. This we think to be better than walking forward when the tenth has been put into the ground, and then draw it out, so that it may be given up with the nine others, and the place of the tenth marked with the foot, as in this way there is the position of the tenth always without its permanent mark: and so great is the convenience of using eleven arrows instead of ten, that we conceive the practice would at once commend itself to any surveyor who should determine to adopt it in preference of using ten only.

It is occasionally found very convenient and advantageous, when chaining over land on which long coarse grass, rushes, or other tall plants are growing, to have a strip of coloured cloth or a small white flag fastened into the handle eye of the arrows, so that they may be the more readily distinguishable amongst the tall herbage; and the practice in such cases affords

a means of frequently expediting the work, in avoiding the loss of time which would sometimes be taken up in searching for the arrows, and enables the eye to catch them readily.

CHAPTER V.

FIELD-BOOK.—ENTRIES, POINTS, ETC.

THE method of keeping the surveying book is a matter deserving of more attention than is often given to it, and a looseness or otherwise in this part of the operations indicates, more or less, the kind of attention bestowed on the business generally.

There is a divided practice in the matter of ruling the Field-book, some using two lines within which to write the chain-lengths, &c., and another and less general method is to have one line only,—this line being an ideal representation of the line chained, and the different lengths being written on it. We must give a decided preference to the latter method, as we have found it more generally convenient to have one line only than two; and in this method the sides of the book are left more clear and open for drawing such lines as may be necessary to illustrate the figures entered down; and the sketching can be more clearly done than when two lines are used.

In underground surveys, having the book lined ready, and being assured of the correct length of the chain, and having an instrument of good quality, the surveyor having selected the first position to set the dial, will carefully level the same and set the needle to zero after it has become steady; and before taking the angles the needle should be disturbed by a slight tap on one of the legs of the stand, or, which is quite as well, by lifting the needle off the pivot with the "rest-piece," and lowering it again, so that it may be seen if it will again return to zero, and if it does the reading off the angle may be proceeded with; if it does not return correctly to zero it should be again slightly disturbed, and if it still varies the first setting

was not correct, or something attracts it, and the zero should be moved a little so as to meet that point at which it will permanently set. The limb will now be worked around so that the light may be correctly seen through the sights, and this being done the angle will be read off and entered in the book, with the number of the line set under it: if the limb has two verniers so much the better (in fact any instrument fit for correct surveying is not without two), and these, divided either to 1′ or 3′, the former being much the best, will enable the angle and its opposite reading to be both taken; and we hold to, and strongly recommend to others, the practice of entering the readings of both verniers in the book, and after they are entered add 180° to the less (mentally), or subtract 180° from the greater reading, and if the sum or difference, as the case may be, makes the other reading as entered in the book, the angles may be considered as correctly entered. Some may think this to be quite unnecessary, but we hold it to be not so, as without it there is no check upon the correct reading of the angle; and there is no person practising this method but will at times find that an error has been made in entering either one or the other of the readings; and without the system of check there would be no certainty, and perhaps no probability, of the error being discovered in time to correct it; but when the two readings are entered, and their difference then computed, there is a strong safeguard against error in this respect.

The two angles of the first two lines having been taken, and the chaining proceeded with, notes are made of anything in the way of headings, faults, or other things occurring in the route, so that the book may be a correct representation of all that will be necessary to be shewn upon the plan: the instrument is then moved forward to beyond the light of the second line, and the operator having gone so far as the back light can be clearly seen, the instrument is again set up, and the operation of setting repeated as before described, and the lines again taken, and so on in succession.

It should be a maxim in working that nothing should be left to memory that can be entered down in the book, for when that is practised the memory will occasionally be overloaded,

and confusion of things will ensue, and error will be the consequence: therefore all things required to be put upon the plan should be distinctly entered in the book, in such order that were it to be put aside for seven years, and then taken up for the purpose of making a Plan of the work described therein, there would be no difficulty in following the particulars entered, and of making a correct plotting of the same.

An abrupt turn to one side or the other will require a mark to indicate the direction in which to continue the plotting or else it may be as natural under certain conditions to think it should be continued in one direction as the other: for the purpose of indicating the change in direction we recommend the making of a mark thus <———, under the figures (for instance <—⑤—) giving the number of the line *commencing at the turn.* The horizontal line is simply that line which is drawn between the figures expressing the *total length* of the preceding, and the *number* of the succeeding line; and the mark either < or > indicates,—the former a turn to the *right* and the latter a turn to the *left*, it being in the latter case placed on *the right* of the horizontal line, and in the former case on *the left.* This mark must not be confounded with the *arrow* or else it will mislead: thus, the mark <———, here recommended, indicating a turn to the right, and ———> indicating a turn to the left, may, if made into arrows thus <———, or ———>, be made to indicate the opposite; but no difficulty on this head will arise when the distinction in the marks is remembered;—the idea being that the point joining the two short lines of the mark <, or >, represents the *position of the instrument* in reference to the two adjoining lines, and the short lines meeting in that point shew the *relative directions* of the lines so meeting, of which they are the representatives.

Note must also be taken of the "rise" or "pitch" of the measures in which the chaining is being done, so as to allow for the same, and to eliminate the true horizontal measure.

An example is given on Plate 4 of a book made upon these principles, with some descriptive entries, and a plotting of the same for the guidance of students.

It will sometimes occur that a survey is required to be made so as to give "points" to headings being driven towards each other, from distant parts of the workings, and it may be of great importance that they should meet well in line, not only to save the cost of driving about (perhaps in hard ground) to get "loose," but to ensure the working being subsequently usable for a heading, engine plane, or some other important work; or it may be to find the relative position which a proposed pit, either for air or otherwise, occupies, or where it will come down when sunk from some overlying measures or veins; or it may be to ascertain the proper position and direction for a drivage into a pit in some measures through which the pit has passed, for drainage or for ventilation: in these and similar cases it will be advisable to make an entire survey around, from one face of the work to the other, or so far as it is practicable to go; and in such surveys it will seldom be necessary to make notes of anything particular on the way, except for the purposes of a check at a few points. These lines will require to be plotted with great care, so as to be able to determine the "point" that will have to be given to the drivages, so that they may meet well. It is in such cases and similar to these, that the method of working out the line of survey by the Traverse Tables is so important, and of such utility; for if the survey has been correctly made, and the lines worked out by the Tables, the *terminal* positions can be determined with much greater accuracy by this combined operation than by plotting. We would not indeed recommend that the plotting should not be done; but where the survey was both worked out by the Tables and plotted, and some little difference resulted in the two methods (which may certainly be looked for), we would unhesitatingly give the preference for accuracy to the result obtained by the Tables. If the difference should be very considerable there may be some error either in the plotting, or in the taking out the figures of the Tables, or in not placing them in their proper columns, and a search should be made to discover it, so that the greater confidence may be given to the result obtained. But we have had a difference between

the plotted and the computed plan (or the terminal points of such plan) of considerable length, of upwards of 30 links; and having failed to discover any error in either the operation of plotting, or working by the Tables, we have decided in favour of the result obtained by using the Tables, and have found the center of the sinking (the matter in hand in that case) to come down within two feet of the mark made for it, although the *route* of the survey lay through very crooked and uneven ground, and through a pit from one measure to the other. This is not given as an unusually good result, for better could be added, but only as a proof that had the plotted survey been acted upon—although it was laid down with the greatest care—instead of the result obtained by the Tables, the error in this case would have been considerable; but as a rule the difference would be less, except in long and difficult surveys when it would perhaps be even more. And it is only what would justly be expected in such cases, if we consider the method of computation to be more correct than plotting could possibly be; for there are no thick compass points, no little accumulating errors in the setting the lines from point to point, the thickness of pencil lines etc., but the rectangular co-ordinates of each line are joined by absolute mathematical points.

If the opposite drivages are to be a heading or drift in hard ground, instead of following a vein of coal or other mineral, it will then be necessary to "level" the headings around from one point of driving to the other, so that the difference of level may be also known, and the "pitch" the new drivages are to be set to, worked out. The miners should then be supplied with teebobs, inclined to the proper angle at the foot, with which they should set the plates or rails of their roads; and as this is only an approximation to the true pitch, the surveyor should take his Spirit-level and try the accuracy of the floors occasionally, as well as examine the "points" by the dial, to ensure against error in either pitch or points.

Great caution should be exercised in making use of timber for holding the points; it will often happen that in con-

sequence of one side of the heading being a little "tighter" than the other, at the place where timber has been selected to hold the point marks, the squeeze of the ground will cause a slight (and sometimes a considerable) lateral movement in one or both of the colar pieces so selected; and possibly they may have motion imparted to them in opposite directions, as is frequently seen; and if such should occur to two colars used for carrying the "point," or even to one of them, the consequence may be serious if not detected in time to rectify the error. Therefore the solid ground should, if possible, carry the points, even if it should be necessary to stand timber for the sole purpose of preventing the roof scaling off, or for supporting the ground from falling while the drivages are in operation, or until the "points" can be moved to more secure ground.

In moving "points" forward great care should be practised, so as to ensure the continuance of the work in the desired alignment. If it is a drivage going to meet another coming from the other side, or a single drivage going to some particular position in some workings ahead of the new heading, the instrument should be used to move the "points" forward when it becomes necessary: and the dial being placed under the first of the former point-marks, others can be set up near the face of the workings for further guidance; or, which is better, a mark should be put at about every 30 yards along the heading or drift, and from these the men should try their work. It is much more sure to drive with points 30 yards apart, if they are carefully put in position, than with those at 10 feet apart; and in the former arrangement it is only necessary to shift one mark at one time, and they are always within easy reach of the workmen.

When two marks are to be shifted forward to the "face" without the application of the dial, the accuracy of the new marks should be carefully tested; and to do this proceed by setting up the new points near the face by plumbing marks to the roof, from lights put in position by looking through the original mark lines: then, first, have a light placed in the "face," by using the *original* mark lines; next, by using the

new lines see if they and that in the "face" still coincide (which they should do); next, take *one* of the old lines and *one* of the new, and again see if they coincide with that in the "face"; if they do this last operation will be a proof that the new lines will correctly continue the alignment given with the old lines, and the marks will have been correctly shifted.

As to the kind of appliance that is best for holding the *light*, we may remark that we have found it the better way to have no appliance at all for suspending the light, but simply to weight the cord, soundly fastened to the roof, sufficiently to draw it out straight and free from kinks. If then the light be held right in front of it, if marks 30 yards distant be used; or by the side and at two inches from it, if the marks are about 10 feet apart,—for at the latter distance the string itself will be plainly visible with the candle two inches off,—the arrangement will admit of very easy and correct working.

If however safety lamps are exclusively used, and open lights are inadmissible, it is then better to suspend the safety lamps from the lines by their handles, and use them in that way to move the mark forward; but in all cases more care will be necessary in driving with safety lamps, than with free lights, as it is more difficult in that case both to "try" and also to keep the point.

The benefit which should and would result to the employer or owner, as well as the satisfaction felt by the surveyor at seeing the work "meet" correctly, will more than compensate for what some may be disposed to look upon, but which we do not, as excessive nicety in the matter of "points" and "pitches"; and not only in these matters, but in every other matter connected with correct surveying, it is the attention to the small details, and the care with which they are executed, that make the difference between a good plan and a comparatively worthless one: for even mining surveying (the roughest-done work of any perhaps in the way of surveying) consists in something more than in being able to tell, within an half degree, the division on the dial to which the needle points, though this is often looked upon as all that is necessary to

enable a person to survey a colliery; but the "snakelike" contortions and windings often to be met with in collieries and mines, as the consequence of not being able to get opposite drivings *loose* in the way they were intended to meet, testify to the rough off-hand methods too often practised, as well as to the expensiveness of such a class of surveying.

CHAPTER VI.

TRAVERSING.

HAVING passed in review the field-book, setting of the dial, reading off the angles, taking the notes, and other things necessary to supply the data and particulars requisite to the making of a correct plan of what will be met with in the mine or colliery, we will next proceed to show what entries are requisite and essential, in order that the Traverse Tables may be used in the computation of a survey.

In the first place we will refer to the accompanying diagram, No. 1, and would recommend a very careful perusal of the whole of the lines comprising the same, and especially of the red lines, and the directions in which the "arrows" point. Let it be supposed that the positions *A* and *B* represent two pits or other objects, and that the black lines represent the lines chained in passing, in the operation of surveying, from *A* to *B*; note also the direction of the meridional line with its transverse axis, and the point that represents the north. The black lines 1, 2, 3 will be observed to go nearly eastward; lines 4, 5, and 6 nearly northward; 7, and 8, have a northern departure from the west, and 9 goes to the south of west; 10, 11, and 12 then go nearly to the north; 13 goes nearly east, and 14 nearly southward. Now it is possible to define the relative positions of *A* and *B* without plotting the lines at all, by the use of the Tables, and the simple operations of addition and subtraction.

In conjunction with each black line representing the chained lengths, will be noticed two red lines, each containing an arrow, and forming with the black line a right-angled triangle. All the *red* lines parallel with the *meridian* line of the diagram are represented by figures in the columns of the Tables headed Lat. (a contraction of "Latitude") and Per. (a contraction of "Perpendicular"), and their values will be taken out from that column; and all red lines parallel with the *transverse* axis of the meridian, being parallel with the equatorial line, are represented by figures in the columns headed Dep. (a contraction of "Departure"), and Base, and their values will be obtained from those columns. Next observe that some of the arrows in the red lines parallel with the meridian point to the North, and some to the South, accordingly as the line to which they stand points northward or southward of the east and west line that may be conceived to pass through the zero or starting-point of the line in question, of the diagram: some of the arrows parallel with the equatorial line point eastward, and some point westward, accordingly as the line to which they stand points eastward, or westward of the corresponding ideal meridian line. Now if we first take the equivalents of the red lines parallel with the meridian, and separate those pointing to the north from those pointing to the south, or, which is the same thing, place them in separate columns, and take the total of each column, we shall see in which direction the working has progressed by observing which column has the higher total; and if the less be subtracted from the greater, the difference or resultant will be the actual distance between the two terminal points, *taken in a line parallel with the meridian.* Also it will be similarly observed that of the lines parallel with the equator —some pointing to the east, and some to the west—if we separate the equivalents of these into distinct columns as we did the other, and take the totals of each, and their difference, we shall find what is the resultant sum of the progress in a line parallel with the equator, and whether it has been to the east or to the west.

These two differences will now give the data with which

to define the relative positions of *A* and *B*. Making a mark to represent *A*, and from this mark drawing a straight line of sufficient length to enable us to prick off on it by some scale (any may be chosen) the first found difference, make a fine and distinct mark; then with a set square or other accurate instrument, draw carefully at right angles from this point a line, either to the right or to the left as required by the *second resultant*, and on this line mark off with the same scale as before the second difference; this distance from the first line will be the position of *B*. The former of these lines will be a north and south line, and the latter will be an east and west line; thus the required relative positions are determined, and the lines with which it is done form the perpendicular and base of a right-angled triangle.

The thorough digestion of the remarks and explanations here given, in reference to the diagram and its several lines, will make the apprehension of what follows comparatively easy; and it is essential that it should be thoroughly understood, before proceeding to attempt to work out any series of lines by the Tables.

Traversing Entries in Field Book. The next thing will be to shew how the angle is to be entered in the book, so that there may be no difficulty in easily determining the proper reading.

The plate forming the *bottom* of a dial is usually divided and figured in each quadrant distinct, from 1^0 to 90^0; two quadrants commencing to reckon from the *N.* point in the box, and two commencing at the *S.* point; two finishing at the *W.* point, and two finishing at *E.* This division of the circle into quadrants shews at once, in their respective initial and terminal points of division, *the points from which*, and the *direction in which* each quadrant must be read, in order to the workings being computed by the Traverse Tables.

The first quadrant therefore, embracing the horizon from north to east, will be read *from* north *to* east; the second quadrant, embracing the horizon from east to south, will be read *from* south *to* east; the third quadrant, embracing the horizon from south to west, will be read *from* south *to* west;

and the fourth quadrant, *from* north *to* west. This must be the *invariable practice* for traverse working, and where the readings cannot be obtained from the verniers, as they cannot be in more than the first quadrant, the angles must be carefully computed: and for this purpose the readings should be severally entered in the surveying-book as before described, as though it was intended to make use of them in plotting a survey as ordinarily: then, the traversing angle in the first quadrant will be the ordinary reading of the instrument; the traversing angle in the second quadrant will be the difference between the ordinary reading and 180°, thus 100° 42′ in the ordinary reading will be 79° 18′ for traversing, and 146° 27′ in the ordinary reading will be 33° 33′ for working by the Tables; the traversing angle in the third quadrant will be the ordinary reading of the instrument, *less* 180°, thus 212° 30′ in the ordinary reading becomes 32° 30′ for traversing, and 254° 12′ will become 74° 12′ for working by the Tables; and the traversing angle in the fourth quadrant will be the difference between the ordinary reading and 360°, thus 293° 45′ will become 66° 15′ for traversing, and 357° 21′ will become 2° 39′ for working with the Tables; and so on of all other angles, in whichever quadrant they may be situate. There is here no great difficulty in comprehending this part of the matter clearly, close diligence and thought being only necessary.

The next thing to be mastered is the *description* to be given in the traversing to the *co-ordinates* of each line: by the description to be given them is to be understood the entering with each angle distinctively whether its co-ordinates are a northing and a westing, or a southing and a westing; a southing and an easting, or a northing and an easting. Thus the angle in the first quadrant (remembering the order given in the instructions above in reference to the angles) will have a northing and an easting for its co-ordinates; the angle in the second quadrant will have a southing and an easting for its co-ordinates; the angle in the third quadrant will have a southing and a westing for its co-ordinates; and the angle in the fourth quadrant will have a northing and a

westing for its co-ordinates; and these rules hold good for every angle in the respective quadrants in succession. Of course if the line should be *exactly* due north, or due south, it will neither have an eastern nor a western co-ordinate, but it will be a northing or a southing entire, without reference to the Tables at all; and in the case of a due east or a due west line, there would be no northing or southing co-ordinate, but the line would be an entire easting or westing as the case may be, without any reference to the Tables, the lengths of the lines to which they refer being entered in full.

These explanations and instructions embrace the whole that need be said to enable any person of ordinary intelligence, and who will try, to understand the theory and practice of working with the Traverse Tables; but the instructions here given should be thoroughly mastered, so that the Tables may be correctly worked, and errors avoided in the operation.

The entry of the particulars for traversing should be made upon the same sheet or leaf of the field-book with the ordinary entries, and will simply consist of the number of the line, the traversing angle, the initials of its descriptive co-ordinates, and the total length; and these can be placed in the side of the book, as in the example we give herewith of a field-book, with both entries placed in juxtaposition.

CHAPTER VII.

THE TRAVERSE TABLES: THEIR CONSTRUCTION, AND APPLICATION.

THESE Tables were constructed by the author some years ago for use in his professional duties, and they are based on the same principles as the Traverse Tables for seamen's practice as given in "Hutton's Mathematical Tables." It will however

be found, on reference to that book, that the Tables there given are worked out for degrees and quarter-points only, and to radii from 1 to 10; and as a Table not less minute than this is inapplicable to surveying we determine to proceed to the construction of tables to every 3 minutes of the quadrant, and to 10 different radii of hundreds, commencing with 100, and ending with 1000. It will be readily seen that thus constructed the Tables are applicable to any base of measurement, either English or Foreign, that may be most convenient; it may be a link, foot, yard, furlong, mile, metre, kilometre, or league, in fact any measure desirable or necessary; but it is of course requisite that the measure of the *angle* should be taken according to the *English* system of 90 degrees in the quadrant, and 60 minutes in the degree.

The figures in the Table, and the positions of the decimal points, are applicable to the lengths represented at the top and bottom of each pair of columns, and marked "Dist."; these are each hundreds. But the computations are perfectly correct and applicable if the "Dist." is multiplied by 10, or considered as thousands; and when this is done it is only requisite to shift the decimal point as in the Tables, or conceive it to be shifted, one figure to the *right*, when the result will be obtained true to one place of decimals. If the "Dist." be *divided* by 10, the figures as in the Tables will in like manner be divided by 10 by shifting the decimal point one figure, or conceiving it to be shifted, to the *left;* and if divided by 100 (say by counting 600 as 6) the figures as in the Tables will be in like manner divided by 100 by shifting the decimal point *two places* to the left: hence the Tables are applicable to the computation of the co-ordinates of a line of any measurement or length whatever, from 1 link to 100,000 links, or from 1 metre to 100,000 metres, or kilometres, or miles. The Tables being computed to every 3 minutes of the quadrant will in general be sufficiently minute for all classes of surveying; but in the case of very long lines, if it should be found or thought necessary to obtain the co-ordinates to a single minute, it can be done to very near the truth (even when the decimal point is moved one figure to the right) by

interpolation, taking either ⅓ or ⅔ of the difference as may be required, between those two lines of figures within which the angle to be computed lies (in doing this remember that the co-ordinates of each line *increase* in opposite directions, one increasing down the column, and the other up the column).

The degrees from 1° to 45° will be found along the top, and the minutes down the left side of the tables; and from 45° to 90° the degrees will be found along the bottom, and the minutes up the right side of the Tables, as in Trigonometrical Tables generally. If the angle therefore is less than 45° it will be found at the top and left side of the Tables, and the proper order for tabulating the co-ordinates of the angle will likewise be found at the top; if the angle is greater than 45° it will be found at the bottom and right side, and the order for tabulating the co-ordinates at the bottom likewise.

Thus, if we wanted to find the co-ordinates of a line 600 links in length, and whose ∠ is 33° 42′, we have on the page for 33° and on the line for 42′ the quantities sought; viz—Per. =499·17., and Base=332·90; also, if we want the co-ordinates of ∠ 56° 18′ (or the comp. of 33° 42′), and a line of the same length as before, we have Per. = 332·90, and Base = 499·17;—being the same figures as by the first operation, *but with their relative positions reversed.*

If the lines had been each of them 666 links, instead of 600, we should have had in the first case, viz. ∠ 33° 42′ and Dist. 666, as follows.

Ex. 1.

	Per.		Base
600 =	499·17	and	332·90
60 =	49·92	„	33·29
6 =	4·99	„	3·33
Totals.	Per. = 554·08		Base = 369·52

And in the second case we should have ∠ 56° 18′ and Dist. 666.

Ex. 2.	Per.		Base
600 =	332·90	and	499·17
60 =	33·29	„	49·92
6 =	3·33	„	4·99
Totals =	369·52	and	554·08

Again, suppose the case of ∠ 20° 39′, and Dist. 745; then look along the Tables until you find 20° and down the left side to 39′, and on that line take out the figures for 700, 40, and 5 respectively, thus:—

Ex. 3. Angle 20° 39′, and Dist. 745.

	Per.		Base
700 =	655·02	and	246·86
40 =	37·43	„	14·11
5 =	4·68	„	1·76
Totals =	697.13	and	262·73

Take one more example. ∠ 76° 45′, Dist. 1789 links. In this case the degrees will be found along the bottom, and the minutes on the right side thus:

Ex. 4.	Per.		Base
1000 =	229·20	and	973·38
700 =	160·44	„	681·36
80 =	18·34	„	77·87
9 =	2·06	„	8·76
Totals =	410·04	and	1741·37

The foregoing examples will clearly indicate the methods of taking out the co-ordinates for traverse surveying.

It will be perhaps advisable to draw attention to the last example, and to point out that in consequence of the Tables having computations for 1000, no shifting of the decimal point is called for in this case in reference to that number; but for every 1000 above the first there will be a shifting of the decimal point;—for instance, the co-ordinates for 2000 will be found in the columns for 200, with the decimal point placed

one figure to the right, and so on for the others as each case may require.

It should also be the invariable practice to place the figures coming under the designation of "Per." *first* in position, as they are above, and those coming under the designation of "Base" *last*. It should also be remembered to consider the co-ordinate designated "Per.", as that ordinate *parallel with the meridian;* and the co-ordinate designated "Base" as that *parallel with the equator:* by following these instructions there will be no confusion of the co-ordinates, nor any misconception of the true positions for the termini to be described from the results obtained.

We will now adduce proof of the foregoing examples by working out corresponding results by the ordinary rules applicable to the computation of right-angled trigonometry, as in the following.

RULE: *Add the logarithm of the given side to the sine of the angle opposite to the side required, and from the sum subtract the sine of the angle opposite to the given side: the remainder will be the logarithm of the side required.*

Bear in mind that the sine of the *angle of the traverse* is the co-ordinate "Base"; and the sine of its complement, the co-ordinate "Per." (or "Perpendicular").

The sine of 90° being equal radius—the angles treated being all right angles—we have

One proof of Ex. 1. (giving the "Base")

As Rad.....	=	10·0000000
: log. of 666	=	2·8234742
:: sine ∠ 33° 42'.....	=	9·7441712
: log. of 369·25	=	2·5676454

Also one proof of Ex. 2. (giving the "Base").

As Rad.....	=	10·0000000
: log. of 666	=	2·8234742
:: sine ∠ 56° 18'.....	=	9·9200994
: log. of 554·08.....	=	2·7435736

Proofs of Ex. 3.

As Rad..... = 10·0000000

: log. of 745 ..... = 2·8721563

:: sine ∠ 20° 39′ ..... = 9·5473542

: log. of 262·73 ..... = 2·4195105

and As Rad..... = 10·0000000

: log. of 745 ..... = 2·8721563

:: sine ∠ 69° 21′..... = 9·9711608

: log. of 697·13..... = 2·8433171

Proofs of Ex. 4.

As Rad. ..... = 10·0000000

: log. of 1789..... = 3·2526103

:: sine ∠ 76° 45′ ..... = 9·9882821

: log. of 1741·37..... = 3·2408924

and As Rad..... = 10·0000000

: log. of 1789..... = 3·2526103

:: sine ∠ 13° 15′..... = 9·3602154

: log of 410·04 ..... = 2·6128257

These proofs shew the construction of the Tables, as well as supplying conclusive evidence of their accuracy, and of the accuracy of the method employed in the operation of extracting the co-ordinates for the different angles, and length of the lines.

There is nothing new in the proposal to apply these principles to surveying. They were, so long ago as A. D. 1635, applied by a Mr Norwood to lines and angles taken with the circumferentor over the public road from York to London, and the difference of latitude of those two cities determined by him, very nearly, from the traverses; and proposals have been made to apply the principles to the planning of large estates; but, so far as the author knows, there have been no

other Tables so far issued, so well calculated as these are to render the application of these principles to surveying both easy and general,—computed as these are to a degree of minuteness sufficient for all practical purposes of correct surveying, and their application duly explained in the accompanying letter-press.

There is no reason why these principles should not be applied extensively to mining surveying; for in large and overlying mining operations there are cases continually arising where it is required to sink from one vein of mineral to another, for ventilation or some other purpose; also to drive from one part of the workings in some mine or colliery across the measures to another; and also when driving in both directions it is highly important that the points given to the drivages should be correct; and sometimes it is required to drift at a more or less steep pitch from one measure to another; and in such operations, when the work proceeds in both directions at the same time, the question of the cost, as well as the matter of convenience, depends greatly upon good surveying and correct points; and it is such cases as these that really put the skill of the surveyor to the test, and they are also those cases in which the application of the principle of traversing is found to yield the most satisfactory results. But cases arise also where it is desirable to ascertain if the boundary has been reached, at some point very remote from the Pit, or level mouth; and sometimes it will require a series of lines of somewhere between 60 and 100 (perhaps more) to reach the supposed position of the boundary: then the survey should be taken with great care, and the application of Traversing should be adopted in such a case, as being much more conducive to accuracy than plotting can be from the same data.

It will likewise occur frequently that in a long underground survey there will occasionally be a prominent turn in the general bearing of the workings; perhaps the general bearing so far has been nearly westward, and at a particular part the direction alters, and the bearings are somewhere to northward. It is very desirable that the point where the change of direction occurs should be very carefully marked down from the traverses

first, and also the terminal point; the plotting should then be proceeded with, and the plan so made should be brought as near as possible at those corresponding parts to the points marked from the traversing computation; and where a little difference is found to have accrued, preference should be given to the traverse points (always bearing in mind that care should be observed to ensure accuracy in taking out the co-ordinates).

If it is suggested that this double operation is unnecessary, and that plotting is sufficient without traversing, we can confidently assert that bad and loose surveying is exceedingly expensive; and that there is no system so conducive to true economy as accuracy, and this cannot be attained without due measures being adopted to ensure it.

The chief utility of the Traverse Tables in surface surveying is the facilities they afford for reducing the principal lines across an estate to one pair of rectangular co-ordinates, and thus increase accuracy.

Although it is still the practice with many surveyors, in making a survey of a property, to make a circuit of the boundary by taking a great number of short lines according to the bends, or outline of the limits of the property,—making these short lines the ground-work of the survey from which to construct the map,—and especially where there is but little more than the boundary to lay down; but it is about the worst system that can be followed, and often results in the production of a Plan not worth the labour expended upon it.

The salient angles in the boundary of the property to be surveyed should first be selected by the surveyor, in sufficient number to enable him to devise such a system of main base-lines and triangles across the property as shall enable him to fix all those points, as well as a sufficient number of objects within the boundary on his Plan, with absolute certainty, by Traversing, or Trigonometry, or both combined, as circumstances require: the intervening lengths of the boundary should then be surveyed by a combination of straight lines and triangles, according to the conformation of the boundary line; and in all cases running the straight lines *where practicable* from one salient point to another, and filling in where neces-

sary with diverging lines to meet bends, and which divergence may be measured as a check with a sextant. In this way, if errors have been committed at any point they are almost sure of being detected, and their locality known; and if they arise between some two of the points selected, they cannot affect the accuracy of the next section; and all the internal objects will be easily taken, in connection with the different internal stations of the triangulation.

As scarcely any two estates are of similar outline, or are affected by similar conformations, so the same system of lines will seldom be found perfectly applicable to any two surveys, and hence the judgment and experience of the surveyor must in each case be relied on to guide him, in the selection of that system which will involve the least amount of labour combined with the greatest amount of attainable accuracy.

Where the estate is tolerably level the selection of these lines, and their measurement, do not present much difficulty; but where the country is mountainous, and steep escarpments have to be crossed and measured it is different, and careful means have to be then adopted to ensure accuracy. Where it appears to be indispensably necessary to have a line, either as a principal, or as a check-line, up a steep and rugged rise, it will generally be found more accurate and preferable to obtain the true length of the line by the application of trigonometry; and which will probably be best accomplished by selecting two stations in the plain, one of which shall be the end of the line to be measured (or even an intermediate point in it, if it is thought to answer the purpose better than the end), and between which and the other station in the plain the ground must be favourable for *correct chaining*,—each visible from the other, and from another station on the top of the hill, which may be the other end of, or an intermediate position in, the line to be measured, and to be selected on favourable ground, from which correct chaining may be continued. The three angles of this triangle should be correctly taken with an accurate instrument, and their sum should equal 180°; and the line in the plain to be measured should be gone over carefully twice, so that the first result may be checked. Then,

by applying the rule of trigonometry previously given in this chapter, the correct horizontal length of the ascending line can be easily determined; and this length should be substituted for the length found by chaining, as being more correct, (even though the horizontal measure may have been procured as near as practicable), and therefore entitled to the preference. If this is a line forming one of a series, for the purpose of determining the positions of the *salient points of the boundary only*, the *chaining* of it will be unnecessary.

When it is thus determined to have a few triangles over the property, to determine the position of the salient points, from which to form the ground-work of the Plan, one base-line should be measured on the most favourable ground obtainable near one side of the property; this should be the "base of computation," on which all the calculations of the Triangulation shall be made; and on the opposite side of the

TRAVERSING COMPUTATION. No. 1.

No. of the line.	Quadrant where found.	Traverse Angle.	Designation of Co-ordinates.		Length in Links.	Perpendicular.		Base.		Remarks.
						Northing.	Southing.	Easting.	Westing.	
		° ′								
1	1st	78·26	N.	E.	208	41·71	...	203·78	...	
2	2nd	76·42	S.	E.	354	...	81·43	344·50	...	
3	1st	3·46	N.	E.	246	245·47	...	16·15	...	
4	4th	5·34	N.	W.	208	207·02	...	...	20·17	
5	do.	11·48	N.	W.	349	341·62	...	...	71·37	
6	do.	15·00	N.	W.	540	521·60	...	...	139·76	
7	do.	81· 9	N.	W.	240	36·92	...	...	237·14	
8	3rd	85·30	S.	W.	320	...	25·10	...	319·01	
9	4th	61·33	N.	W.	528	251·53	...	...	464·22	
10	1st	3·00	N.	E.	340	339·53	...	17·79	...	
			Totals are =			1985·40	106·53	582·22	1251·67	
			Subtract			106·53			582·22	
			Nett results			1878·87 = Northing.			669·45 = Westing.	

property the best line possible for correct measurement should be chained, as a check upon the trigonometrical computation. This will be called the "base of verification," and if the angles

The following will exhibit the operation of taking out the above co-ordinates in succession.

(1) 78°·26′ and 208

Dist.	Per.	Base.
200 =	40·11 —	195·94
8 =	1·60 —	7·84
208 =	41·71 —	203·78

(2) 76°·42′ and 354

Dist.	Per.	Base.
300 =	69·01 —	291·95
50 =	11·50 —	48·66
4 =	·92 —	3·89
354 =	81·43 —	344·50

(3) 3°·46′ and 246

Dist.	Per.	Base.
200 =	199·57 —	13·13
40 =	39·91 —	2·63
6 =	5·99 —	·39
246 =	245·47 —	16·15

(4) 5°·34′ and 208

Dist.	Per.	Base.
200 =	199·06 —	19·40
8 =	7·96 —	·77
208 =	207·02 —	20·17

(5) 11°·48′ and 349

Dist.	Per.	Base.
300 =	293·66 —	61·35
40 =	39·15 —	8·18
9 =	8·81 —	1·84
349 =	341·62 —	71·37

(6) 15° and 540

Dist.	Per.	Base.
500 =	482·96 —	129·41
40 =	38·64 —	10·35
540 =	521·60 —	139·76

(7) 81°·9′ and 240

Dist.	Per.	Base.
200 =	30·77 —	197·62
40 =	6·15 —	39·52
240 =	36·92 —	237·14

(8) 85°·30′ and 320

Dist.	Per.	Base.
300 =	23·53 —	299·07
20 =	1·57 —	19·94
320 =	25·10 —	319·01

(9) 61°·33′ and 528

Dist.	Per.	Base.
500 =	238·19 —	439·61
20 =	9·53 —	17·58
8 =	3·81 —	7·03
528 =	251·53 —	464·22

(10) 3° and 340

Dist.	Per.	Base.
300 =	299·59 —	15·70
40 =	39·94 —	2·09
340 =	339·53 —	17·79

have been taken and the sides calculated, each correctly, this last measured line should not differ from the computed length more than from 2 to 3 links. This satisfactory result being obtained (and with instruments of the best construction for taking the angles a nearer agreement is quite possible, and generally considered essential), the points so determined will form an excellent ground-work for the making of a thoroughly reliable map of the property; and the labour expended in obtaining it will be well repaid, by the confidence with which the Plan may subsequently be referred to at all times; and especially should any question as to infringement of boundary, in the working of the minerals of the property, or a committal of trespass from an adjoining property, at any time arise.

As shewing the application of Traversing to a series of underground lines of a survey, we will now proceed to take the first 10 lines of the preceding example of a Field-book, in which *A* (see the plotted Plan of same), the starting point, may be supposed to represent a Pit or Level Mouth; and *B* to be the "face," and end of the 10th line. *C* may also be considered as the face of a branch part of the same workings; and if we suppose it to be required to drive from *C* to *B*, by having a point given at *C*, we shall be able by the Tables to discover the true distance from *C* to *B*, and the point to be given at *C* to drive into the "face" at *B*.

The nett results of the foregoing traversing operations is a Northing of 1878·87 links, and a Westing of 669·45 links; and if the process as before directed be performed, the terminal point of the (10) line can be exactly described from the above co-ordinates as here found and expressed.

We may now proceed from the two lines here found of the triangle, with the included right angle, to determine the other line and the two other angles, and thus complete the triangle; the rule for which is the following: Having two sides and the included angle,

RULE:—*When the angle included is a right angle, add the radius to the logarithm of the less side, and from the sum subtract the logarithm of the greater side, or add its arith.*

complement: the remainder, or sum, will be the tangent of the angle opposed to the less side.

Then, taking the foregoing sides of the triangle, we have

Rad. + Log. of 669·45 = 12·8257181

Arith. Comp. Log. 1878·87 = 6·7261032

9·5518213 = Tan. 19° 37′ nearly.

Therefore 90° − 19° 37′ = 70° 23′ is = the third angle of the triangle.

By applying Euclid 1, Prop. 47, we shall ascertain the length of the third side: using logarithms, we have

Log. 1878·87 = 3·2738968

2

6·5477936 = 35301533 = $1878{\cdot}87^2$.

Log. 669·45 = 2·8257181

2

5·6514362 = 448163·2 = $669{\cdot}45^2$.

And the sum of the squares are thus = 3978316·5

Then, Log. 3978316·5 = 6·5995993

÷ by 2 = 3·2998496 = 1994·57 = $\sqrt{3978316{\cdot}5}$.

= the side *opposite the right angle.*

We may prove this result by finding the number 1994·57 from the Tables: taking 19° 36′ as being the next nearest number in the minute column we have,

	Per.	Base.	Numbers.
∠ 19° 36′, and the coordinates	1878·87	669·45	
in column of thousands take	942·06	335·45	= 1000
remainder	936·81	334·00	
„ „ of 900 take	847·85	301·91	= 900
remainder	88·96	32·09	
„ „ of 900 take	84·78	30·19	= 90
remainder	4·18	1·90	
„ „ of 400 take	3·76	1·34	= 4
„ „ of 500 approximately	·42	·56	= ·5
			1994·5

These numbers from the Tables being successively subtracted from the preceding remainder, exhaust the co-ordinates; and, adding up the successive corresponding numbers, we have 1994·5.

This last operation proves the correctness of the mode of working, and also the finding the side by Euclid I, Prop. 47; for although the last figure ·5 is not found strictly correct, yet, as the tangent of the angle lay between 19° 36′ and 19° 37′ (nearer the latter than the former), and the co-ordinates here taken out stand on the line for 19° 36′, the agreement is so near that for all practical purposes the difference may be disregarded.

It may sometimes be necessary to find the third side, and the two other angles, so that the position of the third line may be correctly laid out on the surface from one of the termini (which may perhaps be the center of a Pit or the Mouth of a Level) to the other, and the angle set out for finding the position of the other terminal point corresponding with that underground, either by chaining along the line laid out to the angle given, or by the application of trigonometry, or a combination of both: this method is much more correct than that of attempting to repeat on the surface all the lines taken underground from one terminal point to the other. Where an important work, such as the sinking of a Pit or some other matter is to be proceeded with, upon the data arrived at by such methods as these, care and precision, with the aid of good instruments, should yield perfectly satisfactory results.

It will here be proper to remark that the lesser angle, whose tangent is found by the foregoing trigonometrical computation, is not *necessarily* the "traversing" angle, by which to ascertain the length of the third side, as in this example; for "the greater side of a triangle is subtended by the greater angle," *but the traversing angle is always subtended by the co-ordinate* "Base": this will be either the greater or lesser co-ordinate according to the nature of the lines operated on. In this example the co-ordinate "Base" is the lesser, and therefore subtended by the lesser angle; and as this trigonometrical operation is to find the lesser angle, it in *this case*

finds the *traversing angle*. But supposing 1878·87 to have been the co-ordinate "Base", and 669·45 to have been the co-ordinate "Per.", then in that case 70° 23′ would have been

TRAVERSING COMPUTATION. No. 2.

No. of the line.	Quadrant where found.	Traverse Angle.	Designation of Co-ordinates.		Length in Links.	Latitude.		Departure.		Remarks.
						Northing.	Southing.	Easting.	Westing.	
		° ′								
1	1st	78·26	N.	E.	208	41·71	...	203·78	...	
2	2nd	76·42	S.	E.	78	...	17·94	75·90	...	
11	4th	56·42	N.	W.	246	135·05	...	...	205·60	
12	4th	64·50	N.	W.	332	141·18	...	...	300·48	
13	3rd	84·36	S.	W.	450	...	42·34	...	448·00	
14	3rd	74·30	S.	W.	320	...	85·51	...	308·36	
15	Par^{l}.	90·00	...	W.	248	...	...	...	248·00	
16	4th	10·51	N.	W.	675	662·93	...	...	127·06	
17	1st	7·54	N.	E.	436	431·85	...	59·92	...	
18	1st	12·15	N.	E.	94	91·86	...	19·95	...	
19	1st	7·44	N.	E.	216	214·04	...	29·07	...	
						1718·62	145·79	388·62	1637·50	
Subtract^{t}. lesser from greater						145·79			388·62	
Nett result						1572·83	and		1248·88	

COMPUTATION. No. 3.

20	1st	62·9½	N.	E.	655·3	306·04	...	579·43	...	
Add Northing						1878·87	Subt. Easting		669·45	

the angle *on whose line in the Tables* should have been sought the length of the third line; and not on that of its complement 19° 37′: this should be carefully remembered in these operations. It is perfectly true that the "line" in the Tables of an

angle and its complement is always *the same*, but the *designation* of the two co-ordinates are in one case the *reverse* of those of the other.

The preceding traversing operation No. 1, embraces the lines of the plotted example of a survey at page 30 from *A* to *B*, lines 1 to 10; the No. 2 embraces the left-hand series of those lines from *A* to *C*, and diverges from the right-hand series at (2) 78.

On comparing the No. 2 with No. 1 computation, it will be seen that the No. 1 line is identical in both, being in fact the same line; No. 2 line of No. 2 computation is so much of the No. 2 line in the No. 1 computation as lies between the commencement of the line and the point of branching off, viz. at 78. From this point they are quite a distinct series of lines. No. 15 line goes due west, and it has therefore no pair of co-ordinates, but the *whole length* of the line is placed *in the column "Westing."*

The nett results will be found to differ from the results of No. 1 operation in having a less Northing, and a greater Westing; and a little reflection will shew that the co-ordinates of the "point or angle" to be given at *C*, to guide a drivage to *B*, will be a Northing and an Easting; consequently it will be in the first quadrant.

No. 3 computation, line 20, supplies the data to make the results of No. 2 equal to those of No. 1, arriving in fact at the same point; and a comparison will shew that the Northing in No. 1 is 306·04 more than in No. 2, and that the Westing in No. 1 is less than in No. 2 by 579·43; and we must reduce the No. 2 Westing to that extent, by having an Easting co-ordinate; therefore a Northing and an Easting: from these data we must now decide the angle and the distance, according to the method before given: viz.

Rad. + Log. 306·04 = 12·4857782

Log. 579·43 = 2·7630010

9·7227772 = Tan. ∠ 27° 50½.

Therefore the angle opposite the co-ordinate Base is

$90^{\circ} - 27^{\circ}\ 50\frac{1}{2}' = 62^{\circ}\ 9\frac{1}{2}'$; and taking this ∠ and the co-ordinates as follows.

		Per.	Base.	Numbers.
		306·04	579·43	
and finding		280·22	530·54	= 600
	·rem.	25·82	48·89	
„ „		23·35	44·21	= 50
	rem.	2·47	4·68	
„ „		2·33	4·42	= 5
	rem.	·14	·26	
„ „		·14	·26	= ·3

we thus find the length of line No. 20 equal to 655·3 links.

We might have arrived at the same result if we had taken the whole of the lines around from *B* to *C*, and have combined them in one computation, omitting No. 1 line and the first 78 links of No. 2; that is, taking No. 2 line as 276 links instead of 354: in that case however either the right or left-hand series of lines would have *a different designation*, accordingly as the survey was taken from *B* to *C*, or from *C* to *B*; if the former, the *right-hand* series would have opposite designations, and if the latter, the *left-hand* series would have the opposite designations. This operation may easily be performed as an example for practice.

Having gone thus fully into the working of underground traverses, and there being really no difference in the mode of application of these principles to any lines either on the surface, or otherwise, there will now be no difficulty in applying them to any series or combination of lines whatsoever to which they are applicable. It is unnecessary therefore to pursue this part of the subject any further.

CHAPTER VIII.

AN IMPROVED DIAL AND PROTRACTOR.

For the adoption of the true meridian as the working base-line of the Survey and Plan, it is necessary to have an instrument suited to such method of working; for the practice of the process of computing it from the "declination", through all the varied angles of each successive survey, and thereby reducing the several angles to their relation to the true meridian, would be a matter of great labour, and its practical difficulty would, as a rule, prevent its adoption; and the practice of a more correct process thus requires an improved instrument for the direct taking of the angles of a survey, in relation to the true meridian as a base-line or zero.

It would be easy enough, in the matter of a simple surface survey only, to represent upon the plan the true meridian; but in the matter of mining surveys where the Plans are being constantly added to over a long series of years, and where the magnetic needle is the medium (and doubtless the best) of adjustment for the instrument, it seems to be essential, to the adoption of the true meridian as the working base-line, that the instrument itself should contain the appliances and means for the necessary correction; while, at the same time, admitting of the same simple method of reading the angles as in the ordinary make of instruments.

To provide for this we have designed an improved circumferentor, by the use of which the correction is made without in any way complicating the method or operation of working the instrument, or of reading and entering the angles of the survey. This improvement consists of an adjustable vernier, working in the divided ring of the dial-box to which the needle is set, and on which the reading verniers move; and it reads on the inside of the ring, and downwards to the bottom plate of the dial-box: and a small portion of the initial part of

the fourth quadrant is divided on the said plate to half-degrees for this operation; and on this the vernier, which reads to minutes, is adjusted; so that its reading, reckoning from the ordinary zero or 360°, expresses the correct declination of the magnetic meridian for the time being. This adjusting vernier carries a small angle piece which works flush with the inside and top of the ring, and for which a small portion of the ring is sunk, by cutting away; and the divisions so cut away from the top of the ring are recut on the sunk part, so that the instrument may be used by the old methods when desirable or necessary: the small angle piece carries a line, in prolongation of the zero line of the adjusting vernier, on its side and top, and to this line the needle is always set instead of to the ordinary zero of the instrument.

The needle being set to this adjusting zero, the ordinary zero of the instrument will be in the *true meridian*, and all the readings of the survey will express the several angles which the successive lines make with that meridian.

If then the meridional lines laid upon the plan be those of the *true meridian*, the protractor must be laid upon those lines; and the "readings" are to be then taken, or marked off, just in the same manner as though the magnetic meridian was being worked from—there being in this respect no difference in the process; and all computations are thus rendered unnecessary.

It will be necessary to adjust this "declination" or adjusting vernier, either quarterly, half-yearly, or yearly, according to the necessities of each surveyor; but in general it would be sufficient to make the adjustment half-yearly. And the plan should have the dates of the general surveys, and a record of the amount of the declination to which the instrument is successively set, entered in tabular order regularly, so that at any future time the later surveys may be made precisely to accord and compare with those made at former periods.

The adjustment is easily effected by the use of a small key, by which the vernier can be moved either way at pleasure; but it is not liable to get out of order, and will not move except when acted on by the accompanying key.

For the purpose of applying this principle to surveying it is necessary to have the means at hand to correct the instrument periodically to the true meridian. This may entail some little labour and expense at the first, but surely the desirability of having first-class Plans will far outweigh any such consideration. Only those who have been much engaged in mining operations, and have observed the great expense very often attendant on bad surveying and bad Plans, as well as the frequent loss of no inconsiderable quantity of valuable mineral, will for a moment question the desirability of adopting good and correct methods of surveying,—unless indeed such persons are blinded by prejudice against the introduction of improvements, in this, as well as in other matters connected with mining.

The expense of procuring the improved instrument, or of having the improvement applied to an existing instrument, is indeed inconsiderable. For the application of the principle continuously upon any property it is desirable—though not *absolutely* necessary—to have two distinct points of observation, either *corresponding with* the true meridian, or making a *known* angle with it. This having been procured it only remains to take the instrument periodically to one of these points, and observe with the adjusted telescope the pole or other object at the other point: if they are *in* the true meridian, set the zero and telescope of the instrument to the said pole, or, if they make a *known* angle with the true meridian, set the verniers of the instrument to the said required angle, and turn the entire instrument round towards the pole or object until the telescope accords with it, while the verniers read the permanent variation of those points from the true meridian. Then observe if the needle and the adjusting vernier agree with each other; if they do not, take the key of the adjusting vernier and move it until it accords with the needle exactly, and the readjustment is complete. Care must be observed in performing this last operation to guard against any attraction of the needle from any cause, or with the correction of one error another will be committed; proper diligence will however ensure against this.

Now it is not absolutely necessary that the true meridian should be laid out upon the property, in order to practise this improved method of surveying, for we may take *any* line near the true meridian (so that it is distinctly understood), and adjust from that line periodically, in the same way as before mentioned, and so ensure the advantages derivable from a correction for magnetic variation: it is doubtless however preferable to have the line laid out either *in* or *from* the true meridian, so as to apply the principle more completely.

We give subjoined two methods for finding the true meridian at any place whose latitude and longitude are known, or are determined.

1st. By equal altitudes of the sun, or a fixed star.

The theodolite being fixed in a position selected as one of the points in the desired line, measure the horizontal angle from the said line to the sun or fixed star (whichever is used), when it is not near the highest or lowest point of its apparent daily course, and take also the altitude, adjusting the intersection of the hairs of the telescope to it, in some marked and definite manner. Leave the limb clamped and let the instrument remain quite undisturbed until the sun or star is approaching the same altitude at the other side of its apparent circular course. Then, without moving the body of the instrument, unclamp the limb, and direct again the telescope towards the same object, and follow it with the aid of the tangent screw until the cross hairs correspond precisely as before with the same relative position upon the disc of the object, as in the first observation: this being done, read off the horizontal measure of the angle between the assumed line and the new direction of the observed object; the mean between the two horizontal angles will define the position of the true meridian, *if a star has been observed.*

If the *sun* has been observed a correction is required in consequence of the sun's change of declination. When that declination changes towards the north the approximate direction of the meridian as found by the foregoing method is too far to the right; if declination changes towards the south it is too far to the left.

The required correction is found by the following formula:—

$$\frac{\text{change of sun's declination}}{2} \times \text{sec. latitude} \times \text{cosec.} \frac{1}{2} \text{ angular}$$

motion of the sun between the observations.

Example. Given:—Date of observation (say 26 July); sun's declination changing towards the south at the rate of 13′ per day, or 32″·5 per hour; time between the observations, say 6 hours, change of declination for 6 hours 3′ 15″; angular motion of the sun between the observations, say 80°; latitude of place of observation, say 51° 34′ N.

Then the correction becomes

$$\begin{aligned}
\text{Log.} \frac{3'\,15''}{2} = 97''{\cdot}5 &= 1{\cdot}9890046 \\
\text{„ sec. } \angle 51^{0}34' &= 10{\cdot}2064865 \\
\text{„ cosec } \angle 40^{0} &= 10{\cdot}1919325 \\
\text{„ } 244'' &= 2{\cdot}3874236
\end{aligned}$$

therefore the approximate direction as found by the observation is too far to the *left* by 244″, or 4′ 4″,

In the preceding process it must be observed that the mean of the two horizontal angles is their half-sum when they are at the same side of the station line, but their half-difference when they are at opposite sides of the said line.

2ndly. Where Greenwich mean time is given or obtainable, by the aid of the telegraph, procure the exact Greenwich time on some convenient day, and with the "equation of time" change that mean time into true apparent solar time: then take the longitude of the place of observation, and add for each degree of difference in west longitude 4 minutes of time, and for each minute of longitude 4 seconds of time; if the observation is in longitude east of Greenwich the same has to be subtracted instead of added.

Then these differences for longitude having been added to, or subtracted from the Greenwich apparent solar time, gives the apparent solar time at the place of observation, and at 12 of that time the sun is due south.

A chromatic object-glass for the telescope is necessary in making these observations.

The small round legs of the mining dial or circumferentor are not sufficiently strong for out-door work in windy weather, when the telescope is being used; for the action of a slight wind will often be sufficient to cause the needle to be unsteady. There seems to be no practicable remedy for this but to have a strong tripod-stand for use in out-door surveying; and while this entails the expense of two sets of legs for the instrument, the advantage gained is often much more than sufficient to compensate for the extra cost. The tripod-stand cannot be used with advantage underground, in many places, for it is too heavy and unsuitable; while it is by far the best stand for surface operations.

Improved Protractor.

We have also designed an Improved Protractor, in the using of which the successive lines of the survey are run off at once from the instrument to their positions upon the Plan, instead of making punctures in the paper, with pencil marks and numbers to distinguish one angle from the other; the pricking of the paper is also thus avoided.

The instrument is made by lengthening out the central bar that carries the verniers, to about 3 inches each way beyond the outer side of the divided circle, and at each end of this bar there is a transverse bar: and on the ends of these transverse bars and reaching from one to the other, are fixed two ebony or metal bars—one on each side—sufficiently distant apart for the inner edge to just work clear of the divided circle. The ends of the central bar each carry a small glass with a + cut, so that those two and the central + are in exact alignment, and form the zero of the instrument. Each side of the frame is exactly parallel with this zero line.

The outer side of the divided circle has a projecting shoulder cut on it, and is provided with a clamp, and adjusting tangent screw, as in the Theodolite.

In plotting a survey from this Protractor it is first laid

upon the meridional line in the most convenient position, and each line of the survey is then laid down in succession as taken from the instrument, either with the large set-square, or parallel ruler, and at once laid down in its proper position; and thus the work of first marking off and numbering the readings is rendered unnecessary.

With the large set-squares either one side or the other of this frame will be sure to be convenient for plotting from.

These instruments can be procured from Messrs Troughton and Simms the eminent Opticians, who have made them from the designs of the author.

CHAPTER IX.

PLOTTING THE SURVEY, ETC.

If the directions respecting the proper seasoning of the paper, laying down the meridional and transverse lines, &c., have been complied with, and the survey made and duly entered in the field-book, the plotting operation will now be proceeded with.

Unless the position of the boundary, and other objects on the surface, can be so far imagined that the site of the pit, or level, can be with confidence fixed for some particular point on the paper (this can frequently be done from some old plan, where such is in existence), the surface survey should be carefully laid down first; and the position of the pit or level having been marked, the underground workings can then be taken in succession.

Taking the underground survey, and the position of the starting point being known or fixed upon, the first thing will be to fix upon an intersection of base-lines from which conveniently to take, or mark off the compass-readings correctly on the paper with the aid of the Protractor.

To produce satisfactory results the surveyor should be possessed of instruments of the best kind, and should then

take care of them. Good instruments once possessed and taken care of will last a lifetime, and will always have the confidence of the owner; but even the best tools, if roughly and carelessly used, cannot be expected to ensure correct results; and badly made instruments can never be reasonably supposed to do so: and the surveyor may rest assured that should his plans prove to be incorrect, the faults will never be attributed to his tools, but to his own want of skill, for he will be expected, or supposed, to have correct tools to work with.

A good protractor is a very essential thing, and those with double folding arms with verniers divided to minutes (when of the best make) are, next to the improved instrument spoken of in the preceding chapter, the best; this instrument having been correctly fixed on the meridian line, the center coinciding with one of the intersections, the two readings of each line should be made to coincide with the verniers, with the aid of a magnifying glass, and both points marked on the paper with a small pencil circle around the punctures, and with the number of the line to which they refer, or run off at once to their position, as the case may be. This having been done with the whole of the lines (if with a folding arm protractor), or of as many of them as can conveniently be marked off from that part of the paper, the readings should then be taken backwards in succession as a check, to see if each pair has been correctly marked off: this being done the protractor will now be removed, and the laying down the lines proceeded with.

For running off the lines either from the improved instrument, or from their markings as before mentioned to the position they are to occupy on the Plan, the best instruments are a pair of large pear-tree angles or set-squares: these will be found to be far preferable to a parallel ruler, or even to one set-square and a straight-edge; for the large set-square has a much broader bearing on the paper than a straight-edge, and is much steadier and less liable to those small lateral movements which will frequently be found to occur with the straight-edge (unless it is a large steel edge, and then it is too

large to be handy for this operation), when sliding in close contact the one over the other, to the position for the lines.

It will be found conducive to accuracy to mark off no more compass readings in any one place on the paper than can easily be run off to their position with one (except in occasional instances) movement of the set-squares; for it will be better to fix the protractor in a new position, and mark off the readings nearly on the part of the paper the lines are to occupy, than to mark off all in one place, and be under the necessity of running them off across a considerable breadth of paper, to their several positions; and with the meridional and transverse lines before recommended, the protractor can be put at any intersection of those lines desirable.

The first line should now be marked in its position by placing the side of the set-square to the two angle-points marked 1 (and a check upon this operation will be afforded by running the eye along the side of the set-square, and observing if it cuts the intersection of the meridional and transverse lines), then placing the other set-square to that side of the former which will the most readily permit the proper side of it *to come upon the starting point* run the line off to its position: a faint pencil mark should now be drawn along the edge of the set-square through it with a hard pencil, so cut that it shall have a thin wedge-like edge, and a flat side to slide against the set-square; and having drawn a line sufficiently long to embrace the distance to be marked off, run the set-square back again to the points to see if it still remains parallel with them; this being proved, next take the compass or dividers with the adjusting leg and the diagonal divided scale, and set the legs to the length required; or if the line be too long to be marked off at once in this manner a portion of the length may be marked off with a scale, and the remainder, including the part less than 100, taken in the compass and added to the former part; a small puncture should be made to indicate the end of the line, and it should be distinguished by a small pencil circle and numbered; also remember to put the number at the *beginning* of the line, as shewn in, and to correspond with, the field-book; the next and

succeeding lines should then be operated on in the same way, until they are all marked off in their respective positions. In progressing forward it will frequently be found that the next line to be put on runs almost at right angles to that last plotted, and except the workings of the mine are thoroughly well known to the surveyor he will be at a loss to know whether it should be a turn to the right or left hand, if he has not remembered to indicate the turn in the field-book; and for this reason the practice should never be omitted, but by turning to the field-book the mark should at once indicate which way to draw the line, whether to the right or left hand; and so on of all the other turns that are capable of misleading without a distinct mark for guidance.

Having laid on all the lines, and being assured of the correct lengths, etc., the different objects and particulars entered to each line in succession should now be placed upon the plan, as indicated by the entries in the field-book.

When we come to a point where the survey has subsequently been continued in another direction, as for instance at 78 ② of the example of a mining survey before given, the length 78 should be marked with the compasses from the diagonal scale, and the "ribs" sketched with the pencil on the proper side, to indicate the position of the commencement of the subsequent work; but where stall roads, faults, commencement of gobs, ribs, or other things have to be indicated it will be sufficiently accurate to mark them off with a pencil, from a long scale laid upon the line in a correct position for so doing, or with a small set scale where necessary. The matter to which these lengths refer should be marked down to each line in succession (after all the lines themselves have been laid down), as the work of filling up proceeds, so that there may be no error from memory in trying to remember subsequently to what the marks refer.

In this way should all the workings of a colliery, or other mine—with the exception of stall-points—be marked down in succession; and as each page of the field-book is completed a mark should be made upon it distinctly, to shew that it is done with.

In reference to the points of stalls, it will be unnecessary to lay down these points in the same manner as those of the important headings, and other parts of the workings. The handiest way of laying down the points of stall roads is to take the small protractor which usually accompanies the other drawing instruments, and laying it properly on the meridional line most conveniently situate, and at one of the intersections, mark off the point with the pencil, and at once with the parallel ruler run off the line to its proper position, and lay down the stall to which it refers; and so on of the others in succession. This will be sufficiently accurate for these things, and they will be pricked off and laid down at once in the operation of laying down the details, instead of mixing them up with the points of the headings and other more important parts of the survey.

If the mine "pitches" too much to make the correction to reduce the measure to "horizontal" as the work of surveying proceeds, care should be observed to make the reduction before the plotting is proceeded with. Where the pitch is general, one taking of the angle of elevation or depression will mostly be sufficient; but if the pitch varies, it will be necessary to observe the angle of elevation or depression more often, and to enter each distinctly: if it is considerable it will be advisable to reduce the lengths of the whole lines, and of any intermediate particular object by the Traverse Tables, for those tables are as applicable to the reduction of hypo. to base, as to the working of traverses on the flat. But here it must be borne in mind, that the horizontal measure will *invariably* be represented by the figures as found in the column "latitude" or "perpendicular," and the other column will shew the vertical rise or fall in the length of the line: in this application of the tables they form a ready medium for correcting the different rises in mines that pitch much, and by an expeditious and correct operation to facilitate the elimination of the true horizontal measure. And where the pitch is considerable, 10° or upwards, the horizontal measure decreases rapidly, and renders it requisite to observe the more care in obtaining the angle of altitude.

While the correct laying down of mining surveys is not difficult to those who go about it with care and diligence, and with a genuine intention to ensure accuracy, it will seldom occur that carelessness or inattention, or a despising of those small niceties which are essential to ensure precision in working, will result in anything but annoying disappointment.

The whole of the workings having been put upon the plan, together with all the details of which note has been made in the field-book, the matter of "inking in" will be ready to be proceeded with. Where two or more veins of mineral are worked one over the other, and are placed upon the same plan, it will be necessary to distinguish one from the other in the colours assigned to the several separate workings, according as fancy or taste may decide, so that each may easily be recognized, with their several connections. Some use the brush in distinguishing the veins one from the other, using black to mark all the workings in, but painting the spaces of each between the lines with a distinguishing colour: of course, these details are very much matter of taste or opinion, and are not of such importance as those principles upon which *accuracy* depends; but we hold that, so far as possible, it is better to dispense with the brush, and use the colours only with the drawing pen; and we consider that a mining plan will generally look neater and better managed in this way than if the colours are put on with the brush, over all the exhausted space. With a surface plan which is finished at once the colouring, shading, etc. may be put on with the brush, and will if done well look well; but with mining plans which are constantly being *added to* it is different, for in this case if the shades as laid on with the brush are not precisely alike they do not look well; and if the colours selected are not simple colours the difficulty of exactly matching them is increased, therefore we think that as much as possible the brush should be dispensed with on a mining plan.

It is very easy in competent colouring hands to make inaccurate plans *look* well, and it is easy to deceive the inexperienced into saying much in praise of a plan so finished, and of the person who has finished it; but no amount of colour

will make an *accurate* plan: and while every surveyor should seek to finish his work neatly his chief care should be to be accurate, and this can only be attained by constant attention to, and practice of, those essential principles upon which accuracy depends.

The manner of laying down the surface lines, and the details of the surface survey, does not essentially differ from that already indicated for the underground survey. In general the surface lines will be much longer and less numerous than the underground lines, and their accuracy will admit of frequent proof. If the groundwork of a survey has been formed by a series of triangles and long base lines in the way indicated in Chapter XI., the laying down the details and the filling in the shorter lines will be comparatively easy; and by a proper disposition of those lines to easily embrace the fences etc., and distinct memoranda in the field-book shewing where they abut on the main base lines, their accuracy, and the accuracy of the plotting of the main lines, will be constantly and reciprocally checked.

The finished plan will of course bear the descriptive title of the property; the name by which it is distinguished; together with the parish and county in which it is situate, and the name of the owner; also on the different boundaries the names of the owners of the adjoining properties. The scale to which it is made will be indicated, and also the detailed area of the property where necessary, and the total area. It will also bear the names of any particular farms, houses, brooks, rivers, woods, faults, or other objects contained within the property.

If the paper has been divided into 10 acre squares as before indicated these will generally assist in the computation of the area: if the whole squares are taken separately, and the parts separately, and the latter added to the former, the aggregate will be the total area. It will however be well to take also another method of computation, that of computing the property as a whole, without reference to the squares; and by comparing one result with the other a check will be obtained upon each computation, and the result obtained will

be entitled to greater confidence. The area will of course be temporarily divided for computation into trapeziums, triangles, etc. principally, and the rules of practical geometry applied to compute the area of those and other figures necessarily entering into the division.

If the meridional lines upon the plan are those of the True Meridian there should be a columnar table on one margin of the plan, in which should be successively entered the date of the last general survey, and also the amount of "declination" to which the instrument was adjusted for the survey; this will then afford a systematic view of the correction continuously applied, for reducing the angular readings to the True Meridian.

It will be found very useful to have a sheet of good tracing paper or tracing cloth, divided into one acre squares (drawn to the same scale as the plan with which it is to be used), in black lines, and each of these squares bisected in both directions with ink of another colour in faint lines, which subdivisions will consequently be each equal $\frac{1}{4}$ acre. If it is then necessary to see approximately the area of ground exhausted of any particular vein of mineral, it will be only necessary to lay this sheet of divided paper on the exhausted space represented on the plan; and by counting the whole acres, and counting and averaging the $\frac{1}{4}$ acres (which can be done by a practised eye very nearly), and adding these together, the area will be obtained to very near the truth, without using pencil lines about or drawing figures upon the plan. And by doing this at the end of each year, and comparing the area of ground exhausted with the coal or other mineral obtained from it, it will be seen at once whether the vein is producing the quantity of mineral which under the concurrent circumstances it ought to do. The side of a square whose area is equal one acre measures 316·228 links.

As examples of the quantities we have proved different veins of coal to produce per acre, for each 1 foot of thickness of the veins, we beg to refer the reader to what is said on that subject in Chapter XII.

CHAPTER X.

THE BUSINESS AND APPLICATION OF SURVEYING.

THE work and business of a surveyor enters largely into mining operations, and particularly into coal mining, and it may very fairly be said that the extent to which good surveying is applied in mining, goes in some degree to indicate the advance made towards a perfect system of management; for although it can never take the place of, or supply the default of, discipline and good scientific knowledge, yet the application of scientific knowledge will proceed, in no small degree, through the channel of good surveying.

Given: A mineral property to be worked, and the direction in which the measures dip being known, it will be of the first importance to know the figure of the property under consideration, so that the position for the "winning" may be determined upon, in order that it may be so situate as to facilitate the economical and convenient raising and draining of the minerals, as well as their despatch to market; and thus a correct Map will be essentially necessary, so that the plan of working may be determined upon. It is still however often the practice to allow the colliery or other mine to form itself, as it were; there being no prearranged plan of development, but simply leaving the different principal headings to follow what appears to the workmen the most natural course. This plan however generally, or rather *invariably* leads, sooner or later, to a complication of workings ill adapted to ensure either the most perfect ventilation of the mines, or the convenient and economical carriage of the minerals to the bank.

If it is a winning by means of a pit, and it is in contemplation to raise a large daily quantity, the laying out of the pit bottom will be a matter that should be well considered; for as this part of the mine will have to pass the whole daily quantity of mineral raised, economy is sometimes considerably affected by the way in which it is laid out: and the most perfect ar-

rangement for passing the full wagons on to the carriages, as well as the empties off for entering the workings, will be found to be worth all the thought that can be bestowed upon it, so as to ensure convenience and despatch. Where the empties are conducted away by a separate road, which has just sufficient fall to keep them moving on to where they are taken off by the horses, or other hauling power, the road will require a fall of from an half-inch to an inch per yard, according to the state the trams are kept in for easy running.

Engine drawing planes on the levels, self-acting incline planes to the rise, and engine drifts to the dip, are now being more generally used than formerly, to cheapen the cost of haulage by reducing the number of horses, as well as at the same time to keep the air more pure and cool than it can be where so large a number of horses, as some collieries would otherwise require, are passing through all the workings continually; and the correct and well-considered laying out of these works and their "banks" will have a considerable effect upon the subsequent cost of working them; for if they are not (except in regular and intentional curves) practically straight, the wear and tear and consequent cost of the wire ropes are very much increased; so that it becomes a matter of great practical importance to get the engine and incline planes in perfect alignment, thereby to ensure that whenever the ropes are by the strain lifted off the rollers they shall again fall back into their correct position, to prevent chafing and cutting against the ground, and surfaces that are not rolling: and if the supervision of the opening of this work is given into the hands of the surveyor, he should take particular care that it is thoroughly done.

There is again the work of opening the headings so that they may be as near as is practicable parallel with each other, when the best point for them has been determined upon; for few things look worse on a mining plan, or prove more objectionable in the practical working of mines, than having all the headings going at random, some to the left hand and some to the right, some giving the stall roads 40 yards to go, and others giving 140 yards before they reach their end; but if they are

driven parallel the coal can be worked much better, and more systematically, and as a natural consequence cheaper and cleaner. And while we know that it rests with the management as to whether such a system is allowed to obtain, and that it cannot well exist where there is a systematic plan of having "points" put on all the headings (and the workmen made to keep them), yet there is very frequently much less care bestowed on this matter than there should be, even with those who will at once admit the necessity and desirability of the principle.

It is sometimes the practice to have points put upon all the stalls where "long-work" is not adopted, as well as upon the headings, and in some veins there is no expenditure of labour that returns a more ample interest than does this; for it is of the first importance to keep the pillars their proper thickness while the roads are going in, to prevent their being reduced, and often as a consequence destroyed, by being "robbed" and made too weak to sustain the top—occasioned by crooked driving; and not only do the pillars thus get crushed and rendered partly, and occasionally wholly worthless, but heavy falls are often thereby occasioned, and sudden outbursts of explosive gas rendered more frequent, which would have exuded more gradually, but for the fall, and the ordinary currents of air would have been able the more effectually to dilute it, and render it less dangerous than is possible when it comes off in large quantities suddenly. Where long-work is adopted, there is not so much necessity for points on the stalls, as there is a much better chance of keeping the roads nearly parallel, when they are open one to the other along the face.

In the making of quarterly or other periodical surveys, it is of importance to have a distinct knowledge as to the position at which to begin. Where it is possible to mark each survey, by placing a *D* or any other mark determined on upon the rib or roof, there is not much difficulty in the matter; but it is not often that the positions can thus be easily determined, and then the surveyor has to fall back upon his knowledge of the relative position of some heading or stall parting, or other

object, and trace out his last terminal position from that. Where there is not much changing of the workmen the name of the man who was working the last road in a panel of work taken, might be entered in the proper position in the book, and the chainage of this parting "cross" being taken it will generally be a good mark from which to define the position at which a former survey terminated, and from which to continue the work of surveying at a subsequent period: though this is, generally speaking, a very correct way of proceeding, yet at times there is a possibility of the surveyor being misled by having the wrong road pointed out as that for which enquiry is being made; and hence constant vigilance will be essentially necessary to ensure against error in this respect. Sometimes it is necessary to measure a level or other heading from some distinct turn, or other conspicuous object; and the system adopted, and regularity with which a surveyor is able to select these points and mark them in his book with clearness for subsequent reference, will greatly influence the time that he will be occupied in going around a large mine to take the new work opened; while an uncertainty in having positions to start from that are but ill defined in the field-book, will often lead to error by the operator taking a wrong point of departure, instead of spending the requisite time to ascertain the correct: hence nothing will contribute more to accuracy than a thoroughly clear and careful method of keeping the field-book, and of entering in a uniform manner the different matter which pertains to the survey made, and whatever may forward the work of continuing the survey at a *subsequent period.*

To facilitate the work of periodical surveys, we would strongly recommend the surveyor to adopt some particular *route* through the workings, and to keep up that route at all his surveys. Also, before going about the survey of a mine or colliery at which he has been before, to sit down and go over all the principal points of his last survey,—making notes in succession upon the different points from which he is about to continue the work of former surveys; these notes being entered in succession in a condensed form on the first page allotted for the new survey, will be all under his eye, and he will

see them all at a glance when proceeding to the operation of surveying, instead of having to search through the whole of the entries made on the last occasion, at each different point of the work. Much time is thus saved in the underground operations, and this system has the additional advantage of bringing before the mind's eye the whole of the workings, and may perhaps recall to mind several matters of importance which it is advisable to note closely, and which, but for it, would probably have been overlooked. It also induces a correct habit in the matter of making entries in the field-book, and of so placing the particulars that they shall be uniform for extracting the notes when required on the next occasion.

The matter of moving points and setting out the gradients of drivages has been treated of in another part of this work; therefore it is unnecessary to go further into that part of the subject in this place. The learner will have no difficulty in ascertaining the point to be given to a drivage, if the line to be followed be first laid down in position on the plan, and next with the parallel ruler or other instrument it be run off to an intersection of the meridional and transverse lines; after which, placing the protractor in its proper position, see what is the angle made by the line of the proposed drivage, and making a note of the same in the field-book, give that angle with the instrument, or dial, to the proper place underground.

In laying out the siding communication, and railways, to the tips or screens of an extensive colliery, or for Iron Works or other operations of large extent, there is need of great care and discernment, and few things will more effectually test the skill of the surveyor, than the designing and planning of these things—indeed it may be almost considered as an entirely distinct matter from the ability to make an ordinary survey and plan; for the latter is, more simply, the making a plan of things, as they already exist, while the former is the *designing* the means of communication for an extent of operations which demand convenience of arrangement with facilities for despatch, in all its details, and this will call for the exercise of sound engineering skill; and upon the successful manner in which these are laid out to meet the peculiarities of the traffic to be

accommodated, and provided for, will depend the amount of engine or other power that will be required to move about, and to place the same; and it is not too much to say that a bad arrangement of roads will speedily involve the employment of an extra engine for shunting, with all its attendant expense, besides perpetuating a continuous existence of more wear and tear, upon both roads and trucks: therefore, the accomplishment of this part of the surveyor's or engineer's work will demand his best energies, for the successful accommodation of the traffic of large works.

It is of great importance to arrange the gradients, wherever it is practicable, so that the wagons shall pass under and away from the colliery screens by their own gravity, giving just sufficient fall to the road to overcome the friction of the rolling surfaces, and thus avoid the necessity of having horses always in attendance: and while in many places the natural conformation of the ground is such that this can be accomplished without the least difficulty, there are others which require to be carefully thought out, to properly accommodate the requirements of the works. And as the rails of the sidings about works are seldom very clean, consequent upon the nature of the traffic to which they are subject, it will not be advisable to count upon flatter gradients than about 1 in 100 for the wagons to run alone, and if a little more fall can be given so much the better; for while a gradient of 1 in 100 will allow for about 22·4 lb. per ton for friction, and should under good conditions be much more than sufficient, yet if wagons and roads are not kept in good running order there will not be any considerable superfluous margin.

In Iron Works it is often impossible to arrange the roads with falling gradients in either direction, to in any appreciable degree facilitate some portions of the traffic; and occasionally the nature of the traffic less requires it than at collieries; but still in such instances there is always much to be considered in the disposition of the roads and partings, to provide for that convenient despatch which is essentially necessary, where so much is being done constantly and unremittingly, night

and day; and for which no further rule can be given than that each particular case must be considered on its own merits, always bearing in mind the quantity and kind of traffic to be provided for, together with the space at hand that can be assigned for its accommodation: and wherever an advantage is to be gained by the adoption of some particular gradient, and there are no insuperable difficulties in the way of its being carried out, such details should always have due consideration.

The practice of levelling for mining purposes does not much differ from that of taking levels for other works, further than this, that when it has to be done underground it is accomplished under very considerable disadvantages; and great care is necessary in throwing light into the telescope, so as to be able to see the cross-hairs, and to get a steady light, yet only what will be just sufficient for the purpose; and also to hold the light as much as possible in the alignment of the axis of the telescope, or else the cross-hairs will appear very unsteady, and the vision of them will frequently be flickering and uncertain to the eye. A good steady light will also be necessary for the assistant who holds the staff, so that the figures may be seen with ease and certainty. There is often great difficulty in securing anything like a good B. M. (bench mark) underground, in consequence of the movements of the ground, but the inconvenience arising from this is generally much less than in important surface works.

The business of surveying as applied to other operations than that of mining etc., has been so frequently and ably treated of before by others, that those who seek specific information on such branches may safely be referred to those works for the information they are in need of.

In the succeeding chapter will be found a more full explanation and illustration of the application of Triangulation to a considerable surface survey; and as the value of a Map depends upon its accuracy, and in mountainous districts the operation of chaining is rendered much more difficult and less accurate, by reason of the uneven nature of the ground, it seems to be most desirable that the principal groundwork of

every important survey should be formed by the application of those principles, therein explained and applied.

CHAPTER XI.

ON TRIANGULATION, ETC.

We will now proceed to give an example of the application of Trigonometry to a surface survey, shewing how a property of a certain figure may be divided for Triangulation, and computed through the whole series of lines. This example is not from any actual survey, but is given as an instance of the application of the principles enunciated in the foregoing chapters of this work.

The superiority of these principles—if accuracy is considered to be at all desirable—cannot be for a moment doubted, and the slightly increased amount of labour, if indeed there is any increase of labour, is abundantly compensated by the increased value of a thoroughly reliable Plan on which to lay out the workings, and guide them for the proper exhaustion of the minerals throughout the property to the boundaries, and the reservation of proper barriers to ensure against flooding and other mischiefs.

In the example here given, Plate 3, line *a* of No. 1 triangle is that supposed to be measured very accurately, for computing the whole of the lines of the Triangulation; and this line, called the "Base of computation," is here represented to be 2564 links. The process of computation is carried on, first from line *a* to line *j*, embracing *b*, *c*, *d*, *e*, *f*, *g*, *h*, and *i*; then from line *a* to line *k*, embracing *q*, *p*, *o*, *e*, *n*, *m*, *l*, *i*, and *k*; thus the lines *e*, and *i*, come into each of the series of computations, and will each form a check upon both series, one of the other; and if these lines come the same in each (as they ought to do) it is so far satisfactory to the operator. Then, selecting one of the lines—say, for instance, *k*— as a "Base of verifica-

tion," it should be measured on the ground with the chain, and checked with extreme care, to ascertain if it corresponds with the computed length: if it comes within ·5, or 1 link of the computed length, it may be looked upon as being very satisfactory.

Care should be taken to avoid as much as possible the adoption of angles greater than 90°, as they are neither so convenient, nor so easy of computation: it should be also carefully observed that the sum of the angles of each triangle equals 180°; and that the sum of all the central angles of each closed polygon (at *A* and *B* for instance) equals 360°.

An instrument, theodolite or circumferentor, of the best make, and in perfect adjustment, will be requisite for taking these angles; and having carefully levelled the instrument the surveyor should proceed to take the angles, and enter them in a sketch, made for the purpose, of the several triangles in their proper relative positions. In some states of the weather the refraction of light will have a very perceptible influence upon the operation of taking the angles, and this should be carefully noted, and errors as far as is possible guarded against: for instance, if there are passing clouds, and *one* observation (or one of the readings for determining the angle) be taken while the sun is obscured and the other *after* the cloud has passed, there will often be a perceptible difference (dependent upon the state of the atmosphere) in the measure of that angle, compared with the measurement when both observations have been taken either in sunshine or in shade. Therefore it is well and proper to bear this in mind, and to check the readings of each angle by a double operation, before the instrument is removed from the position or station.

At *A*, and *B*, the whole of the angles at the center of each polygon can be taken and checked at one fixing of the instrument.

The trigonometrical Rule by which these computations are made, is that given before:—viz. *Add the logarithm of the given side to the sine of the angle opposite to the side required, and from the sum subtract the sine of the angle opposite to the given side; the remainder will be the logarithm*

of the side required: or, instead of subtracting the sine of the angle opposite to the given side add its co-secant, and the sum will be the logarithm of the side required.

Then, the given side a being 2564 links, we have the following computations.

To find side b.

As sine of $\angle 82^\circ 33'$	$= 9{\cdot}9963183$
: log. side $a = 2564$	$= 3{\cdot}4089180$
:: sine of $\angle 40^\circ 30'$	$= 9{\cdot}8125444$
	$13{\cdot}2214624$
: log. side $b = 1679{\cdot}4$	$= 3{\cdot}2251441$

To find side c.

As sine of $\angle 46^\circ 21'$	$= 9{\cdot}8594804$
: log. side $b = 1679{\cdot}4$	$= 3{\cdot}2251441$
:: sine of $\angle 75^\circ 34'$	$= 9{\cdot}9860720$
	$13{\cdot}2112161$
: log. side $c = 2247{\cdot}7$	$= 3{\cdot}3517357$

To find side d.

As sine of $\angle 75^\circ 34'$	$= 9{\cdot}9860720$
: log. side $c = 2247{\cdot}7$	$= 3{\cdot}3517357$
:: sine of $\angle 58^\circ 5'$	$= 9{\cdot}9288145$
	$13{\cdot}2805502$
: log. side $d = 1970{\cdot}1$	$= 3{\cdot}2944782$

To find side e.

As sine of $\angle 38^\circ 21'$	$= 9{\cdot}7927163$
: log. side $d = 1970{\cdot}1$	$= 3{\cdot}2944782$
:: sine of $\angle 76^\circ 29'$	$= 9{\cdot}9878012$
	$13{\cdot}2822794$
: log. side $e = 3087{\cdot}2$	$= 3{\cdot}4895631$

To find side *f*.

As sine of ∠ 38° 21′	= 9·7927163
: log. side *d* = 1970·1	= 3·2944782
:: sine of ∠ 65° 10′	= 9·9578626
	13·2523408
: log. side *f* = 2881·5	= 3·4596245

To find side *g*.

As sine of ∠ 51° 41′	= 9·8946461
: log. side *f* = 2881·5	= 3·4596245
:: sine of ∠ 68° 35′	= 9·9689262
	13·4285507
: log. side *g* = 3419·	= 3·5339046

To find side *h*.

As sine of ∠ 68° 35′	= 9·9689262
: log. side *g* = 3419·	= 3·5339046
:: sine of ∠ 59° 44′	= 9·9363574
	13·4702620
: log. side *h* = 3172·	= 3·5013358

To find side *i*.

As sine of ∠ 48° 7′	= 9·8718681
: log. side *h* = 3172·	= 3·5013358
:: sine of ∠ 75° 9′	= 9·9852468
	13·4865826
: log. side *i* = 4118·2	= 3·6147145

To find side *j*.

As sine of ∠ 75° 9′	= 9·9852468
: log. side $i = 4118{\cdot}2$	= 3·6147145
:: sine of ∠ 56° 44′	= 9·9222721
	13·5369866
: log. side $j = 3562{\cdot}4$	= 3·5517398

To find side *q*.

As sine of ∠ 40° 30′	= 9·8125444
: log. side $b = 1679{\cdot}4$	= 3·2251441
:: sine of ∠ 56° 57′	= 9·9233450
	13·1484891
: log. side $q = 2167{\cdot}4$	= 3·3359447

To find side *p*.

As sine of ∠ 46° 6′	= 9·8576648
: log. side $q = 2167{\cdot}4$	= 3·3359447
:: sine of ∠ 75° 20′	= 9·9856129
	13·3215576
: log. side $p = 2910$	= 3·4638928

To find side *o*.

As sine of ∠ 75° 20′	= 9·9856129
: log. side $p = 2910$	= 3·4638928
:: sine of ∠ 58° 34′	= 9·9310750
	13·3949678
: log. side $o = 2566{\cdot}6$	= 3·4093549

To find side *n*.

As sine of ∠ 68° 8′	= 9·9675728
: log. side *e* = 3087·2	= 3·4895723
:: sine of ∠ 61° 23′	= 9·9434172
	13·4329895
: log. side *n* = 2920·2	= 3·4654167

To find side *m*.

As sine of ∠ 46° 7′	= 9·8577863
: log. side *n* = 2920·2	= 3·4654167
:: sine of ∠ 86° 15′	= 9·9990691
	13·4644858
: log. side *m* = 4042·9	= 3·6066995

To find side *l*.

As sine of ∠ 86° 15′	= 9·9990691
: log. side *m* = 4042·9	= 3·6066995
:: sine of ∠ 47° 38′	= 9·8685548
	13·4752543
: log. side *l* = 2993·5	= 3·4761852

To find side *k*.

As sine of ∠ 44° 46′	= 9·8477091
: log. side *l* = 2993·5	= 3·4761852
:: sine of ∠ 59° 36′	= 9·9357660
	13·4119512
: log. side *k* = 3666·4	= 3·5642421

To find side *i* by this series.

As sine of ∠ 44° 46′	= 9·8477091
: log. side k = 2993·5	= 3·4761852
:: sine of ∠ 75° 38′	= 9·9862017
	13·4623869
: log. side i = 4118	= 3·6146778

being ·2 difference between the first and second computation to find the side *i*; this is satisfactory.

If it should be required to ascertain what the respective outer angles of No. 9 triangle *should* be, so as to close the polygon composed of triangles 3, 4, 5, 9, 8, and 7, and the length of the line *k* between the two abutting triangles 5 and 8, we must then apply the following rule. Having two sides and the angle between them; to find the two other angles and the third side. Rule:—

If the angle included be oblique, add the logarithm of the difference of the given sides to the tangent of half the sum of the unknown angles, and from the sum subtract the logarithm of the sum of the given sides, or add its complement; the remainder or sum will be the tangent of half their difference.

Applying this rule, we have the two given sides = 4118·3 and 2993·5; and 2993·5 + 4118·3 = 7111·8 = the sum; and 2993·5 − 4118·3 = 1124·8 = their difference.

Also sum of unknown angles = 180° − 59° 36′ = 120° 24′; and 120° 24′ ÷ 2 = 60° 12′ = the half sum.

Then, Log. diff. 1124·8 =	3·0510367
Log. Tangent 60° 12′ =	10·2420687
	13·2931054
Sub. Log. sum = 7111·8 =	3·8519795
Log. Tangent 15° 26′ =	9·4411259.

Therefore the angles will be, 1st, 60° 12′ + 15° 26′ = 75°.

38′; and, 2nd, 60° 12′ − 15° 26′ = 44 46′; and the side *k* will then be computed by the rule given for the other sides.

The diagram of lines of which the foregoing is the computation forms a complete foundation or groundwork on which to proceed to fill up all the details of the estate to which it may refer. Good firm stakes, or some other reliable marks, should clearly distinguish the positions of all the stations, and then poles can be set up for measuring those lines or combining short diverging lines with them, as circumstances may require, in order to take in all the requisite boundaries, fences, brooks, houses, or other matter; and, if found necessary, portions only of some of the lines can be measured from one or other of the ends if complete information for plotting can thus be obtained, and so that the correct particulars are entered in the field-book. In this way awkward and rugged parts of lines, which, with another system, would have to be measured entire to complete the information requisite to plot the work, may be occasionally omitted, without in any manner impairing the accuracy of the Plan: and no error could be continued beyond the triangle in which it arises, and could scarcely arise (with ordinary diligence) without being detected.

The selection of the secondary lines for filling up, and their directions, depend entirely upon the judgment of the surveyor being brought to bear upon the peculiarities and requirements of each particular case, and for which scarcely any two instances will be found requiring the same treatment and division: therefore the better and quicker the judgment exercised in their selection, the more accurate and less laboriously attained will be the result.

The application of these principles is often of great importance also in foreign countries, in the procuring information respecting, and the development of, their mineral resources, and for acquiring a fairly accurate idea of the stratification, correct average line of dip, direction of principal faults, and other information so necessary to guard against error in taking up concessions, and developing the mineral riches of comparatively unknown countries. For while in England,

generally speaking, the boundaries of our deposits of coal &c., are pretty well known, in new and undeveloped countries such information has to be gathered.

The object then to be attained, is to procure the data from which to lay down a connected relative view of different and distant points; for while, often, a consideration of the peculiarities and evidences of any one small district, taken alone, will be found to contribute but little to a general knowledge of the surrounding country, and will often indeed be found, when read *by themselves*, to be completely contradictory of the *general* evidences found elsewhere, they will—when read in the light of the *combined* information to be gathered throughout the area under enquiry, and combined also with a consideration of the relative positions of those places, the character, pitch, and strike of the strata as a whole, line of faults, evidences afforded by denudations of the strata, either in large valleys or deep ravines, together with the fossils to be found in the area being examined—enable the intelligent enquirer to form a tolerably accurate idea as to the capabilities of the district under consideration to yield any of those minerals which have been so much a source of wealth and greatness to the countries possessed of them, and of which he may be in search.

In getting up a Plan of such a district as is here supposed, the application of the science of Trigonometry is generally the only practicable mode of doing so; and at the same time it provides a groundwork for the filling in any details in reference to any particular part deemed desirable, either at that or at any subsequent period.

The rules here given are those most generally employed, in such computations as are here indicated; but there are others which have to be adopted occasionally, under peculiar circumstances, instead of these, and which will be found given in treatises on Trigonometry. Also, when the lines under consideration are so large that the curvature of the earth has an appreciable effect upon them, Spherical Trigonometry must be employed, intead of Plane Trigonometry.

While the peculiar evidences of *large* districts must thus

be sought, if coal, stratified deposits of iron ore, and some other minerals, are being sought after, there are some deposits, and amongst them the *casual* deposits of iron ore, which do not become influenced so much by the general features of large districts, but their evidences are rather to be sought for in a more confined area. Occasionally an unpromising looking outcrop will be found to lead to a very valuable deposit of rich iron ore beneath; while a very considerable area on the slope of a hill may be reddened by the descent of the rain-waters, whose solvent action upon a *slightly* disseminated sprinkling of ore in the strata of the brow of the hill will have brought about a gradual change of colour, and which may lead some persons to fancy that a very valuable deposit of rich iron ore is near at hand. Springs of water holding iron in solution will often give a colour to the surface over which it flows, and which, if taken as indicative of what may be found in the immediate substrata, will be eminently calculated to mislead. We have taken crusts of brown oxyd of iron, upwards of two inches thick, from the face of a bank over which has flowed a stream of water that has contained iron in solution, and which iron was, most probably, originally taken up from decomposing iron pyrites in the strata through which the water had flowed, as that mineral was known to exist near in considerable quantities: and this crust could not have been, from the peculiar nature of the position, many years in forming. Sometimes the material of which houses are constructed (if stone has been employed) will give a clue to deposits in the neighbourhood, by the occasional appearance of a stray lump of ore amongst the building material, especially where, as in primitive countries, the people gather and use the loose material they find lying in their fields, rather than raise it in a quarry. Sometimes the outcrop of a deposit of mineral may be discovered in going along over a country after a very heavy rain, such as falls in southern countries, which has washed the surface of all the loose débris, and left the native strata quite clean and bare—it being, in fact, nature's performance of that operation of "scouring," which is still carried out artificially, occasionally, in many countries, to discover the outcrops of metallic deposits

and other minerals of value. Deep ravines, and the beds of brooks when dry, will also often yield unerring evidence of the presence, or otherwise, of certain metalliferous deposits; as will also the examination of the localities of disruptions, and dislocations of the strata, and also by following the line of contact of rocks which lie unconformably one upon another.

In examining any new country and seeking therein evidence of mineral deposits, these, and many other such-like circumstances, will be made use of by the experienced observer, in endeavouring to ascertain the prospects of success (setting aside for the moment the questions of labour, transit, climate, &c.) of any mining enterprise with which he may be connected and have an interest in establishing upon a profitable basis, and with which the business of surveying must become more or less intimately connected.

CHAPTER XII.

ON THE VALUATION OF MINING PROPERTIES.

As the raising of minerals, and converting them to profitable use, is one of the businesses of life, and not one of its luxuries, except perhaps to an exceptional few, whose luxuries in it consist less of profits derived from the legitimate working of mineral properties than of profits made out of the unwary, by letting them into worthless things at a very high figure under specious colours, and from which no profit perhaps ever has been, nor is ever likely to be made—those who place their capital in mining enterprises do so in the legitimate hope and expectation of reaping a proper return for its employment. There are things in which money is embarked or spent, and in which, if the expectation of profit is not very great, the liability to risk and accident and loss is at least very small; but mining is much too serious a matter to embark in simply

from a love of the adventurous, and therefore those who take, or buy mining property, to work it legitimately, and who place their capital in it, have a right to look for an ample return upon their investment.

Whether the question of the manner of deposition of our splendid deposits of minerals, and especially of coal, will ever be decided to the satisfaction of every enquiring mind is extremely doubtful; nor does the value of the possession depend upon the decision of any such abstract question. The fact of its value is unquestioned, and those countries which possess it in greatest abundance, and also the means for its development, are found, taken at large, to be the most advanced in wealth, and prosperity, and high civilization. It is true that we do not always find an advanced state of civilization amongst the workers in our collieries and mines. The miners are a race of men whom some may judge to be somewhat repulsive and uncivilized, simply from the black appearance the coal-dust, or the red skin that the hematite mine and other accompaniments of mining, give them—adhering to the strong and able frame in consequence of the streams of perspiration that pour out through all the pores of the whole body in the laborious work of mining. Such a judgment would however be very inaccurate, and would, as a rule, convey a most uncharitable idea of the miner. That he is impulsive, and often careless of his own life and that of his fellows, we have frequent evidence of in the melancholy testimony borne to the causes which lead, or have led, to the too frequent recurrence of the sad loss of life in collieries and other mines: but it is by no means just to lay all the blame upon the miner when an accident happens, and come at once to the conclusion that it has necessarily originated in his carelessness or foolishness. Other causes, over which he has no control, and for which he is in no way responsible, but by which he is placed in imminent peril, are often operating to accelerate and bring about destruction to life and property; and some of these it is within the range of possibility to prevent, and some are events which it is to be feared no ordinary forethought could have fully averted.

Placed thus in the midst of daily danger, and labouring in the monotonous toil of the mine, it is not to be wondered at if the miner is sometimes found to be heedless and callous; but such is not the case universally, for there will be found amongst miners men of noble impulse and generosity, and some who take a greater notice of the mineral they raise than that merely involved in the raising and filling it into the trams. There are those who are fond of collecting specimens of some of those beautiful fern fronds, and other fossiliferous impressions so frequently to be found in some strata in and about the minerals, which evince an enquiring and teachable mind; and they have also (some of them) formed some opinions of their own respecting the manner of deposition of the veins, &c. they labour in. These, and similar evidences, tend to shew that if the miner is not so highly civilized as he should be, the fault is frequently less his own than of the concurrent surrounding circumstances of his position, and perhaps also of those who are by their opportunities and education placed by Providence in a position that enables them, if they would, to assist in the work of improving his mental condition.

The many works of practical benevolence and love which the miner is found assisting in, and in some instances to initiate, shew that he has a heart as warm and generous as others more generally recognized; and if his heart is warmly appealed to in the matters of religion, purity, prudence, and charity, it will not infrequently be as warmly responded to. Therefore we would submit that if the miner is not in all things what it would be desirable to have him to be, he is not at the same time what he is capable of being made; and those with superior endowments and positions may at all times do much towards raising him to that standard, to which it is very frequently said the "safe" miner should conform.

It is occasionally said that as men grow in intelligence they dislike the mine, and that the possession of one is inconsistent with the performance of labour in the other: but to apply such judgment as a principle, simply because isolated instances may have arisen in confirmation of such an idea, would be manifestly unfair. And can it be for a moment supposed that

an All-wise Providence has so placed these great blessings and comforts—as our mineral riches are admitted to be—in a position from which they can be brought only by the prevalence of the most ignorant and degraded class of labour? and that its removal can only be properly secured by the existence of that state of being which is utterly repugnant to, and inconsistent with, those higher attainments of our nature to which, in other works of the same All-wise Providence, we are constantly called? Surely this cannot be, and we should be guilty of great indiscretion if we were to suppose that it is not possible to secure these great blessings, and accessions to our comforts, at a less cost than the perpetual existence of a maxima of vice and irreligion.

Doubtless there will always be enough, and too much, of these in our world while the present course of things continues; but at the same time there is no reason why a general advancement in intelligence should not be striven after, for the benefit of each generation as it arises; for the more men grow in intelligence, the less will they become a prey to designing and professional agitators, and the more will the connection between labour and capital be improved. Men will see that it is to the advantage of themselves and of their employers that their labour should be performed with the greatest care, and to secure the best results; and that if capital cannot be equally well remunerated in mining enterprises with labour, it will not continue to be so employed, but will seek new fields of enterprise for its employment.

Thus we submit that this question of the state of intelligence of the labour employed has no slight influence upon the value of mineral properties; for it is well known that where there are steady, and contentedly settled men, with which to carry on mining operations, there is a great advantage in comparison of that where men are unsettled and roving, and consequently less easily controlled and disciplined, and less amenable to reason.

When mining property comes into the market for sale, its real value is a matter not always easily determined. Foreign enterprises are occasionally offered to speculators and capi-

talists, and often consist of concessions held from foreign governments; these concessions we have known in some instances to have been both numerous, and widely dispersed; and instead of a large number of concessions being a source of strength to foreign mining it is much more frequently its weakness, for it is mostly made compulsory upon the holders that a certain amount of work shall be performed at each concession yearly, or the right to hold is endangered, and may be forfeited: and thus are many companies led into attempting too great a series of operations at the first, which, in new countries, and with untrained labour, often involve so heavy an expenditure that not a few collapse under the burden; while if attention had been confined to some two of the most promising and conveniently situate until they were fairly developed and made productive, much less preliminary cost would have been incurred, much capital saved, and the other concessions could then be gradually attended to and developed in a manner most conducive to the permanent success of the entire undertaking; and much fewer companies would then be brought to ruin.

If mining properties in foreign countries are much scattered there is much attendant expense in opening up and maintaining roads, or channels of communication and transit for the minerals; for, most likely, there will often be but few native roads that can be made available in any but detached parts; and where this is so, and native labour is used, though it may be available in sufficient abundance it may still be unacquainted with the class of work to be performed, and thus, while the labour is nominally cheap, it may in reality be very much dearer than a higher paid class of labour in other countries; and, as a consequence, the results of carrying on work in foreign countries seldom realize the expectations raised on the basis of cheap labour. These things will be elements in the calculation, in seeking to form an estimate of the value of foreign mining or other properties for the profitable employment of capital, and great care will be needful in making such valuations: for while doubtless there are many rich properties awaiting the application of capital, to yield a

most remunerative return, there are others from which little can be properly expected.

The same may be said of properties in our own country, coming into the market; for while there are some properties which are thoroughly legitimate and sound, and offer a good security for the investment of capital, there are some occasionally offered which, if they ever have paid a return on capital, have now pretty well ceased to have any prospects of doing so any more; in fact the owners may wish to sell them in consequence of their being of no value to hold. It is, however, certain, that such transactions are not confined to mining, for in many other ways we find the same principle acted on; and if a person with his eyes open buys a worthless thing he has himself principally to blame, if he did not take due precaution to ascertain by the best available means whether it was of the value represented to him.

While coal veins, and other stratified deposits, admit of the application of general estimates of acreage yields of unworked ground, and with the cost at which the mineral can be produced and sold supply general information for computation, casual deposits cannot often have such principles wholly applied to them; for they are in some cases very variable in their features, and so irregular in the quantities yielded by similar areas, that it is but the merest guess-work to assign any quantity as a probable yield.

Doubtless, in some of the hematite iron ore deposits, the further we follow them down the greater we may find their bulk to be, and probably their richness also; but the manner of working imposed by the nature of the deposit, and the precautions necessary to ensure against flooding, will be certain, in these deposits, to affect their value in the course of time; and what will be a good system for one deposit in one country, may in another country, and with precisely similar surroundings of stratification etc., be quite inapplicable: for if we suppose the case of a hematite deposit, very rich in itself, but situate where the Lancashire practice of working downward would be inapplicable in consequence of semi-tropical or violent rains prevailing at times—an instance of which we

have known, and have been connected with, and directing, in Portugal—some other mode of working must be adopted, so as to keep the surface intact; for were it in such cases to be broken by the subsidence brought about by working downward from the surface, the immense rains which in a very short time occasionally convert the surface of the ground into one wide sheet of water, would then quickly bring about the destruction of the mine, not alone by flooding the lower workings, but by carrying down into the workings such great quantities of the small refuse and sand overlying the district, that it would become a matter of *reopening* the mine to get at the former workings again. Of course, when the deposit being worked is situate on a hill, where good natural drainage can be arranged, such inconvenience from this cause does not arise, or is easily provided for. Under the circumstances we have mentioned, the only feasible plan seems to be that of going sufficiently deep to commence operations, and then to work upwards towards the surface, filling in the excavated space with rough material sent down into the mine; and always leaving a sufficient thickness untouched, to ensure an unbroken crust to keep out the flood waters.

These and similar circumstances will all go to tell upon the value of mineral undertakings, of the different kinds to which they respectively apply; so that the correct estimation of the value of a mineral property is often a complicated question, involving several matters which require special consideration,—and can only be duly arrived at by the examination of each property, not only upon general principles, but also with regard to the special features of the nature of the deposit; country in which situate; facilities for transit; abundance of labour; cost of establishing, developing, and conducting the operations; market for produce at a profitable price; and other matters of a relative nature which will suggest themselves to a competent valuer, who may have to entertain the question of the valuation of such properties.

We have found that veins of coal of thicknesses varying from 2 ft. 6 in. to 5 ft. have yielded the following different numbers of tons per foot of thickness per acre: viz. 1100,

1090, 920, 1300, and 1000 tons, statute weight—taking the exhausted ground as measurement; and these numbers will probably be found to be about what it is possible to realize in practice generally, though some of the quantities may be considered to be very low; but several causes go to influence the total quantity to be obtained from any particular vein,—not only the varying specific gravity of coals, but the strength or firmness of the veins, the nature of the superincumbent strata, mode of working, and also, in no slight degree, the system of management.

As being amongst the prominent questions that would have to be entertained and considered, in valuing any colliery that may be offered for sale, are some or most of the following.

1st. The area and compactness of the property, provisions and clauses of lease:—are they reasonably favourable to the Lessee, and what is the royalty reserved, dead rent, average clause, power of assignment, of removal of plant at expiration or other determination of lease etc. etc. etc.?

2nd. What is its relative position in regard to other properties being worked?

3rd. What is its relative position for market purposes for produce; to what markets can its produce be sent; and has it good railway, or other sufficient service?

4th. What are the general known features of the strata in respect of Faults, Springs, Swamps, old Workings (if any) etc., and how situate?

5th. Are any of the veins of mineral—coal or iron—in the same property, that can be worked to the detriment of that whose value is sought, demised to *other* lessees?

6th. Quantity and quality of coal unworked, and its accessibility from Pit, or Level; and will the coal last out the term of the lease, and cover the dead rent to the end?

7th. What is the price at which the coal can be delivered at "bank" in ordinary times, all working charges paid; and what are the royalties, wayleaves, and other charges binding on same?

8th. What profit per ton will this cost admit of in ordinary

states of the market; and what quantity is the colliery capable of producing per diem?

9th. Does the vein of coal give off much explosive gas; is the "top" good; and does it require much timber?

10th. Is there a good sound system of ventilation in operation, with sufficient air, and shafts etc. for same?

11th. How many years will the royalty last to *give profit?*

12th. Is the colliery well provided with machinery, plant etc., requisite for raising and despatching the coal; and are good plans of all the *old* workings obtainable?

13th. Are the boundaries infringed upon at any point; and do any liabilities attach to the property other than those usual and ordinary?

14th. What facilities of access to other veins, or royalties, does the possession of the present field and vein, or veins, give; and what is their money value to an incoming tenant or purchaser?

15th. What convenience, and area of spoil ground, is there for tipping rubbish etc. from the colliery?

16th. Do the minerals underlie any town, village, reservoir or accumulation of water, or any other notable place or thing, which would render the working of the coal liable to, or for, any damages, or actions at law, more than ordinary?

17th. Then:—Under all the circumstances what net profit per annum will the colliery produce, and for how many years, at a certain estimated output per annum?

An amount should be allowed off the profit for an insurance or compensation fund (in case of accident), and the income should provide a sufficient amount, after that deduction, to give a good interest on capital laid out, as well as to provide for the recouping of capital, before the colliery becomes exhausted.

The annual value etc. having been thus obtained, and any other matter observed necessary being duly considered, the present value will have to be worked out by the ordinary rules for money computations of this nature, and the result shewn in a proper tabulated form.

We will conclude by giving a few examples of the valuation of the freehold interest in mineral property; and also of a reserved rent-charge: and these will also, to a considerable extent, illustrate the principle to be followed in estimating the value of a terminable income, annuity, or other interest, arising from a colliery, or other property generally.

Reserved Rent Charge.

EXAMPLE I.

A person is entitled to an annual rent-charge of £140 upon a property, for a remaining term of 17 years; and it is required to clear the property of this charge:—the proprietor is willing to accept an immediate equivalent, and to allow interest at the rate of 3½ per cent. per annum. To what amount does he become immediately entitled?

To solve this, we require to find the value of an immediate annuity of £1 per annum, at that rate of interest; this is found by Smart's Tables to be £12·65132, and if we multiply this amount into the rent-charge we have 12·65132 × 140 = £1771·184 for the value of the immediate equivalent.

We may also work out the same result by Thoman's Tables —using logarithms.

Thus, taking same term and rate,

Comp. log. a^n = 1·1021358
Log. 140 = 2·1461280
Log. 1771·184 = 3·2482638 = the same amount

as that above given, or £1771. 3*s.* 8*d.*

A little consideration will suffice to shew that these formulæ will apply to other questions of money interests. For instance, the computations will apply if the question is:—What is the present value of an immediate annuity for 17 years, in any property, if it is required to make an interest of 3½ per cent. per annum? for a person who is liable to pay an annual rent-charge, if he rids himself of the liability

by paying down a certain sum, does, in effect, purchase to himself an annuity equal in amount to the rent-charge he was before paying. Other applications of the same principle will readily suggest themselves to the enquiring mind.

If the result of the computation be first obtained, while treating the annuity as being one single yearly payment, though it be, in fact, really divided into quarterly payments, or half-yearly payments, we may find the increased value under the latter circumstances very simply by Thoman's Tables.

EXAMPLE 2.

Question. A property yields a yearly rent of £860: what is it worth as a perpetuity when the rent is paid yearly; also when paid half-yearly; and when paid quarterly? Interest at the rate of 7½ per cent. per annum.

1st. When paid yearly.

Comp. log. a^n	= 1·1249387
Log. 860	= 2·9344985
Log. £11,466·66	= 4·0594372 = the Value.

2ndly. When paid half-yearly.

Log. of 1st value = 11,466·66	= 4·0594372
+ Q_2 equivalent	= 0·0080676
Log. £11,681·664	= 4·0675048 = the Value.

To prove the second computation, it is sufficient to shew that the 1st value is increased by an amount = to the value of a perpetuity of ½ year's interest on ½ the annuity, at 7½ per cent. as before; viz. £16·125.

Then	Comp. log. a^n	= 1·1249387
	Log. 16·125	= 1·2074997
	Log. £215	= 2·3324384 = the Value; being

the amount by which the 1st value is increased in the 2nd computation: viz. 11,466·66 + 215 = £11,681·66.

3rdly. What is the value when the annuity is paid quarterly, the same rate of interest being allowed?

Log. of 1st value = 4·0594372
+ Q_4 equivalent = ·0121951
Log. £11,793·216 = 4·0716323 = the Value; being the value when the annuity is paid quarterly.

To prove the 3rd computation the proceeding is more intricate; but it may be shewn that the 1st value, £11,466·66, is increased by the perpetuity of the following amounts, viz.:

1st. ¼ interest on 1 quarter of the annuity
= £12·09375 = in value £161·249

2nd. ½ interest on 1 quarter of the annuity
= £8·0625 = in value £107·500

3rd. ¼ interest on 1 quarter of the annuity
= 4·03125 = in value £53·735

and by the compound interest on the quarter's interest on the quarter's interest on the quarters of the annuity = £4·072

Then, Amount of Value when paid yearly = £11,466·660
Total Value as per 3rd computation = £11,793·216

Example 3.

The Valuation of Freehold Property with Minerals immediately productive.

What is the Value of an Estate of 400 acres, the minerals of which are just beginning to be worked, the fair estimated output per annum of coal being 120,000 tons (=400 tons per day for 300 days) at a royalty of, say, 10*d.* per ton; and the estimated duration of the same being 80 years? This would produce an annuity of £5000 for 80 years. The surface is being let, as farm land and rough pasturage, at £250 a year. Interest on capital to be allowed at 8 per cent. per annum; and capital to be recouped at 3½ per cent. per annum; interest on purchase money of surface being at the rate of 4 per cent. per annum, and in perpetuity.

Then, the surface will thus be worth 25 years' purchase, or a total derived thus: $250 \times 25 = 6{,}250$ £6,250. 0

The annuity of £5000 for 80 years, at 8 per cent. per annum, and to recoup at 3½ per cent. per annum, is worth in present money the sum total of £60,690. 15. 0

Total Value £66,940. 15. 0

The formula by which the second value is derived is that given by Thoman in his treatise, and is as follows.:

$$V = \frac{a}{t' + \frac{a^n}{r^n}},$$

in which V is the value; a the annuity; t' the rate per cent. to be made on purchase money; a^n the annuity which £1 will purchase for that term, and at rate of recoupment; and r^n the amount of £1 at the end of the term, and at the same rate.

Then applying this formula to the foregoing case we have the following values:

	Logarithms.	Nat. Numbers.
Log. a^n	= 8·5726961	
Log. r^n	= 1·1952280	
$a^n r^{-n}$	= 7·3774681	= . . . . . 0·0023845
		$+t'$ = 0·08
Log. $t' + \frac{a^n}{r^n}$	= 8·9158475	= . . . . . = 0·0823845
Log. a = 5,000	= 3·6989700	
Log. of V	= 4·7831225	= £60,690·75, or £60,690. 15*s.* 0*d.*

purchase money of the annuity.

We may check the above computation by working out the question by the rules given by Hardy as in Inwood's Tables at page 179.

1st. by Smart's Tab. 3, $(a)^{80}$ = 419·30678

5000

Product = 2096533·9

2nd. $(a)^{80} = 419{\cdot}30678$
$\times t' = \quad {\cdot}08$
Product $= 33{\cdot}5445424$
$+ \ 1$
Sum $= 34{\cdot}5445424$

3rd.
$\frac{2096533{\cdot}9}{34{\cdot}5445424} = £60{,}690{\cdot}75 = V$ the value as before.

Then, the interest on this value at 8 per cent. per annum will amount to.....4th. ..... 4855·26
giving for amortisement at $3\frac{1}{2}\%$ $(= s)$ 144·74
Total annuity £5,000·00

And to prove the sufficiency of the amortisement we have, by following the same rule,—

5th. Smart's Tab. I.
$r^{80} = 15{\cdot}67573$
$- \ 1{\cdot}$
Diff. $= 14{\cdot}67573$

6th. Log. $14{\cdot}67573 = 1{\cdot}1666016$
Log. $s = 144{\cdot}74 = 2{\cdot}1605886$
C' Log. rate $\cdot035 = 11{\cdot}4559320$
Log. $60{,}690{\cdot}71 = 4{\cdot}7831222$ = the capital recouped

at the end of the term of 80 years.

The foregoing estimate of quantity of coal would require a total thickness of 25 ft. or thereabouts, over the whole area of the property.

Example 4.

Freehold Property with Minerals, being partly immediately productive, and partly deferred, and comprising the following lots, viz.:

Lot A. Surface rental of 350 acres, equal to a perpetuity, *and let at* a yearly sum of £145, net.

Lot B. The upper veins of Coal producing 200 tons per day, or 60,000 tons per annum, let at 9*d.* per ton royalty; and calculated to yield this quantity for a further term of 18 years.

Lot C. The lower veins of Coal which from their favourable position are likely to be productive in 15 years, and then to yield an output of 300 tons per day, or 90,000 tons per annum, for a term of 65 years, and to let at 1*s.* per ton royalty.

Lot D. Freehold groundrents from houses built, amounting to £200 per annum, for 75 years unexpired term of the leases.

Lot E. Royalties or wayleaves yielding, at 1*d.* per ton, £80 per annum; and estimated so to continue for a further term of 12 years.

Then supposing interest to be allowed on the several lots as follows, viz:—

Lot A. 4 per cent. per annum as a perpetuity,
Lot B. 8 per cent. per an. on Capital, and to recoup at 3½ %,
Lot C. 10 „ „ „ „ „ „ 3 %,
Lot D. 7 „ „ „ „ „ „ 3½ %,
Lot E. 10 „ „ „ „ „ „ 3 %,

the following will be the values: the same notation and rules being applied as in Example 3.

The Value of Lot A will then be $145 \times 25 = £3{,}625.\ 0s.\ 0d.$

Lot B. Annuity for 18 years £2,250.
Interest on Capital 8 per cent.
Interest to recoup 3½ per cent.

	Logarithms.	Nat. Numbers.
Log. a^{18}	$= 8{\cdot}8797657$	
Log. r^{18}	$= 0{\cdot}2689262$	
Log. $a^{18}r^{-18}$	$= 8{\cdot}6108395 =$	$\ldots\ldots\ldots\ldots 0{\cdot}0408168$
		$+ t' = 0{\cdot}08$
Log. $t' + \frac{a^{18}}{r^{18}}$	$= 9{\cdot}0821269 =$	$\ldots\ldots\ldots\ldots 0{\cdot}1208168$
Log. 2,250	$= 3{\cdot}3521825$	
Log. of Value	$= 4{\cdot}2700556 =$	£18.623·26.

Checking by Hardy's rules as in Ex. 3,

1st. Smart's Tab. 3, $(a)^{18}$ = 24·49969
2,250
Product = 55124·3025

2nd. $(a)^{18}$ = 24·49969
$\times t'$ = ·08
Prod. = 1·9599752
+ 1·
Sum = 2·9599752

3rd. $\frac{55124·3025}{2·9599752}$ = 18,623·23 = Value.

4th. Int. on Value at 8 % = 1489·8584
Amort. to recoup 3½ % = 760·1416
The annuity = £2,250·0000

5th. Smart's Tab. 1. r^{18} = 1·85749
− 1·
Diff. = 0·85749

Then 6th. Log. ·85749 = 9·9332291
Log. 760·1416 = 2·8808944
C^t Log. ·035 = 11·4559320
Log. 18,623·25 = 4·2700555 = the capital reproduced.

Lot C. The annuity for 65 years equals £4,500.
Interest on Capital = 10 per cent.
Interest to recoup = 3 per cent.
Period the annuity is deferred = 15 years.

	Logarithms.	
	$=8\cdot5458736$	
	$=0\cdot8344196$	Nat. Numbers.
	$7\cdot7114540=$	$\ldots\ldots\ldots\ldots\ldots\ldots0\cdot0051458$
		$+t'=0\cdot10$
$t'+\frac{a^{65}}{r^{65}}=$	$9\cdot0217900=$	$\ldots\ldots\ldots$Sum$=\cdot1051458$
	$4,500=3\cdot6532125$	

Log. of $V=4\cdot6314225=$ Value, treated as "immediate," $=$ £42,797·9.

Checked by Hardy's rules,

1st. Smart's Tab. 3, $(a)^{65}=194\cdot3328$

$\times\ 4,500$

Product $=874497\cdot60$

2nd. $(a)^{65}=194\cdot3328$

$\times t'=\ \cdot10$

Prod. $=19\cdot43328$

$+\ 1\cdot$

Sum $=20\cdot43328$

3rd. $\frac{874497\cdot6}{20\cdot433280}=42,797\cdot71=$ Value, as "immediate."

4th. Int. on Value at $10\frac{0}{0}=4279\cdot771$

Amort. to recoup at $3\frac{0}{0}=\ 220\cdot229$

Total annuity £4,500·000

5th. Smart's Tab. 1. $r^{65}=6\cdot830$

$-\ 1\cdot$

Diff. $=5\cdot830$

6th. Log. 5·83 = 0·7656686
Log. 220·229 = 2·3428746
C^t Log. ·03 = 11·5228787
Log. 42,797·85 = 4·6314219 = the capital reproduced.

But as this annuity is deferred for 15 years (the first payment coming due in the 16th year) the above value has to be discounted for that term; and supposing that 8 per cent. per annum discount is allowed for it we have to proceed as follows. The interest at 8 per cent. per annum on £42,797·9 is £3,423·832.

Then (by Thoman's) Log. 3,423·832 = 3·5345164
÷ by Log. a^{15} = 9·0675527
Log. 29,374· = 4·4669637

∴ 42,797·9 − 29,374 = £13,423·9 = the value of the deferred annuity of Lot C treated on the terms indicated.

Lot D. The annuity for 75 years = £200.
Interest on Capital = 7 per cent.
Interest to recoup = 3½ per cent.

	Logarithms.	Nat. Numbers.
Log. a^{75}	= 8·5782860	
Log. r^{75}	= 1·1205262	
Log. $a^{75}r^{-75}$	= 7·4577598 =	·0028692
		$+t'$ = ·07
Log. $t' + \frac{a^{75}}{r^{75}}$	= 8·8625440 =	Sum = ·0728692
Log. 200	= 2·3010300	
Log. V	= 3·4384860 =	£2,744·64 = Value.

By Hardy's rules,

1st. $(a)^{75}$ = 348·53001
200
Product = 69706·002

2nd. $(a)^{75} = 348{\cdot}53001$
$\times t' = \quad {\cdot}07$
Prod. $= 24{\cdot}3971$
$+ \quad 1{\cdot}$
Sum $= 25{\cdot}3971$

3rd. $\frac{69{,}706{\cdot}002}{25{\cdot}3971} = £2{,}744{\cdot}64 =$ Value as above.

4th. Then Int. on Value at $7\frac{8}{8} = 192{\cdot}1248$
Amort. to recoup at $3\frac{1}{2}\frac{8}{8} = \quad 7{\cdot}8752$
Total annuity $= £200{\cdot}0000$

5th. $r^{75} = 13{\cdot}19855$
$- \quad 1{\cdot}$
Diff. $= 12{\cdot}19855$

6th. Log. $12{\cdot}19855 = 1{\cdot}0863082$
Log. $7{\cdot}8752 = 0{\cdot}8962616$
C^t Log. ${\cdot}035 = 11{\cdot}4559320$
Log. $2{,}744{\cdot}74 = 3{\cdot}4385018 =$ the capital reproduced at the end of 75 years.

Lot E. The annuity for 12 years equals £80.
Interest on Capital = 10 per cent.
Interest to recoup = 3 per cent.

	Logarithms.	
Log. a^{12}	$= 9{\cdot}0020022$	
Log. r^{12}	$= 0{\cdot}1540467$	Nat. Numbers.
Log. $a^{12}r^{-12}$	$= 8{\cdot}8479555 =$	$= {\cdot}070462$
		$+t' = {\cdot}10$
Log. $t' + \frac{a^{12}}{r^{12}}$	$= 9{\cdot}2316273 =$	Sum $= {\cdot}170462$
Log. 80	$= 1{\cdot}9030900$	
Log. V	$= 2{\cdot}6714627 = £469{\cdot}313$ the Value.	

Check by Hardy's rules.

1st. $(a)^{12} = 14{\cdot}192$
$\times \quad 80$
Product $= 1135{\cdot}36$

2nd. $(a)^{12} = 14{\cdot}192$
$\times t' = \quad {\cdot}10$
Prod. $= 1{\cdot}4192$
$+ 1'$
Sum $= 2{\cdot}4192$

3rd. $\frac{1135{\cdot}36}{2{\cdot}4192} = £469{\cdot}313 =$ Value as above.

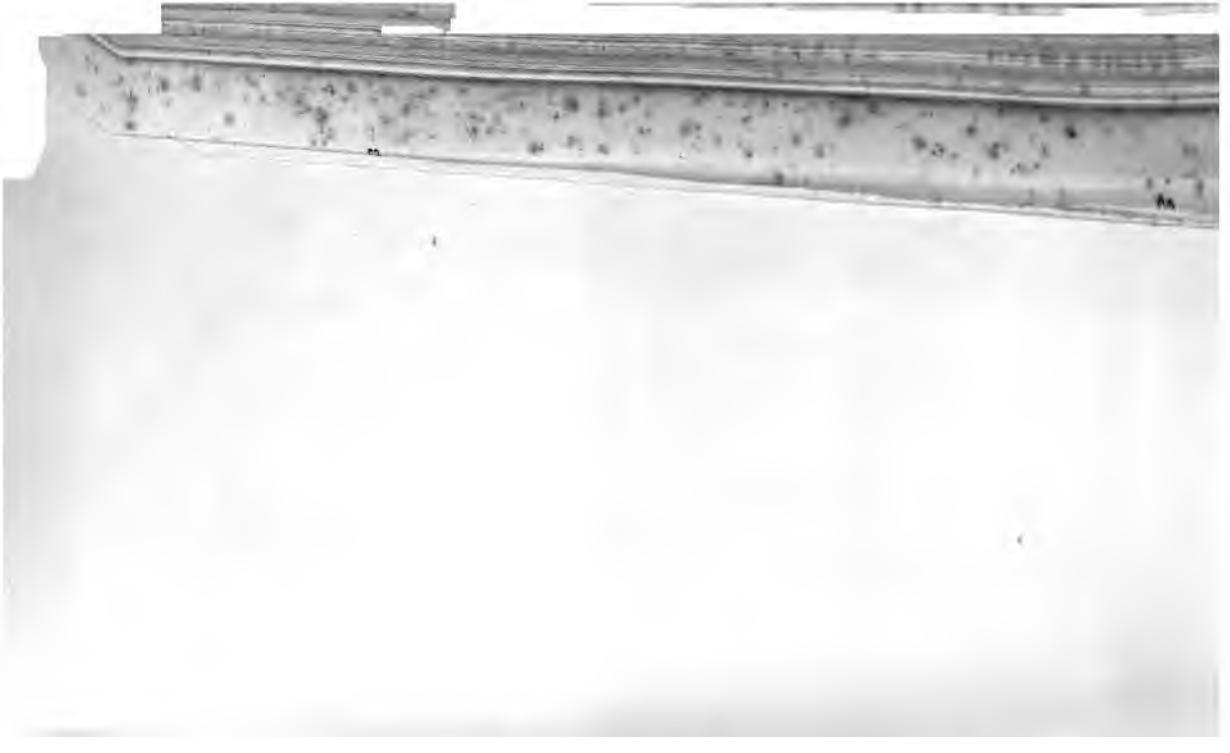
Then 4th. Interest on Value at 10 % $= 46{\cdot}9312$
Amort. to recoup at 3 % $= 33{\cdot}0688$
Total annuity $= £80{\cdot}0000$

5th. $r^{12} = 1{\cdot}4258$
$- 1'$
Diff. $= {\cdot}4258$

6th. Log. ·4258 = 9·6292057
Log. 33·0688 = 1·5194185
C^t Log. ·03 = 11·5228787
Log. 469·35 = 2·6715029 = the capital reprod at the end of the term.

SUMMARY OF VALUES.

	£	s.	d.
The Value of Lot A =	3,625	0	0
„ Value of Lot B =	18,623	5	2
„ Value of Lot C =	13,423	18	0
„ Value of Lot D =	2,744	12	9
„ Value of Lot E =	469	6	3
Total Value =	38,886	2	2



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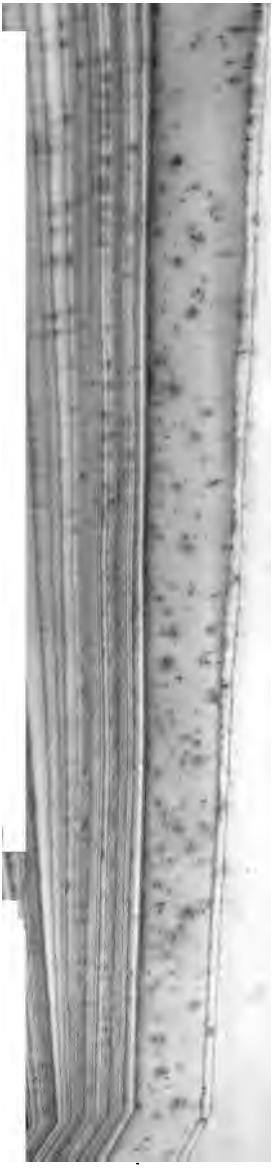

CHAPTER XIII.

THE TRAVERSE TABLES.

(DESCRIBED IN CHAPTER VII, PAGES 21—37.)

0 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	100·00	0·00	200·00	0·00	300·00	0·00	400·00	0·00	500·00	0·00	60
3	99·99	0·08	199·99	0·17	299·99	0·26	399·99	0·35	499·99	0·43	57
6	99·99	0·17	199·99	0·35	299·99	0·52	399·99	0·70	499·99	0·87	54
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12	99·99	0·35	199·99	0·70	299·99	1·04	399·99	1·39	499·99	1·74	48
15	99·99	0·44	199·99	0·87	299·99	1·30	399·99	1·74	499·99	2·17	45
18	99·99	0·52	199·99	1·04	299·99	1·57	399·99	2·09	499·99	2·61	42
21	99·99	0·61	199·99	1·21	299·99	1·83	399·99	2·44	499·99	3·05	39
24	99·99	0·70	199·99	1·39	299·99	2·09	399·98	2·79	499·98	3·49	36
27	99·99	0·78	199·99	1·56	299·99	2·35	399·98	3·14	499·98	3·92	33
30	99·99	0·87	199·99	1·74	299·98	2·61	399·98	3·49	499·98	4·36	30
33	99·99	0·96	199·99	1·91	299·98	2·87	399·98	3·84	499·97	4·79	27
36	99·99	1·05	199·99	2·09	299·98	3·14	399·97	4·18	499·97	5·23	24
39	99·99	1·13	199·98	2·26	299·98	3·40	399·97	4·53	499·96	5·66	21
42	99·99	1·22	199·98	2·44	299·97	3·66	399·96	4·88	499·96	6·10	18
45	99·99	1·31	199·98	2·61	299·97	3·92	399·96	5·23	499·95	6·54	15
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51	99·99	1·48	199·97	2·96	299·96	4·44	399·95	5·93	499·94	7·41	9
54	99·98	1·57	199·97	3·14	299·96	4·71	399·94	6·28	499·93	7·85	6
57	99·98	1·66	199·97	3·31	299·95	4·97	399·94	6·63	499·93	8·28	3
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	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

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9	599·99	1·56	699·99	1·83	799·99	2·09	899·99	2·35	999·99	2·62	51
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18	599·99	3·14	699·99	3·66	799·98	4·19	899·98	4·71	999·98	5·24	42
21	599·99	3·66	699·98	4·27	799·98	4·89	899·98	5·49	999·98	6·11	39
24	599·98	4·18	699·98	4·88	799·98	5·58	899·97	6·28	999·97	6·98	36
27	599·98	4·70	699·97	5·49	799·97	6·28	899·97	7·06	999·96	7·85	33
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33	599·97	5·75	699·96	6·71	799·96	7·68	899·95	8·63	999·95	9·60	27
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54	599·92	9·42	699·91	10·99	799·90	12·57	899·89	14·14	999·87	15·71	6
57	599·91	9·95	699·90	11·61	799·89	13·26	899·87	14·92	999·86	16·58	3
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39	99·96	2·88	199·91	5·75	299·87	8·63	399·83	11·52	499·79	14·39	21
42	99·95	2·97	199·91	5·93	299·86	8·89	399·82	11·86	499·78	14·83	18
45	99·95	3·05	199·90	6·10	299·86	9·15	399·81	12·21	499·76	15·26	15
48	99·95	3·14	199·90	6·28	299·85	9·42	399·80	12·56	499·75	15·70	12
51	99·95	3·23	199·89	6·45	299·84	9·68	399·79	12·91	499·74	16·13	9
54	99·94	3·31	199·89	6·63	299·83	9·94	399·78	13·26	499·72	16·57	6
57	99·94	3·40	199·88	6·80	299·82	10·20	399·76	13·61	499·71	17·01	3
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6	599·89	11·51	699·87	13·44	799·85	15·35	899·83	17·27	999·81	19·19	54
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12	599·86	12·56	699·84	14·66	799·82	16·75	899·80	18·84	999·78	20·94	48
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18	599·84	13·61	699·82	15·88	799·79	18·15	899·76	20·41	999·74	22·68	42
21	599·83	14·13	699·80	16·49	799·78	18·85	899·75	21·20	999·72	23·55	39
24	599·82	14·65	699·79	17·10	799·76	19·54	899·73	21·99	999·70	24·43	36
27	599·81	15·17	699·77	17·71	799·74	20·24	899·71	22·77	999·68	25·30	33
30	599·79	15·70	699·76	18·32	799·72	20·94	899·69	23·56	999·66	26·17	30
33	599·78	16·22	699·74	18·93	799·71	21·64	899·67	24·34	999·63	27·04	27
36	599·76	16·75	699·72	19·54	799·69	22·33	899·65	25·12	999·61	27·92	24
39	599·75	17·27	699·70	20·15	799·67	23·03	899·62	25·91	999·58	28·79	21
42	599·73	17·80	699·69	20·76	799·64	23·73	899·60	26·70	999·56	29·66	18
45	599·72	18·32	699·67	21·37	799·62	24·42	899·57	27·48	999·53	30·53	15
48	599·70	18·84	699·65	21·98	799·60	25·12	899·55	28·27	999·50	31·41	12
51	599·68	19·36	699·63	22·59	799·58	25·82	899·53	29·05	999·48	32·28	9
54	599·67	19·89	699·61	23·20	799·56	26·52	899·50	29·84	999·45	33·15	6
57	599·65	20·41	699·59	23·81	799·54	27·22	899·48	30·62	999·42	34·02	3
60	599·63	20·94	699·57	24·42	799·51	27·92	899·45	31·41	999·39	34·90	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

88 Deg.

2 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	99·94	3·49	199·87	6·98	299·81	10·47	399·75	13·96	499·69	17·45	60
3	99·93	3·58	199·87	7·15	299·80	10·73	399·74	14·31	499·68	17·88	57
6	99·93	3·66	199·86	7·33	299·80	10·99	399·73	14·65	499·66	18·32	54
9	99·93	3·75	199·86	7·50	299·79	11·25	399·72	15·00	499·65	18·75	51
12	99·92	3·84	199·85	7·67	299·78	11·51	399·70	15·35	499·63	19·19	48
15	99·92	3·93	199·84	7·84	299·77	11·77	399·69	15·70	499·61	19·62	45
18	99·92	4·01	199·84	8·02	299·76	12·03	399·67	16·05	499·60	20·06	42
21	99·91	4·10	199·83	8·19	299·75	12·29	399·66	16·40	499·58	20·49	39
24	99·91	4·19	199·82	8·37	299·73	12·56	399·65	16·75	499·56	20·93	36
27	99·91	4·27	199·81	8·54	299·72	12·82	399·63	17·10	499·54	21·36	33
30	99·90	4·36	199·81	8·72	299·71	13·08	399·62	17·44	499·52	21·80	30
33	99·90	4·45	199·80	8·89	299·70	13·34	399·60	17·79	499·50	22·24	27
36	99·89	4·54	199·79	9·07	299·69	13·60	399·59	18·14	499·48	22·68	24
39	99·89	4·62	199·78	9·24	299·68	13·86	399·57	18·49	499·46	23·11	21
42	99·89	4·71	199·78	9·42	299·66	14·13	399·55	18·84	499·44	23·55	18
45	99·88	4·80	199·77	9·59	299·65	14·39	399·54	19·19	499·42	23·98	15
48	99·88	4·88	199·76	9·77	299·64	14·65	399·52	19·54	499·40	24·42	12
51	99·87	4·97	199·75	9·94	299·63	14·91	399·50	19·89	499·38	24·85	9
54	99·87	5·06	199·74	10·11	299·61	15·17	399·49	20·23	499·36	25·29	6
57	99·86	5·15	199·73	10·28	299·60	15·43	399·47	20·58	499·33	25·72	3
60	99·86	5·23	199·72	10·46	299·59	15·70	399·45	20·93	499·31	26·16	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

87 Deg.

2 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	599·63	20·94	699·57	24·42	799·51	27·92	899·45	31·41	999·39	34·90	60
3	599·61	21·46	699·55	25·03	799·49	28·62	899·42	32·19	999·36	35·77	57
6	599·60	21·98	699·53	25·65	799·46	29·31	899·39	32·98	999·33	36·64	54
9	599·58	22·50	699·50	26·26	799·44	30·01	899·36	33·76	999·30	37·51	51
12	599·56	23·03	699·48	26·87	799·41	30·71	899·33	34·54	999·26	38·39	48
15	599·54	23·55	699·46	27·48	799·38	31·41	899·30	35·32	999·23	39·26	45
18	599·52	24·07	699·43	28·09	799·35	32·10	899·27	36·11	999·19	40·13	42
21	599·49	24·59	699·41	28·70	799·32	32·80	899·24	36·89	999·16	41·00	39
24	599·47	25·12	699·38	29·31	799·30	33·50	899·21	37·68	999·13	41·87	36
27	599·45	25·64	699·36	29·92	799·27	34·20	899·17	38·46	999·09	42·75	33
30	599·43	26·17	699·33	30·53	799·24	34·89	899·14	39·25	999·05	43·62	30
33	599·40	26·69	699·30	31·14	799·21	35·59	899·10	40·03	999·01	44·49	27
36	599·38	27·21	699·28	31·75	799·17	36·29	899·07	40·82	998·97	45·36	24
39	599·36	27·74	699·25	32·36	799·14	36·99	899·03	41·60	998·93	46 23	21
42	599·33	28·26	699·22	32·97	799·11	37·68	899·00	42·39	998·89	47·11	18
45	599·31	28·78	699·19	33·58	799·08	38·38	898·96	43·17	998·84	47·98	15
48	599·28	29·31	699·16	34·19	799·04	39·08	898·92	43·96	998·80	48·85	12
51	599·26	29·83	699·13	34·80	799·01	39·78	898·88	44·74	998·76	49·72	9
54	599·23	30·35	699·10	35·41	798·97	40·47	898·84	45·53	998·72	50·59	6
57	599·20	30·88	699·07	36·02	798·94	41·17	898·80	46·31	998·67	51·46	3
60	599·18	31·40	699·04	36·63	798·90	41·86	898·76	47·10	998·63	52·34	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

87 Deg.

3 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	99·86	5·23	199·72	10·46	299·59	15·70	399·45	20·93	499·31	26·16	60
3	99·86	5·32	199·71	10·63	299·57	15·96	399·43	21·28	499·29	26·59	57
6	59·85	5·41	199·71	10·81	299·56	16·22	399·41	21·63	499·26	27·03	54
9	99·85	5·49	199·70	10·98	299·55	16·48	399·39	21·98	499·24	27·47	51
12	99·84	5·58	199·69	11·16	299·53	16·74	399·37	22·32	499·22	27·91	48
15	99·84	5·67	199·68	11·33	299·52	17·00	399·35	22·67	499·19	28·34	45
18	99·83	5·76	199·67	11·51	299·50	17·27	399·33	23·02	499·17	28·78	42
21	99·83	5·84	199·66	11·68	299·49	17·53	399·31	23·37	499·14	29·21	39
24	99·82	5·93	199·65	11·86	299·47	17·79	399·29	23·72	499·12	29·65	36
27	99·82	6·02	199·64	12·03	299·45	18·05	399·27	24·07	499·09	30·08	33
30	99·81	6·10	199·63	12·21	299·44	18·31	399·25	24·41	499·06	30·52	30
33	99·81	6·19	199·61	12·38	299·42	18·57	399·23	24·76	499·04	30·95	27
36	99·80	6·28	199·60	12·55	299·41	18·83	399·21	25·11	499·01	31·39	24
39	99·80	6·36	199·59	12·72	299·39	19·09	399·19	25·46	498·98	31·82	21
42	99·79	6·45	199·58	12·90	299·37	19·35	399·16	25·81	498·96	32·26	18
45	99·78	6·54	199·57	13·07	299·36	19·61	399·14	26·16	498·93	32·69	15
48	99·78	6·63	199·56	13·25	299·34	19·88	399·12	26·51	498·90	33·13	12
51	99·77	6·71	199·55	13·42	299·32	20·14	399·09	26·86	498·87	33·56	9
54	99·77	6·80	199·54	13·60	299·30	20·40	399·07	27·20	498·84	34·00	6
57	99·76	6·89	199·52	13·77	299·29	20·66	399·05	27·55	498·81	34·43	3
60	99·75	6·97	199·51	13·95	299·27	20·92	399·02	27·90	498·78	34·87	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

86 Deg.

3 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	599·18	31·40	699·04	36·63	798·90	41·86	898·76	47·10	998·63	52·34	60
3	599·15	31·92	699·01	37·24	798·86	42·56	898·72	47·88	998·58	53·21	57
6	599·12	32·44	698·97	37·85	798·83	43·26	898·68	48·67	998·54	54·08	54
9	599·09	32·96	698·94	38·46	798·79	43·06	898·64	49·45	998·49	54·95	51
12	599·06	33·49	698·91	39·07	798·75	44·65	898·60	50·24	998·44	55·82	48
15	599·03	34·01	698·87	39·68	798·71	45·35	898·55	51·02	998·39	56·69	45
18	599·00	34·54	698·84	40·29	798·67	46·05	898·51	51·80	998·34	57·56	42
21	598·97	35·06	698·80	40·90	798·63	46·75	898·46	52·58	998·29	58·43	39
24	598·94	35·58	698·76	41·51	798·59	47·44	898·41	53·37	998·24	59·31	36
27	598·91	36·10	698·73	42·12	798·55	48·14	898·37	54·15	998·18	60·18	33
30	598·88	36·63	698·69	42·73	798·51	48·84	898·32	54·94	998·13	61·05	30
33	598·85	37·15	698·65	43·34	798·46	49·54	898·27	55·72	998·08	61·92	27
36	598·81	37·67	698·62	43·95	798·42	50·23	898·22	56·51	998·02	62·79	24
39	598·78	38·19	698·58	44·56	798·38	50·93	898·17	57·29	997·97	63·66	21
42	598·75	38·72	698·54	45·17	798·33	51·62	898·12	58·08	997·91	64·53	18
45	598·71	39·24	698·50	45·78	798·28	52·32	898·07	58·86	997·86	65·40	15
48	598·68	39·76	698·46	46·39	798·24	53·01	898·02	59·64	997·80	66·27	12
51	598·64	40·28	698·42	47·00	798·19	53·71	897·97	60·42	997·74	67·14	9
54	598·61	40·81	698·38	47·61	798·15	54·41	897·92	61·21	997·68	68·01	6
57	598·57	41·33	698·33	48·22	798·10	55·11	897·86	61·99	997·62	68·88	3
60	598·54	41·85	698·29	48·83	798·05	55·80	897·81	62·78	997·56	69·75	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

86 Deg.

4 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	99·75	6·97	199·51	13·95	299·27	20·92	399·02	27·90	498·78	34·87	60
3	99·75	7·06	199·50	14·12	299·25	21·18	399·00	28·25	498·75	35·30	57
6	99·74	7·15	199·49	14·30	299·23	21·45	398·97	28·60	498·72	35·74	54
9	99·74	7·24	199·47	14·47	299·21	21·71	398·95	28·95	498·69	36·18	51
12	99·73	7·32	199·46	14·64	299·19	21·97	398·92	29·30	498·66	36·62	48
15	99·72	7·41	199·45	14·81	299·17	22·23	398·90	29·65	498·62	37·05	45
18	99·72	7·50	199·43	14·99	299·15	22·49	398·87	29·99	498·59	37·49	42
21	99·71	7·58	199·42	15·17	299·13	22·75	398·85	30·34	498·56	37·92	39
24	99·70	7·67	199·41	15·34	299·11	23·01	398·82	30·68	498·53	38·36	36
27	99·70	7·76	199·39	15·51	299·09	23·27	398·79	31·03	498·49	38·79	33
30	99·69	7·85	199·38	15·69	299·07	23·53	398·76	31·38	498·46	39·23	30
33	99·68	7·93	199·37	15·86	299·05	23·79	398·74	31·73	498·42	39·66	27
36	99·68	8·02	199·35	16·03	299·03	24·06	398·71	32·08	498·39	40·10	24
39	99·67	8·11	199·34	16·20	299·01	24·32	398·68	32·43	498·35	40·53	21
42	99·66	8·19	199·33	16·38	298·99	24·58	398·65	32·77	498·32	40·97	18
45	99·65	8·28	199·31	16·55	298·97	24·84	398·62	33·12	498·28	41·40	15
48	99·65	8·37	199·30	16·73	298·95	25·10	398·60	33·47	498·24	41·84	12
51	99·64	8·45	199·28	16·90	298·92	25·36	398·57	33·82	498·21	42·27	9
54	99·63	8·54	199·27	17·08	298·90	25·62	398·54	34·16	498·17	42·71	6
57	99·63	8·63	199·25	17·25	298·88	25·88	398·51	34·51	498·13	43·14	3
60	99·62	8·71	199·24	17·43	298·86	26·14	398·48	34·86	498·10	43·58	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

85 Deg.

4 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	598·54	41·85	698·29	48·83	798·05	55·80	897·81	62·78	997·56	69·75	60
3	598·50	42·37	698·25	49·44	798·00	56·50	897·75	63·56	997·50	70·63	57
6	598·46	42·90	698·21	50·05	797·95	57·19	897·70	64·34	997·44	71·50	54
9	598·42	43·42	698·16	50·66	797·90	57·89	897·64	65·12	997·37	72·37	51
12	598·39	43·94	698·12	51·27	797·85	58·59	897·58	65·91	997·31	73·24	48
15	598·35	44·46	698·07	52·88	797·80	59·29	897·53	66·69	997·25	74·11	45
18	598·31	44·98	698·03	52·49	797·75	59·98	897·47	67·47	997·18	74·98	42
21	598·27	45·50	697·98	53·10	797·69	60·68	897·41	68·25	997·12	75·85	39
24	598·23	46·03	697·94	53·70	797·64	61·37	897·35	69·04	997·05	76·72	36
27	598·19	46·55	697·89	54·31	797·59	62·07	897·29	70·82	996·98	77·59	33
30	598·15	47·07	697·84	54·92	797·53	62·77	897·23	70·61	996·92	78·46	30
33	598·11	47·60	697·79	55·33	797·48	63·47	897·16	71·39	996·85	79·33	27
36	598·06	48·12	697·74	56·14	797·42	64·16	897·10	72·18	996·78	80·20	24
39	598·02	48·64	697·70	56·75	797·36	64·86	897·04	72·96	996·71	81·07	21
42	597·98	49·16	697·65	57·35	797·31	65·55	896·97	73·75	996·64	81·94	18
45	597·94	49·68	697·59	57·96	797·25	66·25	896·91	74·53	996·56	82·81	15
48	597·89	50·20	697·54	58·57	797·19	66·94	896·84	75·31	996·49	83·68	12
51	597·85	50·72	697·49	59·18	797·13	67·64	896·78	76·09	996·42	84·55	9
54	597·80	51·25	697·44	59·79	797·07	68·33	896·71	76·87	996·34	85·42	6
57	597·76	51·77	697·39	60·40	797·01	69·03	896·64	77·65	996·27	86·29	3
60	597·72	52·29	697·33	61·01	796·95	69·72	896·58	78·44	996·19	87·15	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

85 Deg.

5 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	99·62	8·71	199·24	17·43	298·86	26·14	398·48	34·86	498·10	43·58	60
3	99·61	8·80	199·22	17·60	298·83	26·40	398·45	35·21	498·06	44·01	57
6	99·60	8·89	199·21	17·78	298·81	26·66	398·41	35·56	498·02	44·45	54
9	99·60	8·98	199·19	17·95	298·79	26·92	398·38	35·91	497·98	44·88	51
12	99·59	9·06	199·18	18·12	298·76	27·19	398·35	36·25	497·94	45·31	48
15	99·58	9·15	199·16	18·29	298·74	27·45	398·32	36·60	497·90	45·74	45
18	99·57	9·24	199·14	18·47	298·72	27·71	398·29	36·95	497·86	46·18	42
21	99·56	9·32	199·13	18·64	298·69	27·97	398·26	37·30	497·82	46·61	39
24	99·55	9·41	199·11	18·82	298·67	28·23	398·22	37·64	497·78	47·05	36
27	99·55	9·50	199·09	18·99	298·64	28·49	398·19	37·99	497·74	47·48	33
30	99·54	9·58	199·08	19·17	298·62	28·75	398·16	38·34	497·70	47·92	30
33	99·53	9·67	199·06	19·34	298·59	29·01	398·12	38·69	497·66	48·35	27
36	99·52	9·76	199·04	19·52	298·57	29·27	398·09	39·03	497·61	48·79	24
39	99·51	9·84	199·03	19·69	298·54	29·53	398·06	39·38	497·57	49·22	21
42	99·50	9·93	199·01	19·86	298·52	29·79	398·02	39·73	497·53	49·66	18
45	99·50	10·02	198·99	20·03	298·49	30·05	397·99	40·08	497·48	50·09	15
48	99·49	10·10	198·97	20·21	298·46	30·31	397·95	40·42	497·44	50·53	12
51	99·48	10·19	198·96	20·38	298·44	30·57	397·92	40·77	497·40	50·96	9
54	99·47	10·28	198·94	20·55	298·41	30·83	397·88	41·12	497·35	51·40	6
57	99·46	10·36	198·92	20·72	298·38	31·09	397·84	41·47	497·31	51·83	3
60	99·45	10·45	198·90	20·90	298·36	31·36	397·81	41·81	497·26	52·26	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

84 Deg.

5 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	597·72	52·29	697·33	61·01	796·95	69·72	896·58	78·44	996·19	87·15	60
3	597·67	52·81	697·28	61·62	796·89	70·42	896·51	79·22	996·11	88·02	57
6	597·62	53·33	697·23	62·22	796·83	71·11	896·44	80·01	996·04	88·89	54
9	597·58	53·85	697·17	62·83	796·77	71·81	896·37	80·79	995·96	89·76	51
12	597·53	54·38	697·12	63·44	796·70	72·50	896·30	81·57	995·88	90·63	48
15	597·48	55·90	697·06	64·05	796·64	73·20	896·22	82·35	995·80	91·50	45
18	597·43	55·42	697·00	64·66	796·58	73·89	896·15	83·13	995·72	92·37	42
21	597·39	55·94	696·95	65·27	796·51	74·59	896·08	83·91	995·64	93·24	39
24	597·33	56·46	696·89	65·87	796·45	75·28	896·01	84·70	995·56	94·11	36
27	597·29	56·98	696·83	66·48	796·38	75·98	895·93	85·48	995·48	94·98	33
30	597·24	57·51	696·78	67·09	796·32	76·68	895·86	86·26	995·39	95·84	30
33	597·19	58·03	696·72	67·70	796·25	77·38	895·78	87·04	995·31	96·71	27
36	597·13	58·55	696·66	68·31	796·18	78·07	895·70	87·82	995·23	97·58	24
39	597·08	59·07	696·60	68·92	796·11	78·77	895·63	88·60	995·14	98·45	21
42	597·03	59·59	696·54	69·52	796·04	79·46	895·55	89·38	995·05	99·32	18
45	596·98	60·11	696·48	70·13	795·97	80·15	895·47	90·16	994·97	100·19	15
48	596·93	60·63	696·42	70·74	795·90	80·84	895·39	90·95	994·88	101·05	12
51	596·87	61·15	696·35	71·35	795·83	81·54	895·31	91·73	994·79	101·92	9
54	596·82	61·67	696·29	71·95	795·76	82·23	895·23	92·51	994·70	102·79	6
57	596·77	62·19	696·23	72·56	795·69	82·93	895·15	93·29	994·61	103·66	3
60	596·71	62·71	696·16	73·17	795·62	83·62	895·07	94·07	994·52	104·53	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

84 Deg.

6 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	99·45	10·45	198·90	20·90	298·36	31·36	397·81	41·81	497·26	52·26	60
3	99·44	10·54	198·88	21·07	298·33	31·62	397·77	42·16	497·21	52·69	57
6	99·43	10·62	198·87	21·25	298·30	31·88	397·73	42·50	497·17	53·13	54
9	99·42	10·71	198·85	21·42	298·27	32·14	397·70	42·85	497·12	53·56	51
12	99·41	10·80	198·83	21·60	298·24	32·40	397·66	43·20	497·07	54·00	48
15	99·40	10·89	198·81	21·77	298·21	32·66	397·62	43·55	497·03	54·43	45
18	99·39	10·97	198·79	21·95	298·19	32·92	397·58	43·89	496·98	54·87	42
21	99·38	11·06	198·77	22·12	298·16	33·18	397·54	44·24	496·93	55·30	39
24	99·37	11·15	198·75	22·29	298·13	33·44	397·51	44·59	496·88	55·73	36
27	99·36	11·23	198·73	22·46	298·10	33·70	397·47	44·94	496·83	56·16	33
30	99·35	11·32	198·71	22·64	298·07	33·96	397·43	45·28	496·78	56·60	30
33	99·34	11·41	198·69	22·81	298·04	34·22	397·39	45·63	496·73	57·03	27
36	99·33	11·49	198·67	22·98	298·01	34·48	397·35	45·97	496·68	57·47	24
39	99·32	11·58	198·65	23·15	297·98	34·74	397·31	46·32	496·63	57·90	21
42	99·31	11·67	198·63	23·33	297·95	35·00	397·27	46·67	496·58	58·34	18
45	99·30	11·75	198·61	23·50	297·92	35·26	397·23	47·02	496·53	58·77	15
48	99·29	11·84	198·59	23·68	297·89	35·52	397·18	47·36	496·48	59·20	12
51	99·28	11·93	198·57	23·85	297·86	35·78	397·14	47·71	496·43	59·63	9
54	99·27	12·01	198·55	24·03	297·83	36·04	397·10	48·05	496·38	60·07	6
57	99·26	12·10	198·53	24·20	297·79	36·30	397·06	48·40	496·33	60·50	3
60	99·25	12·19	198·51	24·38	297·76	36·56	397·02	48·75	496·27	60·93	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

83 Deg.

6 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	596·71	62·71	696·16	73·17	795·62	83·62	895·07	94·07	994·52	104·53	60
3	596·66	63·23	696·10	73·78	795·54	84·31	894·99	94·85	994·43	105·39	57
6	596·60	63·76	696·04	74·38	795·47	85·01	894·90	95·63	994·34	106·26	54
9	596·55	64·28	695·97	74·99	795·39	85·70	894·82	96·41	994·24	107·13	51
12	596·49	64·80	695·90	75·60	795·32	86·40	894·74	97·20	994·15	108·00	48
15	596·43	65·32	695·84	76·21	795·24	87·09	894·65	97·98	994·05	108·87	45
18	596·37	65·84	695·77	76·81	795·17	87·79	894·56	98·76	993·96	109·73	42
21	596·32	66·36	695·70	77·42	795·09	88·48	894·48	99·54	993·86	110·60	39
24	596·26	66·88	695·64	78·03	795·01	89·17	894·39	100·32	993·77	111·47	36
27	596·20	67·40	695·57	78·64	794·93	89·86	894·30	101·10	993·67	112·33	33
30	596·14	67·92	695·50	79·24	794·86	90·56	894·21	101·88	993·57	113·20	30
33	596·08	68·44	695·43	79·85	794·77	91·25	894·12	102·66	993·47	114·07	27
36	596·02	68·96	695·36	80·46	794·69	91·95	894·03	103·44	993·37	114·94	24
39	595·96	69·48	695·29	81·07	794·61	92·65	893·94	104·22	993·27	115·80	21
42	595·90	70·00	695·22	81·67	794·53	93·34	893·85	105·00	993·17	116·67	18
45	595·84	70·52	695·14	82·28	794·45	94·03	893·76	105·78	993·07	117·54	15
48	595·78	71·04	695·07	82·88	794·37	94·72	893·67	106·56	992·96	118·40	12
51	595·72	71·56	695·00	83·49	794·29	95·42	893·57	107·34	992·86	119·27	9
54	595·65	72·08	694·93	84·10	794·20	96·11	893·48	108·12	992·76	120·13	6
57	595·59	72·60	694·85	84·71	794·12	96·80	893·38	108·90	992·65	121·00	3
60	595·53	73·12	694·78	85·31	794·03	97·50	893·29	109·68	992·54	121·87	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

83 Deg.

7 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base	
0	99·25	12·19	198·51	24·38	297·76	36·56	397·02	48·75	496·27	60·93	60
3	99·24	12·27	198·49	24·55	297·73	36·82	396·97	49·10	496·22	61·37	57
6	99·23	12·36	198·46	24·72	297·70	37·08	396·93	49·44	496·17	61·80	54
9	99·22	12·45	198·44	24·90	297·67	37·34	396·89	49·79	496·11	62·23	51
12	99·21	12·53	198·42	25·07	297·63	37·60	396·84	50·13	496·06	62·67	48
15	99·20	12·62	198·40	25·24	297·60	37·86	396·80	50·48	496·00	63·10	45
18	99·19	12·71	198·38	25·42	297·57	38·12	396·76	50·83	495·95	63·53	42
21	99·18	12·79	198·36	25·59	297·53	38·38	396·71	51·17	495·89	63·97	39
24	99·17	12·88	198·33	25·76	297·50	38·64	396·67	51·52	495·83	64·40	36
27	99·15	12·97	198·31	25·93	297·47	38·90	396·62	51·87	495·78	64·83	33
30	99·13	13·05	198·28	26·11	297·43	39·16	396·58	52·21	495·72	65·26	30
33	99·12	13·14	198·26	26·28	297·40	39·42	396·53	52·56	495·66	65·70	27
36	99·11	13·22	198·24	26·45	297·36	39·68	396·48	52·90	495·61	66·13	24
39	99·10	13·31	198·22	26·63	297·33	39·94	396·44	53·25	495·55	66·56	21
42	99·09	13·40	198·20	26·80	297·29	40·20	396·39	53·60	495·49	66·99	18
45	99·07	13·48	198·17	26·97	297·26	40·46	396·35	53·94	495·43	67·42	15
48	99·06	13·57	198·15	27·14	297·22	40·72	396·30	54·29	495·37	67·86	12
51	99·05	13·66	198·12	27·32	297·19	40·98	396·25	54·63	495·31	68·29	9
54	99·04	13·74	198·10	27·49	297·15	41·23	396·20	54·98	495·25	68·72	6
57	99·03	13·83	198·08	27·66	297·12	41·49	396·15	55·32	495·19	69·15	3
60	99·02	13·92	198·05	27·84	297·08	41·75	396·11	55·67	495·13	69·59	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

82 Deg.

7 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	595·53	73·12	694·78	85·31	794·03	97·50	893·29	109·68	992·54	121·87	60
3	595·46	73·64	694·71	85·92	793·95	98·19	893·19	110·46	992·44	122·73	57
6	595·40	74·16	694·63	86·52	793·86	98·88	893·10	111·24	992·33	123·60	54
9	595·33	74·68	694·55	87·13	793·78	99·57	893·00	112·02	992·22	124·47	51
12	595·27	75·20	694·48	87·73	793·69	100·27	892·90	112·80	992·11	125·33	48
15	595·20	75·72	694·40	88·34	793·60	100·96	892·80	113·58	992·00	126·20	45
18	595·13	76·24	694·32	88·94	793·51	101·65	892·70	114·36	991·89	127·06	42
21	595·07	76·76	694·25	89·55	793·42	102·34	892·60	115·14	991·78	127·93	39
24	595·00	77·28	694·17	90·16	793·33	103·04	892·50	115·92	991·67	128·79	36
27	594·93	77·80	694·09	90·76	793·24	103·73	892·40	116·69	991·56	129·66	33
30	594·86	78·32	694·01	91·37	793·15	104·42	892·30	117·47	991·44	130·52	30
33	594·80	78·83	693·93	91·97	793·06	105·11	892·20	118·25	991·33	131·39	27
36	594·73	79·35	693·85	92·58	792·97	105·80	892·09	119·03	991·21	132·26	24
39	594·66	79·87	693·77	93·19	792·88	106·50	891·99	119·81	991·10	133·12	21
42	594·59	80·39	693·69	93·79	792·78	107·19	891·88	120·59	990·98	133·98	18
45	594·52	80·91	693·60	94·40	792·69	107·88	891·78	121·36	990·86	134·85	15
48	594·45	81·43	693·52	95·00	792·60	108·57	891·67	122·14	990·75	135·71	12
51	594·38	81·95	693·44	95·60	792·50	109·26	891·56	122·92	990·63	136·58	9
54	594·30	82·47	693·35	96·21	792·41	109·95	891·46	123·70	990·51	137·44	6
57	594·23	82·99	693·27	96·82	792·31	110·65	891·35	124·48	990·39	138·31	3
60	594·16	83·50	693·18	97·42	792·21	111·34	891·24	125·26	990·27	139·17	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

82 Deg.

8 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	99·02	13·92	198·05	27·84	297·08	41·75	396·11	55·67	495·13	69·59	60
3	99·01	14·00	198·03	28·01	297·04	42·01	396·06	56·02	495·07	70·02	57
6	99·00	14·09	198·00	28·18	297·01	42·27	396·01	56·36	495·01	70·45	54
9	98·99	14·18	197·98	28·36	296·97	42·53	395·96	56·71	494·95	70·88	51
12	98·98	14·26	197·95	28·53	296·93	42·79	395·91	57·05	494·89	71·31	48
15	98·96	14·35	197·93	28·70	296·89	43·05	395·86	57·40	494·82	71·75	45
18	98·95	14·43	197·90	28·87	296·86	43·31	395·81	57·74	494·76	72·18	42
21	98·94	14·52	197·88	29·05	296·82	43·57	395·76	58·09	494·70	72·61	39
24	98·93	14·61	197·85	29·22	296·78	43·83	395·71	58·44	494·63	73·04	36
27	98·91	14·69	197·83	29·39	296·74	44·08	395·66	58·78	494·57	73·47	33
30	98·90	14·78	197·80	29·57	296·70	44·34	395·61	59·12	494·51	73·90	30
33	98·89	14·87	197·78	29·74	296·66	44·60	395·55	59·47	494·44	74·34	27
36	98·87	14·95	197·75	29·91	296·63	44·86	395·50	59·81	494·38	74·77	24
39	98·86	15·04	197·72	30·08	296·59	45·12	395·45	60·16	494·31	75·20	21
42	98·85	15·13	197·70	30·25	296·55	45·38	395·40	60·51	494·25	75·63	18
45	98·84	15·21	197·67	30·43	296·51	45·64	395·34	60·85	494·18	76·06	15
48	98·82	15·30	197·64	30·60	296·47	45·90	395·29	61·19	494·11	76·49	12
51	98·81	15·38	197·62	30·77	296·43	46·16	395·24	61·54	494·05	76·92	9
54	98·79	15·47	197·59	30·94	296·39	46·41	395·18	61·88	493·98	77·36	6
57	98·78	15·56	197·56	31·12	296·35	46·67	395·13	62·23	493·91	77·79	3
60	98·77	15·64	197·54	31·29	296·30	46·93	395·07	62·57	493·84	78·22	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

81 Deg.

8 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	594·16	83·50	693·18	97·42	792·21	111·34	891·24	125·26	990·27	139·17	60
3	594·09	84·02	693·10	98·03	792·11	112·03	891·13	126·03	990·14	140·04	57
6	594·01	84·54	693·01	98·63	792·02	112·72	891·02	126·81	990·02	140·90	54
9	593·94	85·06	692·93	99·24	791·92	113·41	890·91	127·59	989·90	141·76	51
12	593·86	85·58	692·84	99·84	791·82	114·10	890·80	128·37	989·77	142·63	48
15	593·79	86·09	692·75	100·44	791·72	114·79	890·69	129·14	989·65	143·49	45
18	593·71	86·61	692·67	101·05	791·62	115·48	890·57	129·92	989·52	144·36	42
21	593·64	87·13	692·58	101·65	791·52	116·17	890·46	130·70	989·40	145·22	39
24	593·56	87·65	692·49	102·26	791·42	116·86	890·34	131·47	989·27	146·08	36
27	593·48	88·17	692·40	102·86	791·31	117·56	890·23	132·25	989·14	146·94	33
30	593·41	88·69	692·31	103·47	791·21	118·25	890·11	133·03	989·01	147·81	30
33	593·33	89·20	692·22	104·07	791·11	118·94	890·00	133·80	988·88	148·67	27
36	593·25	89·72	692·13	104·67	791·00	119·63	889·88	134·58	988·75	149·53	24
39	593·17	90·24	692·04	105 28	790·90	120·32	889·76	135·36	988·62	150·40	21
42	593·09	90·76	691·95	105·88	790·80	121·01	889·64	136·13	988·49	151·26	18
45	593·02	91·27	691·85	106·48	790·69	121·70	889·52	136·91	988·36	152·12	15
48	592·94	91·79	691·76	107·09	790·58	122·39	889·41	137·69	988·23	152·98	12
51	592·86	92·31	691·66	107·69	790·47	123·08	889·28	138·46	988·09	153·85	9
54	592·77	92·83	691·57	108·30	790·37	123·77	889·16	139·24	987·96	154·71	6
57	592·69	93·34	691·47	108·90	790·26	124·46	889·04	140·01	987·82	155·57	3
60	592·61	93·86	691·38	109·50	790·15	125·15	888·92	140·79	987·69	156·43	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

81 Deg.

9 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	98·77	15·64	197·54	31·29	296·30	46·93	395·07	62·57	493·84	78·22	60
3	98·75	15·73	197·51	31·46	296·26	47·19	395·02	62·92	493·77	78·65	57
6	98·74	15·81	197·48	31·63	296·22	47·45	394·96	63·26	493·71	79·08	54
9	98·73	15·90	197·45	31·81	296·18	47·71	394·91	63·61	493·64	79·51	51
12	98·71	15·99	197·43	31·98	296·14	47·96	394·85	63·95	493·57	79·94	48
15	98·70	16·07	197·39	32·15	296·10	48·22	394·80	64·30	493·50	80·37	45
18	98·68	16·16	197·37	32·32	296·06	48·48	394·74	64·64	493·43	80·80	42
21	98·67	16·25	197·34	32·50	296·01	48·75	394·68	64·99	493·36	81·23	39
24	98·66	16·33	197·31	32·67	295·97	49·00	394·63	65·33	493·28	81·66	36
27	98·64	16·42	197·28	32·84	295·93	49·25	394·57	65·68	493·21	82·09	33
30	98·63	16·50	197·26	33·01	295·88	49·52	394·51	66·02	493·14	82·52	30
33	98·61	16·59	197·23	33·18	295·84	49·77	394·46	66·36	493·07	82·95	27
36	98·60	16·68	197·20	33·36	295·80	50·03	394·40	66·71	493·00	83·38	24
39	98·58	16·76	197·17	33·53	295·75	50·29	394·34	67·05	492·92	83·81	21
42	98·57	16·85	197·14	33·70	295·71	50·55	394·28	67·40	492·85	84·24	18
45	98·55	16·93	197·11	33·87	295·67	50·81	394·22	67·74	492·78	84·67	15
48	98·54	17·02	197·08	34·04	295·62	51·06	394·16	68·08	492·70	85·10	12
51	98·52	17·11	197·05	34·22	295·58	51·32	394·10	68·43	492·63	85·53	9
54	98·51	17·19	197·02	34·39	295·53	51·58	394·04	68·77	492·55	85·96	6
57	98·49	17·28	196·99	34·56	295·49	51·84	393·98	69·12	492·48	86·39	3
60	98·48	17·36	196·96	34·73	295·44	52·09	393·92	69·46	492·40	86·82	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

80 Deg.

9 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	592·61	93·86	691·38	109·50	790·15	125·15	888·92	140·79	987·69	156·43	60
3	592·53	94·38	691·28	110·11	790·04	125·84	888·80	141·57	987·55	157·30	57
6	592·45	94·89	691·19	110·71	789·93	126·53	888·67	142·34	987·41	158·16	54
9	592·37	95·41	691·09	111·31	789·82	127·21	888·55	143·12	987·27	159·02	51
12	592·28	95·93	690·99	111·92	789·71	127·90	888·42	143·80	987·14	159·88	48
15	592·20	96·45	690·90	112·52	789·60	128·59	888·30	144·67	987·00	160·74	45
18	592·11	96·96	690·80	113·12	789·48	129·28	888·17	145·44	986·85	161·60	42
21	592·03	97·48	690·70	113·72	789·37	129·97	888·04	146·22	986·71	162·46	39
24	591·94	97·99	690·60	114·33	789·26	130·66	887·91	147·00	986·57	163·32	36
27	591·86	98·51	690·50	114·93	789·14	131·35	887·79	147·77	986·43	164·19	33
30	591·77	99·03	690·40	115·53	789·03	132·04	887·66	148·54	986·28	165·05	30
33	591·68	99·54	690·30	116·14	788·91	132·73	887·53	149·32	986·14	165·91	27
36	591·60	100·06	690·20	116·74	788·80	133·41	887·40	150·09	986·00	166·77	24
39	591·51	100·58	690·09	117·34	788·68	134·10	887·27	150·87	985·85	167·63	21
42	591·42	101·09	689·99	117·94	788·56	134·79	887·13	151·64	985·70	168·49	18
45	591·33	101·61	689·89	118·54	788·44	135·48	887·00	152·41	985·55	169·35	15
48	591·24	102·12	689·79	119·15	788·32	136·17	886·86	153·19	985·41	170·21	12
51	591·15	102·64	689·68	119·75	788·21	136·85	886·73	153·96	985·26	171·07	9
54	591·06	103·16	689·58	120·35	788·09	137·54	886·60	154·74	985·11	171·93	6
57	590·97	103·67	689·47	120·95	787·97	138·23	886·46	155·51	984·96	172·79	3
60	590·88	104·19	689·36	121·55	787·84	138·90	886·33	156·28	984·81	173·65	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

80 Deg.

10 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	98·48	17·36	196·96	34·73	295·44	52·09	393·92	69·46	492·40	86·82	60
3	98·46	17·45	196·93	34·90	295·40	52·35	393·86	69·80	492·33	87·25	57
6	98·45	17·54	196·90	35·08	295·35	52·61	393·80	70·15	492·25	87·68	54
9	98·43	17·62	196·87	35·25	295·30	52·87	393·74	70·49	492·17	88·11	51
12	98·42	17·71	196·84	35·42	295·26	53·13	393·68	70·83	492·10	88·54	48
15	98·40	17·80	196·81	35·59	295·21	53·38	393 62	71·18	492·02	88·97	45
18	98·39	17·88	196·78	35·76	295·17	53·64	393·55	71·52	491·94	89·40	42
21	98·37	17·97	196·75	35·93	295·12	53·90	393·49	71·86	491·86	89·83	39
24	98·36	18·05	196·71	36·11	295·07	54·16	393·43	72·21	491·78	90·26	36
27	98·34	18·14	196·68	36·28	295·02	54·41	393·36	72·55	491·71	90·69	33
30	98·32	18·23	196·65	36·45	294·97	54·67	393·30	72·89	491·63	91·12	30
33	98·31	18·31	196·62	36·62	294·93	54·93	393·24	73·24	491·55	91·55	27
36	98·29	18·40	196·59	36·79	294·88	55·19	393·17	73·58	491·47	91·98	24
39	98·28	18·49	196·55	36·96	294·83	55·44	393·11	73·92	491·39	92·40	21
42	98·26	18·57	196·52	37·13	294·78	55·70	393·04	74·27	491·31	92·83	18
45	98·24	18·66	196·49	37·30	294·73	55·96	392·98	74·61	491·22	93·26	15
48	98·23	18·74	196·46	37·48	294·68	56·21	392·91	74·95	491·14	93·69	12
51	98·21	18·83	196·42	37·65	294·63	56·47	392·85	75·30	491·06	94·12	9
54	98·19	18·91	196·39	37·82	294·58	56·73	392·78	75·64	490·98	94·55	6
57	98·18	19·00	196·36	37·99	294·53	56·99	392·72	75·98	490·90	94·98	3
60	98·16	19·08	196·32	38·16	294·48	57·24	392·65	76·32	490·81	95·40	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

79 Deg.

10 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	590·88	104·19	689·36	121·55	787·84	138·92	886·33	156·28	984·81	173·65	60
3	590·79	104·70	689·26	122·15	787·72	139·60	886·19	157·06	984·65	174·51	57
6	590·70	105·22	689·15	122·76	787·60	140·29	886·05	157·83	984·50	175·37	54
9	590·61	105·73	689·04	123·36	787·48	140·98	885·91	158·60	984·35	176·22	51
12	590·52	106·25	688·94	123·96	787·36	141·67	885·78	159·38	984·19	177·08	48
15	590·42	106·76	688·83	124·56	787·23	142·35	885·64	160·15	984·04	177·94	45
18	590·33	107·28	688·72	125·16	787·11	143·04	885·50	160·92	983·88	178·80	42
21	590·23	107·79	688·61	125·76	786·98	143·73	885·35	161·69	983·73	179·66	39
24	590·14	108·31	688·50	126·36	786·86	144·41	885·21	162·47	983·57	180·52	36
27	590·04	108·83	688·39	126·96	786·73	145·10	885·07	163·24	983·41	181·38	33
30	589·95	109·34	688·28	127·56	786·60	145·79	884·93	164·01	983·25	182·23	30
33	589·85	109·86	688·17	128·16	786·48	146·47	884·79	164·78	983·09	183·09	27
36	589·76	110·37	688·05	128·76	786·35	147·16	884·64	165·56	982·94	183·95	24
39	589·66	110·89	687·94	129·36	786·22	147·85	884·50	166·33	982·77	184·81	21
42	589·56	111·40	687·83	129·96	786·09	148·53	884·35	167·10	982·61	185·67	18
45	589·47	111·91	687·71	130·56	785·96	149·22	884·21	167·87	982·45	186·52	15
48	589·37	112·43	687·60	131·17	785·83	149·90	884·06	168·64	982·29	187·38	12
51	589·27	112·94	687·48	131·77	785·70	150·59	883·91	169·41	982·12	188·24	9
54	589·17	113·46	687·37	132·37	785·57	151·28	883·76	170·19	981·96	189·09	6
57	589·07	113·97	687·25	132·97	785·43	151·96	883·61	170·96	981·79	189·95	3
60	588·97	114·48	687·14	133·57	785·30	152·65	883·46	171·73	981·62	190·81	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

79 Deg.

11 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	98·16	19·08	196·32	38·16	294·48	57·24	392·65	76·32	490·81	95·40	60
3	98·14	19·16	196·29	38·33	294·43	57·50	392·58	76·66	490·73	95·83	57
6	98·13	19·25	196·26	38·51	294·39	57·76	392·52	77·01	490·64	96·26	54
9	98·11	19·34	196·22	38·68	294·34	58·01	392·45	77·35	490·56	96·69	51
12	98·09	19·42	196·19	38·85	294·28	58·27	392·38	77·69	490·48	97·12	48
15	98·08	19·51	196·16	39·02	294·23	58·53	392·31	78·04	490·39	97·54	45
18	98·06	19·59	196·12	39·19	294·18	58·78	392·24	78·38	490·31	97·97	42
21	98·04	19·68	196·09	39·36	294·13	59·04	392·18	78·72	490·22	98·40	39
24	98·02	19·76	196·05	39·53	294·08	59·30	392·11	79·06	490·13	98·83	36
27	98·00	19·85	196·02	39·70	294·03	59·55	392·04	79·40	490·05	99·26	33
30	97·99	19·94	195·98	39·88	293·98	59·81	391·97	79·75	489·96	99·68	30
33	97·97	20·02	195·95	40·05	293·92	60·06	391·90	80·09	489·87	100·11	27
36	97·95	20·11	195·91	40·22	293·87	60·32	391·83	80·43	489·79	100·54	24
39	97·94	20·19	195·88	40·39	293·82	60·58	391·76	80·77	489·70	100·96	21
42	97·92	20·28	195·84	40·56	293·77	60·84	391·69	81·11	489·61	101·39	18
45	97·90	20·36	195·81	40·73	293·71	61·09	391·62	81·45	489·52	101·82	15
48	97·89	20·45	195·77	40·90	293·66	61·35	391·54	81·80	489·43	102·25	12
51	97·87	20·53	195·74	41·07	293·61	61·61	391·47	82·14	489·34	102·68	9
54	97·85	20·62	195·70	41·24	293·55	61·86	391·40	82·48	489·25	103·10	6
57	97·83	20·71	195·66	41·14	293·50	62·12	391·33	82·82	489·16	103·53	3
60	97·81	20·79	195·63	41·58	293·44	62·37	391·26	83·16	489·07	103·95	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

78 Deg.

11 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	588·97	114·48	687·14	133·57	785·30	152·65	883·46	171·73	981·62	190·81	60
3	588·87	115·00	687·02	134·17	785·17	153·33	883·31	172·50	981·46	191·66	57
6	588·77	115·51	686·90	134·76	785·03	154·08	883·16	173·27	981·29	192·52	54
9	588·67	116·03	686·79	135·36	784·90	154·70	883·01	174·04	981·12	193·38	51
12	588·57	116·54	686·67	135·96	784·76	155·40	882·86	174·81	980·95	194·23	48
15	583·47	117·05	686·55	136·56	784·63	156·09	882·71	175·58	980·78	195·09	45
18	588·37	117·57	686·43	137·16	784·49	156·76	882·55	176·35	980·61	195·95	42
21	588·27	118·08	686·31	137·76	784·35	157·44	882·40	177·12	980·44	196·80	39
24	588·16	118·59	686·19	138·36	784·22	158·13	882·24	177·89	980·27	197·66	36
27	588·06	119·11	686·07	138·96	784·08	158·81	882·09	178·66	980·10	198·51	33
30	587·95	119·62	685·95	139·56	783·94	159·49	881·93	179·42	979·92	199·37	30
33	587·85	120·13	685·82	140·16	783·80	160·18	881·77	180·19	979·75	200·22	27
36	587·74	120·65	685·70	140·75	783·66	160·86	881·62	180·97	979·57	201·08	24
39	587·64	121·16	685·58	141·35	783·52	161·55	881·46	181·74	979·40	201·93	21
42	587·53	121·67	685·45	141·95	783·38	162·23	881·30	182·51	979·22	202·79	18
45	587·42	122·18	685·33	142·55	783·23	162·91	881·14	183·28	979·04	203·64	15
48	587·32	122·70	685·21	143·15	783·09	163·60	880·98	184·05	978·86	204·49	12
51	587·21	123·21	685·08	143·74	782·95	164·28	880·82	184·82	978·69	205·35	9
54	587·10	123·72	684·95	144·34	782·81	164·96	880·66	185·58	978·51	206·20	6
57	587·00	124·23	684·83	144·94	782·66	165·65	880·49	186·35	978·33	207·06	3
60	586·89	124·75	684·70	145·54	782·52	166·33	880·33	187·12	978·15	207·91	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Minutes.
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

78 Deg.

12 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	97·81	20·79	195·63	41·58	293·44	62·37	391·26	83·16	489·07	103·95	60
3	97·80	20·88	195·59	41·76	293·39	62·63	391·18	83·51	488·98	104·38	57
6	97·78	20·96	195·56	41·92	293·33	62·87	391·11	83·85	488·89	104·81	54
9	97·76	21·05	195·52	42·10	293·28	63·14	391·04	84·19	488·80	105·24	51
12	97·74	21·13	195·48	42·27	293·22	63·40	390·96	84·53	488·71	105·66	48
15	97·72	21·22	195·44	42·44	293·17	63·66	390·89	84·87	488·61	106·09	45
18	97·70	21·30	195·41	42·61	293·11	63·91	390·82	85·21	488·52	106·51	42
21	97·68	21·39	195·37	42·78	293·06	64·17	390·74	85·55	488·43	106·94	39
24	97·67	21·47	195·33	42·95	293·00	64·42	390·67	85·89	488·33	107·37	36
27	97·65	21·56	195·30	43·12	292·94	64·68	390·59	86·24	488·24	107·79	33
30	97·63	21·64	195·26	43·29	292·89	64·93	390·52	86·58	488·15	108·22	30
33	97·61	21·73	195·22	43·46	292·83	65·19	390·44	86·92	488·05	108·64	27
36	97·59	21·81	195·18	43·63	292·77	65·44	390·37	87·26	487·96	109·07	24
39	97·57	21·90	195·14	43·80	292·72	65·70	390·29	87·60	487·86	109·50	21
42	97·55	21·98	195·11	43·97	292·66	65·95	390·21	87·94	487·77	109·93	18
45	97·53	22·07	195·07	44·14	292·60	66·21	390·14	88·28	487·67	110·35	15
48	97·51	22·15	195·03	44·31	292·54	66·47	390·06	88·62	487·57	110·77	12
51	97·49	22·24	194·99	44·48	292·49	66·72	389·98	88·96	487·48	111·20	9
54	97·48	22·33	194·95	44·65	292·43	66·98	389·90	89·30	487·38	111·62	6
57	97·46	22·41	194·91	44·82	292·37	67·23	389·83	89·64	487·28	112·15	3
60	97·44	22·49	194·87	44·99	292·31	67·49	389·75	89·98	487·18	112·47	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

77 Deg.

12 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	586·89	124·75	684·70	145·54	782·52	166·33	880·33	187·12	978·15	207·91	60
3	586·78	125·26	684·57	146·13	782·37	167·01	880·17	187·89	977·96	208·76	57
6	586·67	125·77	684·45	146·73	782·22	167·69	880·00	188·66	977·78	209·62	54
9	586·56	126·28	684·32	147·33	782·08	168·38	879·84	189·42	977·60	210·47	51
12	586·45	126·79	684·19	147·93	781·93	169·06	879·67	190·19	977·41	211·32	48
15	586·34	127·31	684·06	148·52	781·78	169·74	879·51	190·96	977·23	212·18	45
18	586·23	127·82	683·93	149·12	781·63	170·42	879·34	191·73	977·04	213·03	42
21	586·11	128·33	683·80	149·72	781·48	171·10	879·17	192·49	976·86	213·88	39
24	586·00	128·84	683·67	150·31	781·33	171·79	879·00	193·26	976·67	214·73	36
27	585·89	129·35	683·54	150·91	781·18	172·47	878·84	194·03	976·48	215·59	33
30	585·78	129·86	683·41	151·51	781·03	173·15	878·67	194·79	976·29	216·44	30
33	585·66	130·37	683·27	152·10	780·88	173·83	878·50	195·56	976·10	217·29	27
36	585·55	130·88	683·14	152·70	780·73	174·51	878·32	196·33	975·91	218·14	24
39	585·43	131·40	683·01	153·30	780·58	175·19	878·15	197·09	975·72	218·99	21
42	585·32	131·91	682·87	153·89	780·43	175·88	877·98	197·86	975·53	219·85	18
45	585·20	132·42	682·74	154·49	780·27	176·56	877·81	198·63	975·34	220·70	15
48	585·09	132·93	682·60	155·08	780·12	177·24	877·63	199·39	975·15	221·55	12
51	584·97	133·44	682·47	155·68	779·96	177·92	877·46	200·16	974·95	222·40	9
54	584·86	133·95	682·33	156·27	779·81	178·60	877·28	200·92	974·76	223·25	6
57	584·74	134·46	682·19	156·87	779·65	179·28	877·11	201·70	974·56	224·11	3
60	584·62	134·97	682·06	157·46	779·49	179·96	876·93	202·45	974·37	224·95	0
	Base.	Per.	Base.	Per	Base.	Per.	Base.	Per.	Base.	Per.	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Minutes
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

77 Deg.

13 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	97·44	22·49	194·87	44·99	292·31	67·49	389·75	89·98	487·18	112·47	60
3	97·42	22·58	194·83	45·16	292·25	67·74	389·67	90·32	487·09	112·90	57
6	97·40	22·66	194·79	45·33	292·19	67·99	389·59	90·66	486·99	113·32	54
9	97·38	22·75	194·75	45·50	292·13	68·25	389·51	91·00	486·89	113·75	51
12	97·36	22·83	194·71	45·67	292·07	68·51	389·43	91·34	486·79	114·17	48
15	97·34	22·92	194·67	45·84	292·01	68·76	389·35	91·68	486·69	114·60	45
18	97·32	23·00	194·63	46·01	291·95	69·01	389·27	92·02	486·59	115·02	42
21	97·30	23·09	194·59	46·18	291·89	69·27	389·19	92·36	486·49	115·45	39
24	97·28	23·17	194·55	46·35	291·83	69·52	389·11	92·70	486·39	115·87	36
27	97·26	23·26	194·51	46·52	291·77	69·78	389·03	93·04	486·29	116·30	33
30	97·24	23·34	194·47	46·69	291·71	70·03	388·95	93·38	486·18	116·72	30
33	97·22	23·43	194·43	46·86	291·65	70·29	388·86	93·72	486·08	117·15	27
36	97·20	23·51	194·39	47·03	291·59	70·54	388·78	94·06	485·98	117·57	24
39	97·18	23·60	194·35	47·20	291·52	70·80	388·70	94·40	485·88	117·99	21
42	97·15	23·68	194·31	47·37	291·46	71·05	388·62	94·74	485·77	118·42	18
45	97·13	23·77	194·27	47·54	291·40	71·31	388·54	95·07	485·67	118·84	15
48	97·11	23·85	194·23	47·71	291·34	71·56	388·45	95·41	485·57	119·27	12
51	97·09	23·94	194·18	47·88	291·28	71·81	388·37	95·75	485·46	119·69	9
54	97·07	24·02	194·14	48·05	291·21	72·07	388·29	96·09	485·36	120·11	6
57	97·05	24·11	194·10	48·22	291·15	72·32	388·20	96·43	485·25	120·54	3
60	97·03	24·19	194·06	48·39	291·09	72·58	388·12	96·77	485·15	120·96	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

76 Deg.

13 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	584·62	134·97	682·06	157·46	779·49	179·96	876·93	202·45	974·37	224·95	60
3	584·50	135·48	681·92	158·06	779·34	180·64	876·75	203·22	974·17	225·80	57
6	584·38	135·99	681·78	158·65	779·18	181·32	876·58	203·99	973·97	226·65	54
9	584·26	136·50	681·64	159·25	779·02	182·00	876·40	204·75	973·78	227·50	51
12	584·15	137·01	681·50	159·85	778·86	182·68	876·22	205·51	973·58	228·35	48
15	584·03	137·52	681·36	160·44	778·70	183·36	876·04	206·28	973·38	229·20	45
18	583·91	138·03	681·22	161·03	778·54	184·04	875·86	207·04	973·18	230·05	42
21	583·78	138·54	681·08	161·63	778·38	184·72	875·68	207·81	972·98	230·90	39
24	583·66	139·05	680·94	162·22	778·22	185·40	875·50	208·57	972·77	231·75	36
27	583·54	139·56	680·80	162·82	778·06	186·08	875·31	209·34	972·57	232·60	33
30	583·42	140·07	680·66	163·41	777·89	186·76	875·13	210·10	972·37	233·45	30
33	583·30	140·57	680·51	164·00	777·73	187·44	874·95	210·86	972·16	234·29	27
36	583·17	141·08	680·37	164·60	777·57	188·11	874·76	211·62	971·96	235·14	24
39	583·05	141·59	680·23	165·19	777·40	188·79	874·57	212·39	971·75	235·99	21
42	582·93	142·10	680·08	165·79	777·24	189·47	874·39	213·15	971·55	236·84	18
45	582·80	142·61	679·94	166·38	777·07	190·15	874·21	213·92	971·34	237·69	15
48	582·68	143·12	679·80	166·97	776·91	190·83	874·02	214·68	971·13	238·53	12
51	582·55	143·63	679·65	167·57	776·74	191·50	873·83	215·44	970·92	239·38	9
54	582·43	144·14	679·50	168·16	776·57	192·18	873·64	216·20	970·72	240·23	6
57	582·30	144·64	679·35	168·75	776·40	192·86	873·46	216·97	970·51	241·07	3
60	582·18	145·15	679·21	169·34	776·23	193·54	873·27	217·73	970·30	241·92	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

76 Deg.

14 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	97·03	24·19	194·06	48·39	291·09	72·58	388·12	96·77	485·15	120·96	60
3	97·01	24·28	194·02	48·55	291·02	72·83	388·03	97·11	485·04	121·38	57
6	96·99	24·36	193·97	48·72	290·96	73·08	387·95	97·44	484·94	121·81	54
9	96·96	24·45	193·93	48·89	290·90	73·34	387·86	97·78	484·83	122·23	51
12	96·94	24·53	193·89	49·06	290·83	73·59	387·78	98·12	484·72	122·65	48
15	96·92	24·62	193·84	49·23	290·77	73·85	387·69	98·46	484·61	123·08	45
18	96·90	24·70	193·80	49·40	290·70	74·10	387·60	98·80	484·51	123·50	42
21	96·88	24·78	193·76	49·57	290·64	74·35	387·52	99·14	484·40	123·92	39
24	96·86	24·87	193·72	49·74	290·57	74·61	387·43	99·48	484·29	124·34	36
27	96·84	24·95	193·67	49·91	290·51	74·86	387·34	99·81	484·18	124·77	33
30	96·81	25·04	193·63	50·08	290·44	75·11	387·26	100·15	484·07	125·19	30
33	96·79	25·12	193·58	50·25	290·38	75·37	387·17	100·49	483·96	125·61	27
36	96·77	25·21	193·54	50·42	290·31	75·62	387·08	100·83	483·85	126·03	24
39	96·75	25·29	193·50	50·58	290·25	75·87	386·99	101·16	483·74	126·46	21
42	96·73	25·37	193·45	50·75	290·18	76·13	386·91	101·50	483·63	126·88	18
45	96·70	25·46	193·41	50·92	290·11	76·38	386·82	101·84	483·52	127·30	15
48	96·68	25·54	193·36	51·09	290·05	76·63	386·73	102·18	483·41	127·72	12
51	96·66	25·63	193·32	51·26	289·98	76·89	386·64	102·51	483·30	128·14	9
54	96·64	25·71	193·27	51·43	289·91	77·14	386·55	102·85	483·19	128·57	6
57	96·62	25·80	193·23	51·60	289·84	77·39	386·46	103·19	483·07	128·99	3
60	96·59	25·88	193·18	51·76	289·78	77·65	386·37	103·53	482·96	129·41	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

75 Deg.

14 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	582·18	145·15	679·21	169·34	776·23	193·54	873·27	217·73	970·29	241·92	60
3	582·05	145·66	679·06	169·94	776·07	194·21	873·07	218·49	970·08	242·77	57
6	581·92	146·17	678·91	170·53	775·90	194·89	872·88	219·25	969·87	243·61	54
9	581·79	146·68	678·76	171·12	775·73	195·57	872·69	220·01	969·66	244·46	51
12	581·67	147·18	678·61	171·71	775·55	196·24	872·50	220·77	969·44	245·31	48
15	581·54	147·69	678·46	172·31	775·38	196·92	872·31	221·54	969·23	246·15	45
18	581·41	148·20	678·31	172·90	775·21	197·60	872·11	222·30	969·01	247·00	42
21	581·28	148·71	678·16	173·49	775·04	198·27	871·92	223·06	968·80	247·84	39
24	581·15	149·21	678·01	174·08	774·86	198·95	871·72	223·82	968·58	248·69	36
27	581·02	149·72	677·85	174·67	774·69	199·63	871·53	224·58	968·36	249·53	33
30	580·89	150·23	677·70	175·27	774·52	200·30	871·33	225·34	968·14	250·38	30
33	580·75	150·73	677·55	175·86	774·34	200·98	871·13	226·10	967·93	251·22	27
36	580·62	151·24	677·39	176·45	774·17	201·66	870·94	226·86	967·71	252·07	24
39	580·49	151·75	677·24	177·04	773·99	202·33	870·74	227·62	967·49	252·91	21
42	580·36	152·25	677·09	177·63	773·81	203·01	870·54	228·38	967·27	253·76	18
45	580·23	152·76	676·93	178·22	773·64	203·68	870·34	229·14	967·04	254·60	15
48	580·09	153·27	676·77	178·81	773·46	204·36	870·14	229·90	966·82	255·45	12
51	579·96	153·77	676·62	179·40	773·28	205·03	869·94	230·66	966·60	256·29	9
54	579·82	154·28	676·46	179·99	773·10	205·71	869·74	231·43	966·37	257·13	6
57	579·69	154·78	676·30	180·58	772·92	206·38	869·54	232·19	966·15	257·97	3
60	579·55	155·29	676·15	181·17	772·74	207·05	869·33	232·94	965·92	258·82	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

75 Deg.

15 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	96·59	25·88	193·18	51·76	289·78	77·65	386·37	103·53	482·96	129·41	60
3	96·57	25·97	193·14	51·93	289·71	77·90	386·28	103·86	482·85	129·83	57
6	96·55	26·05	193·09	52·10	289·64	78·15	386·19	104·20	482·74	130·25	54
9	96·52	26·13	193·05	52·27	289·57	78·40	386·10	104·54	482·62	130·67	51
12	96·50	26·22	193·00	52·44	289·50	78·66	386·01	104·87	482·51	131·09	48
15	96·48	26·30	192·96	52·61	289·44	78·91	385·91	105·21	482·40	131·52	45
18	96·45	26·39	192·91	52·78	289·37	79·16	385·82	105·55	482·28	131·94	42
21	96·43	26·47	192·86	52·94	289·30	79·41	385·73	105·87	482·16	132·36	39
24	96·41	26·55	192·82	53·11	289·23	79·67	385·64	106·22	482·05	132·78	36
27	96·39	26·64	192·77	53·28	289·16	79·92	385·55	106·56	481·93	133·20	33
30	96·36	26·72	192·73	53·45	289·09	80·17	385·45	106·89	481·81	133·62	30
33	96·34	26·81	192·68	53·62	289·02	80·42	385·36	107·23	481·70	134·04	27
36	96·32	26·89	192·63	53·78	288·95	80·67	385·26	107·57	481·58	134·46	24
39	96·29	26·98	192·58	53·95	288·88	80·93	385·17	107·90	481·46	134·88	21
42	96·27	27·06	192·54	54·12	288·81	81·18	385·08	108·24	481·34	135·30	18
45	96·24	27·14	192·49	54·29	288·73	81·43	384·98	108·58	481·23	135·72	15
48	96·22	27·23	192·44	54·46	288·66	81·68	384·89	108·91	481·11	136·14	12
51	96·20	27·31	192·40	54·62	288·59	81·94	384·79	109·25	480·99	136·56	9
54	96·17	27·39	192·35	54·79	288·52	82·19	384·70	109·58	480·87	136·98	6
57	96·15	27·48	192·30	54·96	288·45	82·44	384·60	109·92	480·75	137·40	3
60	96·13	27·56	192·25	55·13	288·38	82·69	384·50	110·25	480·63	137·82	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

74 Deg.

15 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	579·55	155·29	676·15	181·17	772·74	207·05	869·33	232·94	965·92	258·82	60
3	579·42	155·80	675·99	181·76	772·56	207·73	869·13	233·70	965·70	259·66	57
6	579·28	156·30	675·83	182·35	772·38	208·40	868·93	234·45	965·47	260·50	54
9	579·14	156·81	675·67	182·94	772·19	209·08	868·72	235·21	965·24	261·35	51
12	579·01	157·31	675·51	183·53	772·01	209·75	868·51	235·97	965·01	262·19	48
15	578·87	157·82	675·35	184·12	771·83	210·42	868·31	236·73	964·77	263·03	45
18	578·73	158·32	675·19	184·71	771·65	211·10	868·10	237·49	964·56	263·87	42
21	578·60	158·83	675·03	185·30	771·46	211·77	867·89	238·24	964·33	264·71	39
24	578·46	159·33	674·87	185·89	771·28	212·44	867·69	239·00	964·09	265·55	36
27	578·32	159·84	674·70	186·48	771·09	213·12	867·48	239·76	963·86	266·40	33
30	578·18	160·34	674·54	187·07	770·90	213·79	867·27	240·51	963·63	267·24	30
33	578·04	160·85	674·38	187·65	770·72	214·46	867·06	241·27	963·40	268·08	27
36	577·90	161·35	674·21	188·24	770·53	215·13	866·85	242·03	963·16	268·92	24
39	577·75	161·86	674·05	188·83	770·34	215·81	866·63	242·78	962·93	269·76	21
42	577·61	162·36	673·88	189·42	770·15	216·48	866·42	243·54	962·69	270·60	18
45	577·47	162·86	673·72	190·01	769·96	217·15	866·21	244·30	962·45	271·44	15
48	577·33	163·37	673·55	190·60	769·77	217·82	866·00	245·05	962·22	272·28	12
51	577·19	163·87	673·38	191·19	769·58	218·50	865·78	245·81	961·98	273·12	9
54	577·04	164·37	673·22	191·77	769·39	219·17	865·57	246·56	961·74	273·96	6
57	576·90	164·88	673·05	192·36	769·20	219·84	865·35	247·32	961·50	274·80	3
60	576·76	165·38	672·88	192·95	769·01	220·51	865·14	248·07	961·26	275·64	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

74 Deg.

16 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	96·13	27·56	192·25	55·13	288·38	82·69	384·50	110·25	9 3	137·82	60
3	96·10	27·65	192·20	55·29	288·30	82·94	384·41	110·5	97 51	138·24	57
6	96·08	27·73	192·15	55·46	288·23	83·19	384·31	110·99	97 39	138·66	54
9	96·05	27·81	192·11	55·63	288·16	83·44	384·21	111·2	97 27	139·08	51
12	96·03	27·90	192·06	55·80	288·09	83·72	384·12	111	980·15	139·49	48
15	96·00	27·98	192·01	55·97	288·01	83·97	384·02	111· 8	9 03	139·91	45
18	95·98	28·07	191·96	56·13	287·94	84·20	383·92	112·27	979·90	140·33	42
21	95·95	28·15	191·91	56·30	287·87	84·45	383·82	112·	979·78	140·75	39
24	95·93	28·23	191·86	56·47	287·79	84·70	383·73	112·94	979·66	141·17	36
27	95·91	28·32	191·81	56·63	287·72	84·95	383·63	113·22	979·53	141·59	33
30	95·88	28·40	191·76	56·80	287·65	85·20	383·53	113·60	979·41	142·01	30
33	95·86	28·48	191·71	56·97	287·57	85·45	383·43	113·86	979·28	142·43	27
36	95·83	28·57	191·66	57·14	287·50	85·71	383·33	114·27	979·16	142·84	24
39	95·81	28·65	191·61	57·30	287·42	85·96	383·23	114·39	979·03	143·26	21
42	95·78	28·74	191·56	57·47	287·35	86·21	383·13	114·	978·91	143·68	18
45	95·76	28·82	191·51	57·64	287·27	86·46	383·03	115·2	978·78	144·10	15
48	95·73	28·90	191·46	57·81	287·20	86·71	382·93	115·61	978·66	144·51	12
51	95·71	28·99	191·41	57·97	287·12	86·96	382·83	115·95	978·53	144·93	9
54	95·68	29·07	191·36	58·14	287·04	87·21	382·72	116·28	978·40	145·35	6
57	95·65	29·15	191·31	58·31	286·97	87·46	382·62	116·62	78·28	145·77	3
60	95·63	29·24	191·26	58·47	286·89	87·71	382·52	116·95	78·15	146·18	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

73 Deg.

16 Deg.

t. 600.	Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
165·38	672·88	192·95	769·01	220·51	865·14	248·07	961·26	275·64	60
165·88	672·71	193·53	768·82	221·18	864·92	248·83	961·02	276·48	57
166·39	672·54	194·12	768·62	221·85	864·70	249·58	960·78	277·31	54
166·89	672·37	194·71	768·43	222·52	864·48	250·34	960·53	278·15	51
167·39	672·20	195·29	768·23	223·19	864·27	251·09	960·29	278·99	48
167·90	672·03	195·88	768·04	223·86	864·05	251·84	960·05	279·83	45
168·40	671·86	196·47	767·84	224·53	863·83	252·60	959·80	280·67	42
168·90	671·69	197·05	767·65	225·20	863·61	253·35	959·56	281·50	39
169·40	671·52	197·64	767·45	225·87	863·39	254·11	959·31	282·34	36
169·91	671·35	198·22	767·25	226·54	863·16	254·86	959·07	283·18	33
170·41	671·17	198·81	767·05	227·21	862·94	255·61	958·82	284·01	30
170·91	671·00	199·40	766·85	227·88	862·71	256·38	958·57	284·85	27
171·41	670·82	199·98	766·66	228·55	862·49	257·12	958·32	285·69	24
171·91	670·65	200·57	766·46	229·22	862·27	257·87	958·07	286·52	21
172·42	670·47	201·15	766·26	229·89	862·04	258·62	957·82	287·36	18
172·92	670·30	201·74	766·05	230·56	861·81	259·37	957·57	288·20	15
173·42	670·12	202·32	765·85	231·23	861·59	260·13	957·32	289·03	12
173·92	669·95	202·91	765·65	231·89	861·36	260·88	957·07	289·87	9
174·42	669·77	203·49	765·45	232·56	861·13	261·63	956·81	290·70	6
174·92	669·59	204·08	765·25	233·23	860·90	262·38	956·56	291·54	3
175·42	669·41	204·66	765·04	233·90	860·68	263·13	956·30	292·37	0
Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
t. 600.	Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

73 Deg.

17 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	95·63	29·24	191·26	58·47	286·89	87·71	382·52	116·95	478·15	146·18	60
3	95·60	29·32	191·21	58·64	286·81	87·96	382·42	117·28	478·02	146·60	57
6	95·58	29·40	191·16	58·81	286·74	88·21	382·32	117·62	477·89	147·02	54
9	95·55	29·49	191·11	58·97	286·66	88·46	382·21	117·95	477·77	147·44	51
12	95·53	29·57	191·05	59·14	286·58	88·71	382·11	118·28	477·64	147·85	48
15	95·50	29·65	191·00	59·31	286·50	88·96	382·01	118·62	477·54	148·27	45
18	95·47	29·74	190·95	59·47	286·43	89·21	381·90	118·95	477·38	148·69	42
21	95·45	29·82	190·90	59·64	286·35	89·46	381·80	119·28	477·25	149·10	39
24	95·42	29·90	190·85	59·81	286·27	89·71	381·70	119·62	477·12	149·52	36
27	95·40	29·99	190·80	59·97	286·19	89·96	381·59	119·95	476·99	149·94	33
30	95·37	30·07	190·74	60·14	286·11	90·21	381·49	120·28	476·86	150·35	30
33	95·34	30·15	190·69	60·31	286·03	90·46	381·38	120·62	476·73	150·77	27
36	95·32	30·24	190·64	60·47	285·96	90·71	381·27	120·95	476·59	151·18	24
39	95·29	30·32	190·58	60·64	285·88	90·96	381·17	121·28	476·46	151·60	21
42	95·26	30·40	190·53	60·81	285·80	91·21	381·06	121·61	476·33	152·02	18
45	95·24	30·49	190·48	60·97	285·72	91·46	380·96	121·94	476·20	152·43	15
48	95·21	30·57	190·43	61·14	285·64	91·71	380·85	122·27	476·06	152·85	12
51	95·18	30·65	190·37	61·30	285·56	91·96	380·74	122·61	475·93	153·26	9
54	95·16	30·73	190·32	61·47	285·48	92·21	380·64	122·94	475·80	153·68	6
57	95·13	30·82	190·26	61·64	285·40	92·45	380·53	123·27	475·66	154·09	3
60	95·10	30·90	190·21	61·80	285·32	92·70	380·42	123·61	475·53	154·51	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

72 Deg.

17 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	573·78	175·42	669·41	204·66	765·04	233·90	860·68	263·13	956·30	292·37	60
3	573·63	175·92	669·23	205·24	764·84	234·56	860·44	263·88	956·05	293·20	57
6	573·47	176·42	669·05	205·83	764·63	235·23	860·21	264·64	955·79	294·04	54
9	573·32	176·92	668·87	206·41	764·43	235·90	859·98	265·39	955·53	294·87	51
12	573·16	177·42	668·69	207·00	764·22	236·56	859·75	266·14	955·28	295·71	48
15	573·01	177·92	668·51	207·58	764·01	237·23	859·52	266·89	955·02	296·54	45
18	572·85	178·42	668·33	208·16	763·81	237·90	859·28	267·64	954·76	297·37	42
21	572·70	178·92	668·15	208·74	763·60	238·56	859·05	268·39	954·50	298·21	39
24	572·54	179·42	667·97	209·33	763·39	239·23	858·82	269·14	954·24	299·04	36
27	572·39	179·92	667·78	209·91	763·18	239·90	858·58	269·88	953·98	299·87	33
30	572·23	180·42	667·60	210·49	762·98	240·56	858·34	270·63	953·72	300·70	30
33	572·07	180·92	667·42	211·08	762·76	241·23	858·11	271·38	953·45	301·53	27
36	571·91	181·42	667·23	211·66	762·55	241·90	857·87	272·13	953·19	302·37	24
39	571·75	181·92	667·05	212·24	762·34	242·56	857·63	272·88	952·92	303·20	21
42	571·60	182·42	666·86	212·82	762·13	243·22	857·39	273·63	952·66	304·03	18
45	571·44	182·92	666·68	213·40	761·92	243·89	857·15	274·38	952·39	304·86	15
48	571·28	183·42	666·49	213·99	761·70	244·55	856·91	275·13	952·13	305·69	12
51	571·12	183·92	666·30	214·57	761·49	245·22	856·67	275·87	951·86	306·53	9
54	570·95	184·41	666·11	215·15	761·27	245·88	856·43	276·62	951·59	307·36	6
57	570·79	184·91	665·93	215·73	761·06	246·55	856·19	277·37	951·32	308·19	3
60	570·63	185·41	665·74	216·31	760·84	247·21	855·95	278·11	951·05	309·02	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Minutes.
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

72 Deg.

18 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	95·10	30·90	190·21	61·80	285·32	92·71	380·42	123·61	475·53	154·51	60
3	95·08	30·98	190·16	61·97	285·23	92·95	380·31	123·94	475·39	154·92	57
6	95·05	31·07	190·10	62·13	285·15	93·20	380·21	124·27	475·26	155·34	54
9	95·02	31·15	190·05	62·30	285·07	93·45	380·10	124·60	475·12	155·75	51
12	95·00	31·23	189·99	62·47	284·99	93·70	379·99	124·93	474·98	156·17	48
15	94·97	31·32	189·94	62·63	284·91	93·95	379·88	125·26	474·85	156·58	45
18	94·94	31·40	189·88	62·80	284·83	94·20	379·77	125·60	474·71	156·99	42
21	94·91	31·48	189·83	62·96	284·74	94·44	379·66	125·93	474·57	157·41	39
24	94·88	31·56	189·77	63·13	284·66	94·69	379·55	126·26	474·44	157·82	36
27	94·86	31·65	189·72	63·29	284·58	94·94	379·44	126·59	474·30	158·24	33
30	94·83	31·73	189·66	63·46	284·50	95·19	379·33	126·92	474·16	158·65	30
33	94·80	31·81	189·61	63·63	284·41	95·44	379·22	127·25	474·02	159·07	27
36	94·78	31·89	189·55	63·79	284·33	95·69	379·11	127·58	473·88	159·48	24
39	94·75	31·98	189·50	63·96	284·25	95·94	378·99	127·91	473·74	159·89	21
42	94·72	32·06	189·44	64·12	284·16	96·18	378·88	128·24	473·60	160·31	18
45	94·69	32·14	189·38	64·29	284·08	96·43	378·77	128·58	473·46	160·72	15
48	94·66	32·23	189·33	64·45	283·99	96·68	378·66	128·91	473·32	161·13	12
51	94·64	32·31	189·27	64·62	283·91	96·93	378·55	129·24	473·18	161·54	9
54	94·61	32·39	189·22	64·78	283·82	97·17	378·43	129·57	473·04	161·96	6
57	94·58	32·47	189·16	64·95	283·74	97·42	378·32	129·90	472·90	162·37	3
60	94·55	32·56	189·10	65·11	283·65	97·67	378·21	130·23	472·76	162·78	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

71 Deg.

18 Deg.

Minutes.		Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
		Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
		Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
5	0	70·62	185·41	665·74	176·31	760·84	167·21	855·95	158·11	951·05	149·02	60
5	3	70·47	185·91	665·58	176·89	760·63	167·88	855·71	158·86	950·78	149·85	57
5	6	70·33	186·41	665·36	177·47	760·40	168·54	855·46	159·61	950·51	150·68	54
5	9	70·14	186·90	665·19	178·05	760·19	169·20	855·22	160·36	950·24	151·50	51
5	12		187·40	664·98	178·63	759·98	169·87	854·97	161·10	949·97	152·33	48
	15		187·90	664·79	179·21	759·76	170·53	854·73	161·85	949·70	153·16	45
5	18	569·65	188·40	664·60	179·79	759·54	171·19	854·48	162·59	949·42	153·99	42
5	21	493	188·89	664·45	180·37	759·32	171·86	854·23	163·34	949·15	154·82	39
	24	32	189·39	664·29	180·95	759·14	172·52	853·99	164·08	948·87	155·65	36
5	27	59	189·89	664·08	181·53	758·88	173·18	853·74	164·83	948·60	156·48	33
5	30	34	190·38	663·83	182·11	758·66	173·84	853·49	165·57	948·32	157·30	30
	33	3	190·88	663·63	182·69	758·44	174·50	853·24	166·32	948·04	158·13	27
5	36	568·66	191·37	663·44	183·27	758·22	175·17	852·99	167·06	947·77	158·96	24
5	39	·49	191·87	663·24	183·85	757·99	175·83	852·74	167·81	947·49	159·78	21
5	42	·33	192·37	663·05	184·43	757·76	176·49	852·49	168·55	947·21	160·61	18
5	45	568·18	192·86	662·85	185·01	757·54	177·15	852·23	169·29	946·93	161·44	15
5	48	7·99	193·36	662·65	185·58	757·32	177·81	851·98	170·04	946·65	162·26	12
	51	·82	193·85	662·46	186·16	757·09	178·47	851·73	170·78	946·37	163·09	9
	54	·65	194·35	662·26	186·74	756·87	179·13	851·48	171·52	946·08	163·92	6
5	57	·48	194·84	662·06	187·32	756·64	179·79	851·22	172·27	945·80	164·74	3
5	60	67·31	195·34	661·86	187·90	756·41	180·45	850·97	173·01	945·52	165·57	0
		Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	
		Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Minutes.
		Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

71 Deg.

19 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	94·55	32·56	189·10	65·11	283·65	97·67	378·21	130·23	472·76	162·78	60
3	94·52	32·64	189·05	65·28	283·57	97·92	378·09	130·56	472·62	163·20	57
6	94·49	32·72	188·99	65·44	283·48	98·16	377·98	130·89	472·47	163·61	54
9	94·47	32·80	188·93	65·61	283·40	98·41	377·86	131·22	472·33	164·02	51
12	94·44	32·89	188·87	65·77	283·31	98·66	377·75	131·55	472·19	164·43	48
15	94·41	32·97	188·82	65·94	283·23	98·91	377·63	131·88	472·04	164·84	45
18	94·38	33·05	188·76	66·10	283·14	99·15	377·52	132·20	471·90	165·26	42
21	94·35	33·14	188·70	66·27	283·05	99·40	377·40	132·53	471·75	165·67	39
24	94·32	33·22	188·64	66·43	282·97	99·65	377·29	132·86	471·61	166·08	36
27	94·29	33·30	188·59	66·60	282·88	99·89	377·17	133·19	471·46	166·49	33
30	94·26	33·38	188·53	66·76	282·79	100·14	377·06	133·52	471·32	166·90	30
33	94·23	33·46	188·47	66·93	282·70	100·39	376·94	133·85	471·17	167·31	27
36	94·20	33·54	188·41	67·09	282·62	100·63	376·82	134·18	471·03	167·72	24
39	94·18	33·63	188·35	67·25	282·53	100·88	376·70	134·51	470·88	168·14	21
42	94·15	33·71	188·29	67·42	282·44	101·13	376·59	134·84	470·73	168·55	18
45	94·12	33·79	188·23	67·58	282·35	101·37	376·47	135·17	470·59	168·96	15
48	94·09	33·87	188·18	67·75	282·26	101·62	376·35	135·49	470·44	169·37	12
51	94·06	33·95	188·12	67·91	282·17	101·87	376·23	135·82	470·29	169·78	9
54	94·03	34·04	188·06	68·07	282·09	102·11	376·11	136·15	470·14	170·19	6
57	94·00	34·12	188·00	68·24	282·00	102·36	375·99	136·48	469·99	170·60	3
60	93·97	34·20	187·94	68·40	281·91	102·60	375·87	136·81	469·84	171·01	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

70 Deg.

19 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	567·31	195·34	661·86	227·90	756·41	260·45	850·97	293·01	945·52	325·57	60
3	567·14	195·83	661·66	228·47	756·19	261·11	850·71	293·75	945·23	326·39	57
6	566·97	196·33	661·46	229·05	755·96	261·77	850·45	294·49	944·95	327·22	54
9	566·80	196·83	661·26	229·63	755·73	262·43	850·20	295·24	944·66	328·04	51
12	566·62	197·32	661·06	230·21	755·50	263·09	849·94	295·98	944·38	328·87	48
15	566·45	197·81	660·86	230·78	755·27	263·75	849·68	296·72	944·09	329·69	45
18	566·28	198·31	660·66	231·36	755·04	264·41	849·42	297·46	943·80	330·51	42
21	566·11	198·80	660·46	231·94	754·81	265·07	849·16	298·20	943·51	331·34	39
24	565·93	199·30	660·25	232·51	754·58	265·73	848·90	298·94	943·22	332·16	36
27	565·76	199·79	660·05	233·09	754·34	266·39	848·64	299·69	942·93	332·98	33
30	565·58	200·28	659·85	233·66	754·11	267·04	848·38	300·43	942·64	333·80	30
33	565·41	200·78	659·64	234·24	753·88	267·70	848·11	301·17	942·35	334·63	27
36	565·23	201·27	659·44	234·81	753·64	268·36	847 85	301·91	942·06	335·45	24
39	565·06	201·76	659·23	235·39	753·41	269·02	847·59	302·65	941·76	336·27	21
42	564·88	202·26	659·03	235·97	753·17	269·67	847·32	303·39	941·47	337·09	18
45	564·70	202·75	658·82	236·54	752·94	270·33	847·06	304·13	941·17	337·91	15
48	564·53	203·24	658·62	237·11	752·70	270·99	846·79	304·86	940·88	338·74	12
51	564·35	203·73	658·41	237·69	752·47	271·65	846·53	305·60	940·58	339·56	9
54	564·17	204·23	658·20	238·26	752·23	272·30	846·26	306·34	940·29	340·38	6
57	563·99	204·72	657·99	238·84	751·99	272·96	845·99	307·08	939·99	341·20	3
60	563·81	205·21	657·78	239·41	751·75	273·61	845·72	307·82	939·69	342·02	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

70 Deg.

20 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	93·97	34·20	187·94	68·40	281·91	102·60	375·87	136·81	469·84	171·01	60
3	93·94	34·28	187·88	68·57	281·82	102·85	375·76	137·13	469·70	171·42	57
6	93·91	34·36	187·82	68·73	281·73	103·10	375·64	137·46	469·55	171·83	54
9	93·88	34·45	187·76	68·89	281·64	103·34	375·52	137·79	469·40	172·24	51
12	93·85	34·53	187·70	69·06	281·55	103·59	375·40	138·12	469·25	172·65	48
15	93·82	34·61	187·64	69·22	281·46	103·84	375·28	138·45	469·10	173·06	45
18	93·79	34·69	187·58	69·39	281·37	104·08	375·15	138·77	468·94	173·47	42
21	93·76	34·77	187·52	69·55	281·27	104·33	375·03	139·10	468·79	173·88	39
24	93·73	34·86	187·46	69·71	281·18	104·57	374·91	139·43	468·64	174·28	36
27	93·70	34·94	187·39	69·88	281·09	104·82	374·79	139·75	468·49	174·69	33
30	93·67	35·02	187·33	70·04	281·00	105·06	374·67	140·08	468·34	175·10	30
33	93·64	35·10	187·27	70·20	280·91	105·31	374·55	140·41	468·18	175·51	27
36	93·61	35·18	187·21	70·37	280·82	105·55	374·42	140·74	468·03	175·92	24
39	93·57	35·26	187·15	70·53	280·72	105·80	374·30	141·06	467·87	176·33	21
42	93·54	35·35	187·09	70·69	280·63	106·04	374·18	141·39	467·72	176·74	18
45	93·51	35·43	187·03	70·86	280·54	106·29	374·05	141·72	467·57	177·15	15
48	93·48	35·51	186·96	71·02	280·45	106·53	373·93	142·04	467·41	177·55	12
51	93·45	35·59	186·90	71·18	280·35	106·78	373·80	142·37	467·26	177·96	9
54	93·42	35·67	186·84	71·35	280·26	107·02	373·68	142·69	467·10	178·37	6
57	93·39	35·75	186·78	71·51	280·17	107·27	373·56	143·02	466·94	178·77	3
60	93·36	35·84	186·71	71·67	280·07	107·51	373·43	143·35	466·79	179·18	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Minutes.
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

69 Deg.

20 Deg.

Minutes.	Dist. 600. Lat. Per.	Dist. 600. Dep. Base.	Dist. 700. Lat. Per.	Dist. 700. Dep. Base.	Dist. 800. Lat. Per.	Dist. 800. Dep. Base.	Dist. 900. Lat. Per.	Dist. 900. Dep. Base.	Dist. 1000. Lat. Per.	Dist. 1000. Dep. Base.	
0	563·81	205·21	657·78	239·41	751·75	273·61	845·72	307·82	939·69	342·02	60
3	563·63	205·70	657·57	239·99	751·51	274·27	845·45	308·55	939·39	342·84	57
6	563·45	206·19	657·36	240·56	751·27	274·93	845·19	309·29	939·09	343·66	54
9	563·28	206·69	657·15	241·13	751·03	275·58	844·92	310·03	938·79	344·48	51
12	563·10	207·18	656·94	241·71	750·79	276·24	844·64	310·77	938·49	345·30	48
15	562·91	207·67	656·73	242·28	750·55	276·89	844·37	311·50	938·19	346·12	45
18	562·73	208·16	656·52	242·85	750·31	277·55	844·10	312·24	937·89	346·93	42
21	562·55	208·65	656·31	243·43	750·07	278·20	843·83	312·98	937·59	347·75	39
24	562·37	209·14	656·10	244·00	749·83	278·86	843·55	313·71	937·28	348·57	36
27	562·18	209·63	655·88	244·57	749·58	279·51	843·28	314·45	936·98	349·39	33
30	562·00	210·12	655·67	245·14	749·34	280·16	843·00	315·19	936·67	350·21	30
33	561·82	210·61	655·45	245·72	749·09	280·82	842·73	315·92	936·37	351·02	27
36	561·64	211·10	655·24	246·29	748·85	281·47	842·45	316·66	936·06	351·84	24
39	561·45	211·59	655·02	246·86	748·60	282·13	842·17	317·39	935·75	352·66	21
42	561·26	212·08	654·81	247·43	748·35	282·78	841·90	318·13	935·44	353·47	18
45	561·08	212·57	654·59	248·00	748·11	283·43	841·62	318·86	935·13	354·29	15
48	560·89	213·06	654·38	248·57	747·86	284·08	841·34	319·60	934·82	355·11	12
51	560·71	213·55	654·16	249·14	747·61	284·74	841·06	320·33	934·51	355·92	9
54	560·52	214·04	653·94	249·72	747·36	285·39	840·78	321·06	934·20	356·74	6
57	560·33	214·53	653·72	250·29	747·11	286·04	840·50	321·80	933·89	357·55	3
60	560·15	215·02	653·50	250·86	746·86	286·69	840·22	322·53	933·58	358·37	0
	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Minutes.
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

69 Deg.

21 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	93·36	35·84	186·71	71·67	280·07	107·51	373·43	143·34	466·79	179·18	60
3	93·32	35·91	186·65	71·83	279·98	107·75	373·31	143·67	466·63	179·59	57
6	93·29	35·99	186·59	71·99	279·88	107·99	373·18	143·99	466·47	179·99	54
9	93·26	36·08	186·53	72·15	279·79	108·23	373·06	144·32	466·32	180·40	51
12	93·23	36·16	186·46	72·32	279·69	108·48	372·93	144·65	466·16	180·81	48
15	93·20	36·24	186·40	72·48	279·60	108·72	372·80	144·97	466·00	181·22	45
18	93·16	36·32	186·33	72·65	279·50	108·97	372·67	145·30	465·84	181·62	42
21	93·13	36·40	186·27	72·81	279·41	109·21	372·55	145·63	465·68	182·03	39
24	93·10	36·48	186·21	72·97	279·31	109·46	372·42	145·95	465·52	182·43	36
27	93·07	36·56	186·15	73·13	279·22	109·70	372·29	146·27	465·36	182·84	33
30	93·04	36·65	186·08	73·30	279·12	109·94	372·16	146·60	465·20	183·25	30
33	93·00	36·73	186·02	73·46	279·03	110·19	372·04	146·92	465·04	183·65	27
36	92·97	36·81	185·95	73·62	278·93	110·43	371·91	147·24	464·88	184·06	24
39	92·94	36·89	185·89	73·78	278·83	110·67	371·78	147·57	464·72	184·47	21
42	92·91	36·97	185·82	73·94	278·73	110·92	371·65	147·89	464·56	184·87	18
45	92·88	37·05	185·76	74·10	278·64	111·16	371·52	148·22	464·40	185·28	15
48	92·84	37·13	185·69	74·27	278·54	111·41	371·39	148·54	464·24	185·68	12
51	92·81	37·21	185·63	74·43	278·45	111·65	371·26	148·87	464·08	186·09	9
54	92·78	37·29	185·56	74·59	278·35	111·89	371·13	149·19	463·92	186·49	6
57	92·75	37·37	185·50	74·75	278·25	112·14	371·00	149·52	463·76	186·90	3
60	92·71	37·46	185·43	74·92	278·15	112·38	370·87	149·84	463·59	187·30	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

68 Deg.

21 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	560·14	215·02	653·50	250·85	746·86	286·69	840·22	322·53	933·58	358·37	60
3	559·96	215·50	653·28	251·42	746·61	287·34	839·94	323·26	933·26	359·18	57
6	559·77	215·99	653·06	251·99	746·36	287·99	839·66	323·99	932·95	359·99	54
9	559·58	216·48	652·84	252·56	746·11	288·64	839·38	324·72	932·64	360·81	51
12	559·39	216·97	652·62	253·13	745·86	289·29	839·09	325·46	932·32	361·62	48
15	559·20	217·46	652·40	253·70	745·61	289·94	838·81	326·19	932·01	362·44	45
18	559·01	217·95	652·18	254·27	745·35	290·60	838·52	326·92	931·69	363·25	42
21	558·82	218·44	651·96	254·84	745·10	291·25	838·24	327·65	931·37	364·06	39
24	558·63	218·92	651·73	255·41	744·84	291·90	837·95	328·38	931·05	364·87	36
27	558·44	219·41	651·51	255·98	744·59	292·55	837·66	329·11	930·73	365·69	33
30	558·24	219·90	651·29	256·55	744·33	293·20	837·37	329·85	930·41	366·50	30
33	558·05	220·39	651·07	257·12	744·08	293·85	837·08	330·58	930·09	367·31	27
36	557·86	220·87	650·84	257·68	743·82	294·50	836·79	331·31	929·77	368·12	24
39	557·67	221·36	650·62	258·25	743·56	295·15	836·50	332·04	929·45	368·93	21
42	557·47	221·84	650·39	258·82	743·30	295·79	836·21	332·77	929·13	369·74	18
45	557·28	222·33	650·17	259·39	743·04	296·44	835·92	333·50	928·81	370·55	15
48	557·09	222·82	649·94	259·95	742·78	297·09	835·63	334·23	928·48	371·36	12
51	556·90	223·31	649·71	260·52	742·52	297·74	835·34	334·96	928·16	372·17	9
54	556·70	223·79	649·48	261·09	742·26	298·39	835·05	335·68	927·83	372·98	6
57	556·50	224·28	649·25	261·66	742·00	299·04	834·75	336·41	927·51	373·79	3
60	556·30	224·76	649·02	262·22	741·74	299·68	834·46	337·14	927·18	374·60	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

68 Deg.

22 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	92·71	37·46	185·43	74·92	278·15	112·38	370·87	149·84	463·59	187·30	60
3	92·68	37·54	185·37	75·08	278·05	112·62	370·74	150·16	463·43	187·70	57
6	92·65	37·62	185·30	75·24	277·95	112·86	370·61	150·48	463·26	188·11	54
9	92·62	37·70	185·24	75·40	277·86	113·10	370·48	150·81	463·10	188·51	51
12	92·58	37·78	185·17	75·56	277·76	113·35	370·34	151·13	462·93	188·92	48
15	92·55	37·86	185·11	75·72	277·66	113·59	370·21	151·45	462·77	189·32	45
18	92·52	37·94	185·04	75·88	277·56	113·83	370·08	151·78	462·60	189·72	42
21	92·48	38·02	184·97	76·04	277·46	114·07	369·95	152·10	462·44	190·13	39
24	92·45	38·10	184·90	76·21	277·36	114·32	369·81	152·42	462·27	190·53	36
27	92·42	38·18	184·84	76·37	277·26	114·56	369·68	152·74	462·10	190·93	33
30	92·38	38·26	184·77	76·53	277·16	114·80	369·55	153·07	461·93	191·34	30
33	92·35	38·34	184·71	76·69	277·06	115·04	369·42	153·39	461·77	191·74	27
36	92·32	38·42	184·64	76·85	276·96	115·29	369·28	153·71	461·60	192·14	24
39	92·28	38·50	184·57	77·01	276·86	115·53	369·15	154·03	461·44	192·54	21
42	92·25	38·59	184·50	77·18	276·76	115·77	369·01	154·36	461·27	192·95	18
45	92·22	38·67	184·44	77·34	276·66	116·01	368·88	154·68	461·10	193·35	15
48	92·18	38·75	184·37	77·50	276·56	116·25	368·74	155·00	460·93	193·75	12
51	92·15	38·83	184·30	77·66	276·45	116·49	368·61	155·32	460·76	194·15	9
54	92·11	38·91	184·23	77·82	276·35	116·73	368·47	155·64	460·59	194·56	6
57	92·08	38·99	184·17	77·98	276·25	116·97	368·34	155·96	460·42	194·96	3
60	92·05	39·07	184·10	78·14	276·15	117·21	368·20	156·29	460·25	195·36	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

67 Deg.

22 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	556·30	224·76	649·02	262·22	741·74	299·68	834·46	337·14	927·18	374·60	60
3	556·11	225·24	648·79	262·79	741·48	300·33	834·17	337·87	926·85	375·41	57
6	555·91	225·73	648·56	263·35	741·22	300·97	833·87	338·60	926·52	376·22	54
9	555·72	226·21	648·33	263·92	740·96	301·62	833·58	339·33	926·19	377·03	51
12	555·52	226·70	648·10	264·48	740·69	302·27	833·28	340·05	925·86	377·84	48
15	555·32	227·18	647·87	265·05	740·43	302·92	832·99	340·78	925·53	378·64	45
18	555·12	227·67	647·64	265·61	740·16	303·56	832·69	341·51	925·20	379·45	42
21	554·92	228·15	647·41	266·18	739·90	304·21	832·39	342·24	924·87	380·26	39
24	554·72	228·64	647·18	266·74	739·63	304·85	832·09	342·96	924·54	381·07	36
27	554·52	229·12	646·95	267·31	739·37	305·50	831·79	343·69	924·21	381·88	33
30	554·32	229·61	646·71	267·87	739·10	306·14	831·49	344·41	923·87	382·68	30
33	554·12	230·09	646·48	268·44	738·83	306·79	831·19	345·14	923·54	383·48	27
36	553·92	230·57	646·24	269·00	738·56	307·43	830·89	345·86	923·20	384·28	24
39	553·72	231·06	646·01	269·57	738·30	308·08	830·59	346·59	922·87	385·09	21
42	553·52	231·54	645·77	270·13	738·08	308·72	830·28	347·31	922·53	385·90	18
45	553·32	232·03	645·54	270·70	737·76	309·37	829·98	348·04	922·20	386·70	15
48	553·11	232·51	645·30	271·26	737·49	310·01	829·67	348·76	921·86	387·51	12
51	552·91	233·00	645·06	271·82	737·22	310·66	829·37	349·49	921·52	388·31	9
54	552·71	233·47	644·82	272·38	736·94	311·30	829·06	350·21	921·18	389·12	6
57	552·51	233·95	644·59	272·94	736·67	311·94	828·76	350·93	920·84	389·92	3
60	552·30	234·43	644·35	273·51	736·40	312·58	828·45	351·65	920·50	390·73	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

67 Deg.

23 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	92·05	39·07	184·10	78·14	276·15	117·21	368·20	156·29	460·25	195·36	60
3	92·01	39·15	184·03	78·30	276·05	117·45	368·06	156·61	460·08	195·76	57
6	91·98	39·23	183·96	78·46	275·94	117·70	367·92	156·93	459·91	196·16	54
9	91·94	39·31	183·89	78·62	275·84	117·94	367·79	157·25	459·74	196·56	51
12	91·91	39·39	183·82	78·78	275·74	118·18	367·65	157·57	459·56	196·97	48
15	91·87	39·47	183·75	78·94	275·64	118·42	367·51	157·89	459·39	197·37	45
18	91·84	39·55	183·68	79·10	275·53	118·66	367·37	158·21	459·22	197·77	42
21	91·81	39·63	183·62	79·26	275·43	118·90	367·24	158·53	459·05	198·17	39
24	91·77	39·71	183·55	79·42	275·32	119·14	367·10	158·85	458·87	198·57	36
27	91·74	39·79	183·48	79·58	275·22	119·38	366·96	159·17	458·70	198·97	33
30	91·70	39·87	183·41	79·74	275·11	119·62	366·82	159·49	458·53	199·37	30
33	91·67	39·95	183·34	79·90	275·01	119·86	366·68	159·81	458·36	199·77	27
36	91·63	40·03	183·27	80·06	274·90	120·10	366·54	160·13	458·18	200·17	24
39	91·60	40·11	183·20	80·22	274·79	120·34	366·40	160·45	458·01	200·57	21
42	91·56	40·19	183·13	80·38	274·69	120·58	366·26	160·77	457·83	200·97	18
45	91·53	40·27	183·06	80·54	274·58	120·82	366·12	161·09	457·66	201·37	15
48	91·49	40·35	182·99	80·70	274·48	121·06	365·98	161·41	457·48	201·77	12
51	91·46	40·43	182·92	80·86	274·37	121·30	365·84	161·73	457·30	202·17	9
54	91·42	40·51	182·85	81·02	274·27	121·54	365·70	162·05	457·12	202·57	6
57	91·39	40·59	182·78	81·18	274·16	121·78	365·56	162·37	456·95	202·97	3
60	91·35	40·67	182·70	81·34	274·06	122·02	365·42	162·69	456·77	203·36	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

66 Deg.

23 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	552·30	234·43	644·35	273·51	736·40	312·58	828·45	351·65	920·50	390·73	60
3	552·10	234·91	644·11	274·07	736·13	313·22	828·15	352·37	920·16	391·53	57
6	551·89	235·40	643·87	274·63	735·85	313·86	827·84	353·10	919·82	392·33	54
9	551·68	235·88	643·63	275·19	735·58	314·50	827·53	353·82	919·48	393·14	51
12	551·47	236·36	643·39	275·75	735·30	315·15	827·22	354·54	919·13	393·94	48
15	551·27	236·84	643·15	276·31	735·03	315·79	826·91	355·26	918·79	394·74	45
18	551·06	237·32	642·91	276·88	734·75	316·43	826·60	355·99	918·44	395·54	42
21	550·86	237·80	642·67	277·44	734·48	317·07	826·29	356·71	918·10	396·34	39
24	550·65	238·28	642·42	278·00	734·20	317·71	825·98	357·43	917·75	397·14	36
27	550·44	238·76	642·18	278·56	733·92	318·35	825·67	358·15	917·40	397·94	33
30	550·23	239·24	641·93	279·12	733·64	318·99	825·35	358·87	917·05	398·74	30
33	550·02	239·72	641·69	279·68	733·36	319·63	825·04	359·59	916·71	399·54	27
36	549·81	240·20	641·45	280·24	733·08	320·27	824·72	360·31	916·36	400·34	24
39	549·60	240·68	641·21	280·80	732·80	320·91	824·41	361·03	916·01	401·14	21
42	549·39	241·16	640·96	281·36	732·52	321·55	824·09	361·75	915·66	401·94	18
45	549·18	241·64	640·72	281·92	732·24	322·19	823·78	362·47	915·31	402·74	15
48	548·97	242·12	640·47	282·48	731·96	322·83	823·46	363·19	914·95	403·54	12
51	548·76	242·60	640·22	283·04	731·68	323·47	823·14	363·91	914·60	404·34	9
54	548·55	243·08	639·97	283·60	731·40	324·11	822·82	364·62	914·25	405·14	6
57	548·34	243·56	639·73	284·16	731·12	324·75	822·51	365·34	913·90	405·94	3
60	548·12	244·04	639·48	284·71	730·83	325·38	822·19	366·06	913·54	406·73	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

66 Deg.

24 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	91·35	40·67	182·70	81·34	274·06	122·02	365·42	162·69	456·77	203·36	60
3	91·31	40·75	182·63	81·50	273·95	122·26	365·28	163·01	456·59	203·76	57
6	91·28	40·82	182·56	81·66	273·84	122·49	365·13	163·33	456·41	204·16	54
9	91·24	40·91	182·49	81·82	273·74	122·73	364·99	163·65	456·24	204·56	51
12	91·21	40·99	182·42	81·98	273·63	122·97	364·84	163·96	456·06	204·96	48
15	91·17	41·07	182·35	82·14	273·53	123·21	364·70	164·28	455·88	205·36	45
18	91·14	41·15	182·28	82·30	273·42	123·45	364·56	164·60	455·70	205·75	42
21	91·10	41·23	182·21	82·46	273·31	123·69	364·42	164·92	455·52	206·15	39
24	91·06	41·31	182·13	82·62	273·20	123·93	364·27	165·24	455·34	206·55	36
27	91·03	41·39	182·06	82·78	273·09	124·17	364·13	165·56	455·16	206·95	33
30	90·99	41·46	181·99	82·93	272·98	124·40	363·98	165·87	454·98	207·34	30
33	90·95	41·54	181·92	83·09	272·88	124·64	363·84	166·19	454·80	207·74	27
36	90·92	41·62	181·84	83·25	272·77	124·88	363·69	166·51	454·61	208·14	24
39	90·88	41·70	181·77	83·41	272·66	125·12	363·55	166·83	454·43	208·54	21
42	90·85	41·78	181·70	83·57	272·55	125·36	363·41	167·14	454·25	208·93	18
45	90·81	41·86	181·63	83·73	272·44	125·60	363·26	167·46	454·07	209·33	15
48	90·77	41·94	181·55	83·89	272·33	125·83	363·11	167·78	453·88	209·72	12
51	90·74	42·02	181·48	84·05	272·22	126·07	362·96	168·10	453·70	210·12	9
54	90·70	42·10	181·40	84·20	272·11	126·31	362·82	168·41	453·52	210·51	6
57	90·66	42·18	181·33	84·36	272·00	126·55	362·67	168·73	453·34	210·91	3
60	90·63	42·26	181·26	84·52	271·89	126·78	362·52	169·04	453·15	211·30	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

65 Deg.

24 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	548·12	244·04	639·48	284·71	730·83	325·38	822·19	366·06	913·54	406·73	60
3	547·91	244·52	639·23	285·27	730·55	326·02	821·87	366·78	913·19	407·53	57
6	547·70	244·99	638·98	285·83	730·26	326·66	821·54	367·49	912·83	408·32	54
9	547·49	245·47	638·73	286·39	729·98	327·30	821·22	368·21	912·48	409·13	51
12	547·27	245·95	638·48	286·94	729·69	327·93	820·90	368·93	912·12	409·92	48
15	547·06	246·43	638·23	287·50	729·41	328·57	820·58	369·65	911·76	410·72	45
18	546·84	246·90	637·98	288·06	729·12	329·21	820·26	370·36	911·40	411·51	42
21	546·62	247·38	637·73	288·62	728·83	329·85	819·94	371·08	911·04	412·31	39
24	546·40	247·86	637·47	289·17	728·54	330·48	819·61	371·79	910·67	413·10	36
27	546·19	248·34	637·22	289·73	728·25	331·12	819·29	372·51	910·31	413·90	33
30	545·97	248·81	636·97	290·28	727·96	331·75	818·96	373·22	909·95	414·69	30
33	545·76	249·29	636·72	290·84	727·67	332·39	818·64	373·94	909·59	415·49	27
36	545·54	249·76	636·46	291·39	727·38	333·02	818·31	374·65	909·23	416·28	24
39	545·32	250·24	636·21	291·95	727·09	333·66	817·98	375·37	908·87	417·07	21
42	545·10	250·72	635·95	292·50	726·80	334·29	817·65	376·08	908·50	417·86	18
45	544·88	251·20	635·70	293·06	726·51	334·93	817·33	376·79	908·14	418·66	15
48	544·66	251·67	635·44	293·61	726·22	335·56	817·00	377·50	907·77	419·45	12
51	544·44	252·15	635·19	294·17	725·93	336·19	816·67	378·22	907·41	420·24	9
54	544·22	252·62	634·93	294·72	725·63	336·82	816·34	378·93	907·04	421·03	6
57	544·00	253·10	634·67	295·28	725·34	337·46	816·01	379·64	906·67	421·82	3
60	543·78	253·57	634·41	295·83	725·04	338·09	815·67	380·35	906·30	422·61	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

65 Deg.

25 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	90 63	42·26	181·26	84·52	271·89	126·78	362·52	169·04	453·15	211·30	60
3	90·59	42·34	181·19	84·68	271·78	127·02	362·37	169·36	452·97	211·70	57
6	90·55	42·42	181·11	84·84	271·67	127·25	362·22	169·68	452·78	212·10	54
9	90·51	42·49	181·04	84·99	271·56	127·49	362·08	170·00	452·60	212·49	51
12	90·48	42·57	180·96	85·15	271·44	127·73	361·93	170·31	452·41	212·88	48
15	90·44	42·65	180·89	85·31	271·33	127·97	361·78	170·63	452·23	213·28	45
18	90·40	42·73	180·81	85·47	271·22	128·20	361·63	170·94	452·04	213·67	42
21	90·37	42·81	180·74	85·63	271·11	128·44	361·48	171·26	451·85	214·07	39
24	90·33	42·89	180·66	85·78	271·00	128·68	361·33	171·57	451·66	214·46	36
27	90·29	42·97	180·59	85·94	270·89	128·92	361·18	171·89	451·48	214·86	33
30	90·25	43·05	180·51	86·10	270·77	129·15	361·03	172·20	451·29	215·25	30
33	90·22	43·13	180·44	86·26	270·66	129·39	360·88	172·52	451·10	215·65	27
36	90·18	43·20	180·36	86·41	270·55	129·62	360·73	172·83	450·91	216·04	24
39	90·14	43·28	180·29	86·57	270·44	129·86	360·58	173·15	450·72	216·44	21
42	90·10	43·36	180·21	86·73	270·32	130·09	360·43	173·46	450·53	216·83	18
45	90·06	43·44	180·14	86·89	270·21	130·33	360·28	173·78	450·34	217·22	15
48	90·03	43·52	180·06	87·04	270·09	130·56	360·12	174·09	450·15	217·61	12
51	89·99	43·59	179·99	87·20	269·98	130·80	359·97	174·41	449·96	218·01	9
54	89·95	43·67	179·91	87·36	269·86	131·04	359·82	174·72	449·77	218·40	6
57	89·91	43·75	179·83	87·52	269·75	131·27	359·67	175·03	449·58	218·79	3
60	89·87	43·83	179·75	87·67	269·63	131·51	359·51	175·34	449·39	219·18	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

64 Deg.

25 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	543·78	253·57	634·41	295·83	725·04	338·09	815·67	380·35	906·30	422·61	60
3	543·56	254·04	634·15	296·38	724·75	338·73	815·34	381·06	905·93	423·40	57
6	543·34	254·51	633·89	296·94	724·45	339·36	815·01	381·78	905·56	424·20	54
9	543·12	254·99	633·63	297·49	724·16	340·00	814·68	382·49	905·19	424·99	51
12	542·89	255·46	633·37	298·04	723·86	340·63	814·34	383·20	904·82	425·78	48
15	542·67	255·94	633·11	298·60	723·56	341·26	814·01	383·91	904·45	426·57	45
18	542·44	256·41	632·85	299·15	723·26	341·89	813·67	384·62	904·08	427·35	42
21	542·22	256·89	632·59	299·70	722·96	342·52	813·34	385·33	903·71	428·14	39
24	542·00	257·36	632·33	300·25	722·66	343·15	813·00	386·04	903·33	428·93	36
27	541·78	257·83	632·07	300·80	722·36	343·78	812·66	386·75	902·96	429·72	33
30	541·55	258·30	631·81	301·35	722·06	344·40	812·32	387·46	902·58	430·51	30
33	541·32	258·77	631·55	301·90	721·76	345·03	811·98	388·17	902·21	431·30	27
36	541·09	259·25	631·28	302·45	721·46	345·66	811·64	388·87	901·83	432·08	24
39	540·87	259·72	631·02	303·00	721·16	346·29	811·30	389·58	901·45	432·87	21
42	540·64	260·19	630·75	303·56	720·86	346·92	810·96	390·29	901·07	433·65	18
45	540·42	260·67	630·49	304·11	720·56	347·55	810·62	391·00	900·69	434·44	15
48	540·19	261·14	630·22	304·66	720·25	348·18	810·28	391·70	900·31	435·23	12
51	539·96	261·61	629·96	305·21	719·95	348·81	809·94	392·41	899·93	435·92	9
54	539·73	262·08	629·69	305·76	719·64	349·44	809·60	393·12	899·55	436·80	6
57	539·50	262·55	629·42	306·31	719·34	350·07	809·26	393·83	899·17	437·59	3
60	539·27	263·02	629·15	306·86	719·03	350·69	808·91	394·53	898·79	438·37	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

64 Deg.

26 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	89·87	43·83	179·75	87·67	269·63	131·51	359·51	175·34	449·39	219·18	60
3	89·84	43·91	179·68	87·83	269·52	131·75	359·36	175·65	449·20	219·57	57
6	89·80	43·99	179·60	87·98	269·40	131·98	359·21	175·97	449·01	219·96	54
9	89·76	44·07	179·53	88·14	269·29	132·22	359·06	176·28	448·82	220·35	51
12	89·72	44·15	179·45	88·30	269·17	132·45	358·90	176·60	448·62	220·75	48
15	89·68	44·22	179·37	88·46	269·06	132·69	358·75	176·91	448·43	221·14	45
18	89·64	44·30	179·29	88·61	268·94	132·92	358·59	177·22	448·24	221·53	42
21	89·61	44·38	179·22	88·77	268·83	133·16	358·44	177·53	448·05	221·92	39
24	89·57	44·46	179·14	88·92	268·71	133·39	358·28	177·85	447·85	222·31	36
27	89·53	44·54	179·06	89·08	268·59	133·63	358·13	178·16	447·66	222·70	33
30	89·49	44·61	178·98	89·23	268·47	133·86	357·97	178·47	447·46	223·09	30
33	89·45	44·69	178·91	89·39	268·36	134·09	357·82	178·78	447·27	223·48	27
36	89·41	44·77	178·83	89·55	268·24	134·32	357·66	179·10	447·07	223·87	24
39	89·37	44·85	178·75	89·71	268·13	134·56	357·50	179·41	446·88	224·26	21
42	89·33	44·93	178·67	89·86	268·01	134·79	357·34	179·72	446·68	224·65	18
45	89·29	45·00	178·59	90·02	267·89	135·03	357·19	180·03	446·49	225·04	15
48	89·25	45·08	178·51	90·17	267·77	135·26	357·03	180·35	446·29	225·43	12
51	89·21	45·16	178·43	90·33	267·65	135·50	356·87	180·66	446·10	225·82	9
54	89·17	45·24	178·35	90·48	267·53	135·73	356·71	180·97	445·90	226·21	6
57	89·14	45·32	178·28	90·64	267·42	135·96	356·56	181·28	445·70	226·60	3
60	89·10	45·39	178·20	90·79	267·30	136·19	356·40	181·59	445·50	226·99	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

63 Deg.

26 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	539·27	263·02	629·15	306·86	719·03	350·69	808·91	394·53	898·79	438·37	60
3	539·04	263·49	628·88	307·41	718·73	351·32	808·57	395·24	898·41	439·15	57
6	538·81	263·96	628·61	307·95	718·42	351·95	808·22	395·94	898·02	439·93	54
9	538·58	264·43	628·34	308·50	718·11	352·58	807·88	396·65	897·64	440·71	51
12	538·35	264·90	628·07	309·05	717·80	353·20	807·53	397·35	897·25	441·50	48
15	538·12	265·37	627·81	309·60	717·49	353·83	807·18	398·06	896·87	442·28	45
18	537·89	265·84	627·54	310·14	717·18	354·45	806·83	398·76	896·48	443·06	42
21	537·66	266·31	627·27	310·69	716·87	355·08	806·49	399·47	896·10	443·84	39
24	537·42	266·78	626·99	311·24	716·56	355·70	806·14	400·17	895·71	444·63	36
27	537·19	267·25	626·72	311·79	716·25	356·33	805·79	400·88	895·32	445·41	33
30	536·96	267·71	626·45	312·33	715·94	356·95	805·44	401·58	894·93	446·19	30
33	536·73	268·18	626·18	312·88	715·63	357·58	805·09	402·28	894·54	446·97	27
36	536·49	268·65	625·90	313·43	715·32	358·20	804·73	402·98	894·15	447·75	24
39	536·26	269·12	625·63	313·97	715·01	358·83	804·38	403·68	893·76	448·53	21
42	536·02	269·59	625·36	314·52	714·69	359·45	804·03	404·38	893·37	449·31	18
45	535·79	270·06	625·09	315·07	714·38	360·08	803·68	405·08	892·98	450·09	15
48	535·55	270·52	624·81	315·61	714·06	360·70	803·32	405·78	892·58	450·87	12
51	535·31	270·99	624·53	316·16	713·75	361·32	802·97	406·48	892·19	451·65	9
54	535·07	271·46	624·25	316·70	713·43	361·94	802·61	407·19	891·79	452·43	6
57	534·84	271·93	623·98	317·25	713·12	362·57	802·26	407·89	891·40	453·21	3
60	534·60	272·39	623·70	317·79	712·80	363·19	801·90	408·59	891·00	453·99	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

63 Deg.

27 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	89·10	45·39	178·20	90·79	267·30	136·19	356·40	181·59	445·50	226·99	60
3	89·06	45·47	178·12	90·94	267·18	136·42	356·24	181·90	445·30	227·38	57
6	89·02	45·55	178·04	91·10	267·06	136·66	356·08	182·21	445·10	227·77	54
9	88·98	45·63	177·96	91·25	266·94	136·89	355·92	182·52	444·90	228·16	51
12	88·94	45·70	177·88	91·41	266·82	137·12	355·76	182·83	444·70	228·54	48
15	88·90	45·78	177·80	91·57	266·70	137·35	355·60	183·14	444·50	228·93	45
18	88·86	45·86	177·72	91·73	266·58	137·59	355·44	183·46	444·30	229·32	42
21	88·82	45·94	177·64	91·88	266·46	137·82	355·28	183·77	444·10	229·71	39
24	88·78	46·01	177·56	92·04	266·34	138·06	355·12	184·08	443·90	230·10	36
27	88·74	46·09	177·48	92·19	266·22	138·29	354·96	184·39	443·70	230·49	33
30	88·70	46·17	177·40	92·35	266·10	138·52	354·80	184·70	443·50	230·87	30
33	88·66	46·25	177·32	92·50	265·98	138·76	354·64	185·01	443·30	231·26	27
36	88·62	46·32	177·24	92·65	265·86	138·99	354·48	185·31	443·10	231·64	24
39	88·58	46·40	177·16	92·81	265·74	139·22	354·32	185·62	442·90	232·03	21
42	88·53	46·48	177·07	92·96	265·61	139·45	354·15	185·93	442·69	232·42	18
45	88·49	46·56	176·99	93·11	265·49	139·68	353·99	186·24	442·49	232·81	15
48	88·45	46·63	176·91	93·27	265·37	139·91	353·83	186·55	442·29	233·19	12
51	88·41	46·71	176·83	93·42	265·25	140·14	353·67	186·86	442·09	233·58	9
54	88·37	46·79	176·75	93·58	265·12	140·37	353·50	187·17	441·88	233·96	6
57	88·33	46·86	176·67	93·73	265·00	140·60	353·34	187·48	441·68	234·35	3
60	88·29	46·94	176·59	93·89	264·88	140·84	353·18	187·78	441·47	234·73	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

62 Deg.

27 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	534·60	272·39	623·70	317·79	712·80	363·19	801·90	408·59	891·00	453·99	60
3	534·36	272·86	623·42	318·34	712·48	363·81	801·55	409·29	890·60	454·77	57
6	534·12	273·32	623·14	318·88	712·16	364·43	801·19	409·99	890·20	455·54	54
9	533·88	273·79	622·87	319·42	711·85	365·05	800·83	410·69	889·80	456·32	51
12	533·64	274·25	622·59	319·96	711·53	365·67	800·47	411·38	889·40	457·09	48
15	533·40	274·72	622·31	320·51	711·21	366·29	800·11	412·08	889·01	457·87	45
18	533·16	275·18	622·03	321·05	710·89	366·91	799·75	412·78	888·61	458·64	42
21	532·92	275·65	621·75	321·59	710·57	367·53	799·39	413·48	888·21	459·42	39
24	532·68	276·11	621·47	322·13	710·25	368·15	799·03	414·18	887·81	460·19	36
27	532·44	276·58	621·19	322·67	709·93	368·77	798·67	414·88	887·41	460·97	33
30	532·20	277·04	620·90	323·22	709·60	369·39	798·31	415·57	887·00	461·74	30
33	531·96	277·51	620·62	323·76	709·28	370·01	797·95	416·27	886·60	462·51	27
36	531·72	277·97	620·34	324·30	708·96	370·63	797·58	416·96	886·20	463·29	24
39	531·48	278·44	620·06	324·84	708·64	371·25	797·22	417·66	885·80	464·07	21
42	531·23	278·90	619·77	325·38	708·31	371·87	796·85	418·35	885·39	464·84	18
45	530·99	279·37	619·49	325·92	707·99	372·49	796·49	419·05	884·98	465·61	15
48	530·74	279·83	619·20	326·47	707·66	373·10	796·12	419·74	884·57	466·38	12
51	530·50	280·29	618·92	327·01	707·34	373·72	795·76	420·44	884·17	467·15	9
54	530·25	280·75	618·63	327·55	707·01	374·34	795·39	421·13	883·76	467·92	6
57	530·01	281·22	618·35	328·09	706·69	374·96	795·02	421·83	883·36	468·69	3
60	529·77	281·68	618·06	328·63	706·36	375·57	794·65	422·52	882·95	469·47	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

62 Deg.

28 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	88 29	46·94	176·59	93·89	264·88	140·84	353·18	187·78	441·47	234·73	60
3	88·25	47·02	176·51	94·04	264·76	141·07	353·02	188·09	441·27	235·12	57
6	88·21	47·10	176·42	94·20	264·63	141·30	352·85	188·40	441·06	235·50	54
9	88·17	47·17	176·34	94·35	264·51	141·53	352·69	188·71	440·86	235·89	51
12	88·13	47·25	176·26	94·51	264·39	141·76	352·52	189·02	440·65	236·27	48
15	88·08	47·33	176·18	94·66	264·27	141·99	352·35	189·33	440·44	236·66	45
18	88·04	47·40	176·09	94·81	264·14	142·22	352·18	189·63	440·23	237·04	42
21	88·00	47·48	176·01	94·96	264·02	142·45	352·02	189·94	440·03	237·43	39
24	87·96	47·56	175·92	95·12	263·89	142·68	351·85	190·24	439·82	237·81	36
27	87·92	47·63	175·84	95·27	263·77	142·91	351·69	190·55	439·61	238·19	33
30	87·88	47·71	175·76	95·43	263·64	143·13	351·52	190·86	439·40	238·57	30
33	87·84	47·79	175·68	95·58	263·52	143·36	351·35	191·17	439·20	238·95	27
36	87·79	47·86	175·59	95·73	263·39	143·60	351·18	191·47	438·99	239·34	24
39	87·75	47·94	175·51	95·88	263·27	143·83	351·02	191·78	438·78	239·72	21
42	87·71	48·02	175·42	96·04	263·14	144·06	350·85	192·08	438·57	240·11	18
45	87·67	48·09	175·34	96·19	263·02	144·29	350·69	192·39	438·36	240·49	15
48	87·63	48·17	175·26	96·35	262·89	144·52	350·52	192·70	438·15	240·87	12
51	87·58	48·25	175·18	96·50	262·76	144·75	350·35	193·01	437·94	241·25	9
54	87·54	48·32	175·09	96·65	262·63	144·98	350·18	193·31	437·73	241·64	6
57	87·50	48·40	175·01	96·80	262·51	145·21	350·01	193·62	437·52	242·02	3
60	87·46	48·48	174·92	96·96	262·38	145·44	349·84	193·92	437·31	242·40	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Minutes.
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

61 Deg.

28 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	529·77	281·68	618·06	328·63	706·36	375·57	794·65	422·52	882·95	469·47	60
3	529·52	282·14	617·77	329·17	706·03	376·19	794·28	423·22	882·54	470·24	57
6	529·27	282·60	617·48	329·70	705·70	376·80	793·91	423·91	882·12	471·01	54
9	529·03	283·06	617·20	330·24	705·37	377·42	793·54	424·60	881·71	471·78	51
12	528·78	283·53	616·91	330·78	705·04	378·04	793·17	425·29	881·30	472·55	48
15	528·53	283·99	616·62	331·32	704·71	378·66	792·80	425·98	880·89	473·32	45
18	528·28	284·45	616·33	331·86	704·38	379·27	792·43	426·67	880·47	474·08	42
21	528·03	284·91	616·04	332·40	704·05	379·89	792·06	427·36	880·06	474·85	39
24	527·78	285·37	615·75	332·93	703·71	380·50	791·68	428·06	879·64	475·62	36
27	527·54	285·83	615·46	333·47	703·38	381·11	791·31	428·75	879·23	476·39	33
30	527·29	286·29	615·16	334·01	703·05	381·72	790·93	429·44	878·81	477·15	30
33	527·04	286·75	614·87	334·55	702·71	382·33	790·56	430·13	878·40	477·92	27
36	526·79	287·21	614·58	335·08	702·38	382·95	790·18	430·82	877·98	478·69	24
39	526·54	287·67	614·29	335·62	702·05	383·56	789·81	431·51	877·56	479·46	21
42	526·28	288·13	614·00	336·15	701·71	384·17	789·43	432·20	877·14	480·22	18
45	526·03	288·59	613·71	336·69	701·38	384·78	789·05	432·89	876·72	480·99	15
48	525·78	289·05	613·41	337·22	701·04	385·40	788·67	433·57	876·30	481·76	12
51	525·53	289·51	613·12	337·76	700·70	386·01	788·29	434·26	875·88	482·52	9
54	525·27	289·96	612·82	338·29	700·37	386·62	787·91	434·95	875·46	483·28	6
57	525·02	290·42	612·53	338·83	700·03	387·23	787·53	435·64	875·04	484·04	3
60	524·77	290·88	612·23	339·36	699·69	387·84	787·15	436·33	874·62	484·80	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

61 Deg.

29 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	87·46	48·48	174·92	96·96	262·38	145·44	349·84	193·92	437·31	242·40	60
3	87·42	48·55	174·84	97·11	262·26	145·67	349·67	194·22	437·10	242·78	57
6	87·37	48·63	174·75	97·26	262·13	145·90	349·50	194·53	436·88	243·16	54
9	87·33	48·70	174·67	97·41	262·00	146·13	349·33	194·83	436·67	243·54	51
12	87·29	48·78	174·58	97·57	261·87	146·35	349·16	195·14	436·46	243·92	48
15	87·25	48·86	174·50	97·72	261·75	146·58	348·99	195·44	436·25	244·30	45
18	87·20	48·93	174·41	97·87	261·62	146·81	348·82	195·75	436·03	244·69	42
21	87·16	49·01	174·33	98·02	261·49	147·04	348·65	196·05	435·82	245·07	39
24	87·12	49·09	174·24	98·18	261·36	147·27	348·48	196·36	435·60	245·45	36
27	87·07	49·16	174·16	98·33	261·23	147·50	348·31	196·66	435·39	245·83	33
30	87·03	49·24	174·07	98·48	261·10	147·72	348·14	196·96	435·17	246·21	30
33	86·99	49·31	173·98	98·63	260·97	147·95	347·97	197·26	434·96	246·59	27
36	86·94	49·39	173·89	98·78	260·84	148·18	347·79	197·57	434·74	246·97	24
39	86·90	49·47	173·81	98·93	260·71	148·41	347·62	197·87	434·53	247·35	21
42	86·86	49·54	173·72	99·09	260·58	148·63	347·45	198·18	434·31	247·72	18
45	86·82	49·62	173·64	99·24	260·45	148·86	347·28	198·48	434·10	248·10	15
48	86·77	49·69	173·55	99·39	260·32	149·09	347·10	198·78	433·88	248·48	12
51	86·73	49·77	173·46	99·54	260·19	149·32	346·93	199·08	433·66	248·86	9
54	86·68	49·84	173·37	99·69	260·06	149·54	346·75	199·39	433·44	249·24	6
57	86·64	49·92	173·29	99·84	259·93	149·77	346·58	199·69	433·23	249·62	3
60	86·60	50·00	173·20	100·00	259·80	150·00	346·40	200·00	433·01	250·00	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Minutes.
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

60 Deg.

29 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	524·77	290·88	612·23	339·36	699·69	387·84	787·15	436·33	874·62	484·80	60
3	524·52	291·34	611·94	339·89	699·35	388·45	786·77	437·02	874·20	485·56	57
6	524·26	291·80	611·64	340·43	699·01	389·06	786·39	437·70	873·77	486·33	54
9	524·01	292·26	611·34	340·96	698·67	389·67	786·01	438·39	873·35	487·09	51
12	523·75	292·71	611·04	341·50	698·33	390·28	785·63	439·07	872·92	487·85	48
15	523·50	293·17	610·74	342·03	697·99	390·89	785·25	439·76	872·50	488·61	45
18	523·24	293·62	610·44	342·56	697·65	391·50	784·86	440·44	872·07	489·38	42
21	522·98	294·08	610·14	343·09	697·31	392·11	784·48	441·13	871·64	490·14	39
24	522·72	294·54	609·84	343·63	696·97	392·72	784·09	441·81	871·21	490·90	36
27	522·47	295·00	609·54	344·16	696·63	393·33	783·71	442·50	870·78	491·66	33
30	522·21	295·45	609·24	344·69	696·28	393·93	783·32	443·18	870·35	492·42	30
33	521·95	295·91	608·94	345·22	695·94	394·54	782·93	443·87	869·92	493·18	27
36	521·69	296·36	608·64	345·75	695·59	395·15	782·54	444·55	869·49	493·94	24
39	521·43	296·82	608·34	346·28	695·25	395·76	782·15	445·23	869·06	494·70	21
42	521·17	297·27	608·04	346·82	694·90	396·36	781·76	445·91	868·63	495·45	18
45	520·91	297·73	607·74	347·35	694·56	396·96	781·37	446·59	868·20	496·21	15
48	520·65	298·18	607·43	347·88	694·21	397·57	780·98	447·27	867·76	496·97	12
51	520·39	298·64	607·13	348·41	693·86	398·18	780·59	447·95	867·33	497·73	9
54	520·13	299·09	606·82	348·94	693·51	398·78	780·20	448·63	866·89	498·48	6
57	519·87	299·55	606·52	349·47	693·16	399·39	779·81	449·31	866·46	499·24	3
60	519·61	300·00	606·21	350·00	692·81	400·00	779·42	450·00	866·02	500·00	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

60 Deg.

30 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	86·60	50·00	173·20	100·00	259·80	150·00	346·40	200·00	433·01	250·00	60
3	86·55	50·07	173·12	100·15	259·67	150·23	346·23	200·30	432·79	250·38	57
6	86·51	50·15	173·03	100·30	259·54	150·45	346·05	200·60	432·57	250·75	54
9	86·47	50·22	172·94	100·45	259·41	150·68	345·88	200·90	432·35	251·13	51
12	86·42	50·30	172·85	100·60	259·28	150·90	345·70	201·20	432·13	251·50	48
15	86·38	50·37	172·76	100·75	259·15	151·13	345·53	201·50	431·91	251·88	45
18	86·33	50·45	172·67	100·90	259·01	151·35	345·35	201·81	431·69	252·26	42
21	86·29	50·52	172·59	101·05	258·88	151·58	345·18	202·11	431·47	252·64	39
24	86·25	50·60	172·50	101·20	258·75	151·80	345·00	202·41	431·25	253·01	36
27	86·20	50·67	172·41	101·35	258·62	152·03	344·83	202·71	431·03	253·39	33
30	86·16	50·75	172·32	101·50	258·48	152·26	344·65	203·01	430·81	253·76	30
33	86·11	50·82	172·23	101·65	258·35	152·49	344·47	203·31	430·59	254·14	27
36	86·07	50·90	172·14	101·80	258·22	152·71	344·29	203·61	430·37	254·52	24
39	86·03	50·97	172·06	101·95	258·09	152·94	344·12	203·91	430·15	254·90	21
42	85·98	51·05	171·97	102·10	257·95	153·16	343·94	204·21	429·92	255·27	18
45	85·94	51·12	171·88	102·25	257·82	153·39	343·76	204·51	429·70	255·65	15
48	85·89	51·20	171·79	102·40	257·68	153·61	343·58	204·81	429·48	256·02	12
51	85·85	51·27	171·70	102·55	257·55	153·84	343·40	205·11	429·26	256·40	9
54	85·80	51·35	171·61	102·70	257·41	154·06	343·22	205·41	429·03	256·77	6
57	85·76	51·42	171·52	102·85	257·28	154·29	343·04	205·71	428·81	257·14	3
60	85·71	51·50	171·43	103·00	257·15	154·51	342·86	206·01	428·58	257·51	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

59 Deg.

30 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	519·61	300·00	606·21	350·00	692·81	400·00	779·42	450·00	866·02	500·00	60
3	519·35	300·45	605·91	350·53	692·47	400·60	779·03	450·68	865·59	500·76	57
6	519·09	300·90	605·60	351·05	692·12	401·20	778·63	451·36	865·15	501·51	54
9	518·83	301·35	605·30	351·58	691·77	401·80	778·24	452·04	864·71	502·26	51
12	518·56	301·81	604·99	352·11	691·41	402·41	777·84	452·71	864·27	503·01	48
15	518·30	302·26	604·68	352·64	691·06	403·01	777·45	453·39	863·83	503·76	45
18	518·03	302·71	604·37	353·16	690·71	403·62	777·05	454·07	863·39	504·52	42
21	517·77	303·16	604·06	353·69	690·36	404·22	776·66	454·75	862·95	505·27	39
24	517·50	303·61	603·75	354·22	690·01	404·82	776·26	455·43	862·51	506·03	36
27	517·24	304·06	603·45	354·75	689·66	405·42	775·86	456·11	862·07	506·78	33
30	516·97	304·52	603·14	355·27	689·30	406·03	775·46	456·78	861·62	507·53	30
33	516·71	304·97	602·83	355·80	688·95	406·63	775·06	457·46	861·18	508·28	27
36	516·44	305·42	602·52	356·32	688·59	407·23	774·66	458·13	860·74	509·04	24
39	516·17	305·87	602·21	356·85	688·24	407·83	774·26	458·81	860·30	509·79	21
42	515·90	306·33	601·89	357·38	687·88	408·43	773·86	459·48	859·85	510·54	18
45	515·64	306·78	601·58	357·91	687·52	409·03	773·46	460·16	859·40	511·29	15
48	515·37	307·23	601·27	358·43	687·16	409·63	773·06	460·83	858·95	512·04	12
51	515·10	307·68	600·96	358·95	686·81	410·23	772·66	461·51	858·51	512·79	9
54	514·83	308·13	600·64	359·47	686·45	410·83	772·25	462·18	858·06	513·54	6
57	514·57	308·58	600·33	359·99	686·09	411·43	771·85	462·86	857·61	514·29	3
60	514·30	309·02	600·01	360·52	685·73	412·03	771·45	463·53	857·16	515·04	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

59 Deg.

31 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	85·71	51·50	171·43	103·00	257·15	154·51	342·86	206·01	428·58	257·51	60
3	85·67	51·57	171·34	103·15	257·02	154·73	342·68	206·31	428·36	257·88	57
6	85·62	51·65	171·25	103·30	256·88	154·95	342·50	206·61	428·13	258·26	54
9	85·58	51·72	171·16	103·45	236·74	155·17	342·32	206·91	427·91	258·63	51
12	85·53	51·80	171·07	103·60	256·60	155·40	342·14	207·21	427·68	259·01	48
15	85·49	51·87	170·98	103·75	256·47	155·62	341·96	207·51	427·46	259·38	45
18	85·44	51·95	170·89	103·90	256·33	155·85	341·78	207·80	427·23	259·75	42
21	85·40	52·02	170·80	104·05	256·20	156·07	341·60	208·10	427·00	260·12	39
24	85·35	52·10	170·70	104·20	256·06	156·30	341·41	208·40	426·77	260·50	36
27	85·31	52·17	170·61	104·35	255·93	156·52	341·23	208·70	426·55	260·87	33
30	85·26	52·24	170·52	104·50	255·79	156·74	341·05	209·00	426·32	261·24	30
33	85·21	52·32	170·43	104·65	255·65	156·96	340·87	209·30	426·09	261·62	27
36	85·17	52·39	170·34	104·79	255·51	157·19	340·69	209·59	425·86	261·99	24
39	85·12	52·47	170·25	104·94	255·38	157·41	340·51	209·89	425·63	262·36	21
42	85·08	52·54	170·16	105·09	255·24	157·64	340·32	210·19	425·40	262·73	18
45	85·03	52·62	170·07	105·24	255·10	157·86	340·14	210·49	425·17	263·10	15
48	84·98	52·69	169·97	105·39	254·96	158·08	339·95	210·79	424·94	263·47	12
51	84·94	52·76	169·88	105·54	254·83	158·30	339·77	211·08	424·71	263·84	9
54	84·89	52·84	169·79	105·68	254·69	158·53	339·58	211·37	424·48	264·21	6
57	84·85	52·91	169·70	105·83	254·55	158·75	339·40	211·67	424·28	264·58	3
60	84·80	52·99	169·60	105·98	254·41	158·97	339·21	211·96	424·02	264·95	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

58 Deg.

31 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	514·30	309·02	600·01	360·52	685·73	412·03	771·45	463·53	857·16	515·04	60
3	514·03	309·47	599·70	361·04	685·37	412·63	771·05	464·20	856·71	515·79	57
6	513·76	309·91	599·38	361·57	685·01	413·22	770·64	464·88	856·26	516·53	54
9	513·49	310·36	599·07	362·09	684·65	413·82	770·23	465·55	855·81	517·28	51
12	513·21	310·81	598·75	362·61	684·29	414·42	769·83	466·22	855·36	518·02	48
15	512·94	311·26	598·44	363·13	683·93	415·02	769·42	466·89	854·91	518·77	45
18	512·67	311·71	598·12	363·66	683·56	415·61	769·01	467·56	854·45	519·51	42
21	512·40	312·15	597·80	364·18	683·20	416·21	768·60	468·23	854·00	520·26	39
24	512·13	312·60	597·48	364·70	682·83	416·80	768·19	468·90	853·55	521·00	36
27	511·86	313·05	597·16	365·22	682·47	417·40	767·78	469·57	853·10	521·75	33
30	511·58	313·49	596·84	365·74	682·11	418·00	767·37	470·24	852·64	522·49	30
33	511·31	313·94	596·52	366·27	681·75	418·59	766·94	470·91	852·18	523·24	27
36	511·03	314·39	596·20	366·79	681·38	419·18	766·55	471·58	851·72	523·98	24
39	510·76	314·84	595·88	367·31	681·01	419·78	766·14	472·25	851·27	524·73	21
42	510·48	315·28	595·56	367·83	680·64	420·37	765·73	472·92	850·81	525·47	18
45	510·21	315·73	595·24	368·35	680·28	420·97	765·32	473·59	850·35	526·21	15
48	509·93	316·17	594·92	368·87	679·91	421·56	764·90	474·26	849·89	526·95	12
51	509·66	316·62	594·60	369·39	679·54	422·16	764·49	474·93	849·43	527·69	9
54	509·38	317·06	594·28	369·90	679·17	422·75	764·07	475·59	848·97	528·43	6
57	509·10	317·51	593·96	370·42	678·80	423·34	763·66	476·26	848·51	529·17	3
60	508·82	317·95	593·63	370·94	678·43	423·93	763·24	476·92	848·04	529·91	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

58 Deg.

32 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	84 80	52·99	169·60	105·98	254·41	158·97	339·21	211·96	424·02	264·95	60
3	84·75	53·06	169·51	106·13	254·27	159·19	339·03	212·25	423·79	265·32	57
6	84·71	53·13	169·42	106·27	254·13	159·41	338·84	212·55	423·56	265·69	54
9	84·66	53·21	169·33	106·42	253·99	159·63	338·66	212·85	423·33	266·06	51
12	84·61	53·28	169·23	106·57	253·85	159·86	338·47	213·15	423·09	266·43	48
15	84·57	53·36	169·14	106·72	253·71	160·08	338·29	213·45	422·86	266·80	45
18	84·52	53·43	169·05	106·87	253·57	160·30	338·10	213·74	422·63	267·17	42
21	84·47	53·50	168·96	107·02	253·43	160·52	337·92	214·04	422·40	267·54	39
24	84·43	53·58	168·86	107·16	253·29	160·74	337·73	214·33	422·16	267·91	36
27	84·38	53·65	168·77	107·31	253·15	160·96	337·54	214·63	421·93	268·28	33
30	84·33	53·73	168·67	107·46	253·01	161·18	337·35	214·92	421·69	268·65	30
33	84·29	53·80	168·58	107·61	252·87	161·40	337·17	215·21	421·46	269·02	27
36	84·24	53·87	168·49	107·75	252·73	161·62	336·98	215·50	421·22	269·38	24
39	84·19	53·95	168·40	107·90	252·59	161·85	336·79	215·80	420·99	269·75	21
42	84·15	54·02	168·30	108·04	252·45	162·07	336·60	216·09	420·75	270·12	18
45	84·10	54·09	168·21	108·19	252·31	162·29	336·41	216·39	420·52	270·49	15
48	84·05	54·17	168·11	108·34	252·17	162·51	336·22	216·68	420·28	270·85	12
51	84·01	54·24	168·02	108·49	252·03	162·73	336·03	216·97	420·05	271·22	9
54	83·96	54·31	167·92	108·63	251·88	162·95	335·84	217·26	419·81	271·58	6
57	83·91	54·39	167·83	108·78	251·74	163·17	335·65	217·56	419·57	271·95	3
60	83·86	54·46	167·73	108·92	251·60	163·38	335·46	217·85	419·33	272·31	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

57 Deg.

32 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	508·82	317·95	593·63	370·94	678·43	423·93	763·24	476·92	848·04	529·91	60
3	508·55	318·39	593·31	371·46	678·06	424·52	762·82	477·58	847·58	530·65	57
6	508·27	318·83	592·98	371·97	677·69	425·11	762·40	478·25	847·11	531·39	54
9	507·99	319·27	592·66	372·49	677·32	425·70	761·99	478·92	846·65	532·13	51
12	507·71	319·72	592·33	373·01	676·95	426·30	761·57	479·59	846·19	532·87	48
15	507·43	320·16	592·01	373·53	676·58	426·89	761·15	480·25	845·73	533·61	45
18	507·15	320·61	591·68	374·04	676·20	427·48	760·73	480·91	845·26	534·35	42
21	506·87	321·05	591·36	374·56	675·83	428·07	760·31	481·57	844·79	535·09	39
24	506·59	321·49	591·03	375·07	675·46	428·66	759·89	482·24	844·32	535·82	36
27	506·31	321·93	590·70	375·59	675·09	429·25	759·47	482·90	843·86	536·56	33
30	506·03	322·37	590·37	376·10	674·71	429·84	759·05	483·56	843·39	537·30	30
33	505·75	322·81	590·04	376·62	674·34	430·43	758·63	484·23	842·92	538·04	27
36	505·47	323·26	589·71	377·13	673·96	431·01	758·20	484·89	842·45	538·77	24
39	505·19	323·70	589·38	377·65	673·58	431·60	757·78	485·55	841·98	539·51	21
42	504·90	324·14	589·05	378·16	673·20	432·19	757·36	486·21	841·51	540·24	18
45	504·62	324·58	588·72	378·68	672·83	432·78	756·94	486·89	841·04	540·97	15
48	504·33	325·02	588·39	379·19	672·45	433·36	756·51	487·53	840·56	541·70	12
51	504·05	325·46	588·06	379·71	672·07	433·95	756·08	488·19	840·09	542·44	9
54	503·77	325·90	587·73	380·22	671·69	434·53	755·65	488·85	839·61	543·17	6
57	503·49	326·34	587·40	380·73	671·31	435·12	755·23	489·51	839·14	543·90	3
60	503·20	326·78	587·07	381·24	670·93	435·71	754·80	490·17	838·67	544·63	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

57 Deg.

33 Deg.

Minutes.	Dist. 100. Lat. Per.	Dist. 100. Dep. Base.	Dist. 200. Lat. Per.	Dist. 200. Dep. Base.	Dist. 300. Lat. Per.	Dist. 300. Dep. Base.	Dist. 400. Lat. Per.	Dist. 400. Dep. Base.	Dist. 500. Lat. Per.	Dist. 500. Dep. Base.	
0	83·86	54·46	167·73	108·92	251·60	163·38	335·46	217·85	419·33	272·31	60
3	83·81	54·53	167·64	109·07	251·46	163·60	335·27	218·14	419·09	272·68	57
6	83·77	54·61	167·54	109·22	251·31	163·82	335·08	218·44	418·85	273·05	54
9	83·72	54·68	167·45	109·37	251·17	164·04	334·89	218·73	418·62	273·42	51
12	83·67	54·75	167·35	109·51	251·02	164·26	334·70	219·02	418·38	273·78	48
15	83·62	54·82	167·26	109·66	250·88	164·48	334·51	219·31	418·14	274·15	45
18	83·58	54·90	167·16	109·80	250·74	164·70	334·32	219·60	417·90	274·51	42
21	83·53	54·97	167·06	109·95	250·60	164·92	334·13	219·89	417·66	274·88	39
24	83·48	55·04	166·96	110·09	250·45	165·14	333·93	220·19	417·42	275·24	36
27	83·43	55·12	166·87	110·24	250·31	165·36	333·74	220·48	417·18	275·60	33
30	83·38	55·19	166·77	110·38	250·16	165·58	333·55	220·77	416·94	275·96	30
33	83·34	55·26	166·68	110·53	250·02	165·80	333·36	221·06	416·70	276·33	27
36	83·29	55·33	166·58	110·67	249·87	166·01	333·16	221·35	416·46	276·69	24
39	83·24	55·41	166·49	110·82	249·73	166·23	332·97	221·64	416·22	277·06	21
42	83·19	55·48	166·39	110·96	249·58	166·45	332·78	221·93	415·97	277·42	18
45	83·14	55·55	166·29	111·11	249·44	166·67	332·59	222·22	415·73	277·78	15
48	83·09	55·62	166·19	111·25	249·29	166·88	332·39	222·51	415·49	278·14	12
51	83·05	55·70	166·10	111·40	249·15	167·10	332·20	222·80	415·25	278·50	9
54	83·00	55·77	166·00	111·54	249·00	167·32	332·00	223·09	415·00	278·87	6
57	82·95	55·84	165·90	111·69	248·86	167·54	331·81	223·38	414·76	279·23	3
60	82·90	55·91	165·80	111·83	248·71	167·75	331·61	223·67	414·51	279·59	0
	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Minutes.
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

56 Deg.

33 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	503·20	326·78	587·07	381·24	670·93	435·71	754·80	490·17	838·67	544·63	60
3	502·92	327·22	586·74	381·76	670·55	436·29	754·37	490·83	838·19	545·36	57
6	502·63	327·66	586·40	382·27	670·17	436·88	753·94	491·49	837·71	546·10	54
9	502·34	328·10	586·07	382·79	669·79	437·46	753·51	492·15	837·24	546·83	51
12	502·05	328·53	585·73	383·30	669·41	438·05	753·08	492·80	836·76	547·56	48
15	501·77	328·97	585·40	383·81	669·03	438·63	752·65	493·46	836·28	548·29	45
18	501·48	329·41	585·06	384·32	668·64	439·21	752·22	494·12	835·80	549·02	42
21	501·19	329·85	584·72	384·83	668·26	439·79	751·79	494·78	835·32	549·75	39
24	500·90	330·28	584·39	385·34	667·87	440·38	751·36	495·43	834·84	550·48	36
27	500·62	330·72	584·05	385·85	667·49	440·96	750·93	496·09	834·36	551·21	33
30	500·33	331·16	583·71	386·36	667·10	441·54	750·49	496·74	833·88	551·93	30
33	500·04	331·60	583·38	386·87	666·72	442·12	750·06	497·40	833·40	552·66	27
36	499·75	332·03	583·04	387·38	666·33	442·70	749·62	498·05	832·91	553·39	24
39	499·46	332·47	582·70	387·89	665·95	443·29	749·19	498·71	832·43	554·12	21
42	499·17	332·90	582·36	388·39	665·56	443·87	748·75	499·36	831·95	554·84	18
45	498·88	333·34	582·02	388·90	665·17	444·45	748·32	500·01	831·47	555·57	15
48	498·58	333·77	581·68	389·40	664·78	445·03	747·88	500·66	830·98	556·29	12
51	498·29	334·21	581·34	389·91	664·39	445·61	747·45	501·32	830·50	557·02	9
54	498·00	334·64	581·00	390·42	664·00	446·19	747·01	501·97	830·01	557·74	6
57	497·71	335·08	580·66	390·92	663·61	446·77	746·57	502·62	829·52	558·47	3
60	497·42	335·51	580·32	391·43	663·22	447·35	746·13	503·27	829·03	559·19	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

56 Deg.

34 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	82 90	55·91	165·80	111·83	248·71	167·75	331·61	223·67	414·51	279·59	60
3	82·85	55·99	165·71	111·97	248·56	167·97	331·42	223·96	414·27	279·95	57
6	82·80	56·06	165·61	112·12	248·41	168·19	331·22	224·25	414·03	280·31	54
9	82·75	56·13	165·51	112·26	248·27	168·41	331·03	224·54	413·78	280·67	51
12	82·70	56·20	165·41	112·41	248·12	168·62	330·83	224·83	413·53	281·04	48
15	82·65	56·28	165·31	112·55	247·97	168·84	330·63	225·12	413·29	281·40	45
18	82·60	56·35	165·21	112·70	247·82	169·05	330·43	225·41	413·04	281·76	42
21	82·56	56·42	165·12	112·84	247·68	169·27	330·24	225·70	412·80	282·12	39
24	82·51	56·49	165·02	112·99	247·53	169·48	330·04	225·98	412·55	282·48	36
27	82·46	56·56	164·92	113·13	247·38	169·70	329·85	226·27	412·31	282·84	33
30	82·41	56·64	164·82	113·28	247·23	169·92	329·65	226·56	412·06	283·20	30
33	82·36	56·71	164·72	113·42	247·09	170·14	329·45	226·85	411·81	283·56	27
36	82·31	56·78	164·62	113·56	246·94	170·35	329·25	227·13	411·56	283·92	24
39	82·26	56·85	164·52	113·70	246·79	170·57	329·05	227·42	411·32	284·28	21
42	82·21	56·92	164·42	113·85	246·64	170·78	328·85	227·71	411·07	284·63	18
45	82·16	56·99	164·33	113·99	246·49	170·99	328·65	227·99	410·82	284·99	15
48	82·11	57·07	164·23	114·14	246·34	171·21	328·45	228·28	410·57	285·35	12
51	82·06	57·14	164·13	114·28	246·19	171·43	328·26	228·57	410·32	285·71	9
54	82·01	57·21	164·03	114·42	246·04	171·64	328·06	228·85	410·07	286·07	6
57	81·96	57·28	163·93	114·56	245·89	171·86	327·86	229·14	409·82	286·43	3
60	81·91	57·35	163·83	114·71	245·74	172·07	327·66	229·43	409·57	286·78	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

55 Deg.

34 Deg.

Minutes.	Dist. 600. Lat. Per.	Dist. 600. Dep. Base.	Dist. 700. Lat. Per.	Dist. 700. Dep. Base.	Dist. 800. Lat. Per.	Dist. 800. Dep. Base.	Dist. 900. Lat. Per.	Dist. 900. Dep. Base.	Dist. 1000. Lat. Per.	Dist. 1000. Dep. Base.	
0	497·42	335·51	580·32	391·43	663·22	447·35	746·13	503·27	829·03	559·19	60
3	497·13	335·94	579·98	391·93	662·83	447·93	745·69	503·92	828·54	559·91	57
6	496·83	336·38	579·64	392·44	662·44	448·51	745·25	504·57	828·05	560·63	54
9	496·54	336·81	579·30	392·94	662·05	449·09	744·81	505·22	827·56	561·35	51
12	496·24	337·24	578·95	393·45	661·66	449·66	744·37	505·87	827·07	562·08	48
15	495·95	337·67	578·61	393·95	661·27	450·24	743·93	506·52	826·58	562·80	45
18	495·65	338·11	578·26	394·46	660·87	450·82	743·48	507·17	826·09	563·52	42
21	495·36	338·54	577·92	394·96	660·48	451·40	743·04	507·82	825·60	564·24	39
24	495·06	338·97	577·57	395·47	660·09	451·97	742·60	508·47	825·11	564·96	36
27	494·77	339·40	577·23	395·97	659·70	452·55	742·16	509·12	824·62	565·68	33
30	494·47	339·84	576·88	396·48	659·30	453·12	741·71	509·76	824·12	566·40	30
33	494·18	340·27	576·54	396·98	658·90	453·70	741·27	510·41	823·63	567·12	27
36	493·88	340·70	576·19	397·49	658·50	454·27	740·82	511·05	823·13	567·84	24
39	493·58	341·13	575·85	397·99	658·11	454·85	740·37	511·70	822·64	568·56	21
42	493·28	341·56	575·50	398·49	657·71	455·42	739·92	512·35	822·14	569·27	18
45	492·99	341·99	575·15	398·99	657·32	455·99	739·48	512·99	821·64	569·99	15
48	492·69	342·42	574·80	399·49	656·92	456·57	739·03	513·64	821·14	570·71	12
51	492·39	342·85	574·45	399·99	656·52	457·14	738·58	514·29	820·65	571·43	9
54	492·09	343·28	574·10	400·50	656·12	457·71	738·13	514·93	820·15	572·14	6
57	491·79	343·71	573·75	401·00	655·72	458·28	737·68	515·57	819·65	572·86	3
60	491·49	344·14	573·40	401·50	655·32	458·86	737·23	516·21	819·15	573·57	0
	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Base. Dep.	Per. Lat.	Minutes.
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

55 Deg.

35 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	81·91	57·35	163·83	114·71	245·74	172·07	327·66	229·43	409·57	286·78	60
3	81·86	57·42	163·72	114·85	245·59	172·28	327·46	229·72	409·32	287·14	57
6	81·81	57·50	163·62	115·00	245·44	172·50	327·26	230·00	409·07	287·50	54
9	81·76	57·57	163·52	115·14	245·29	172·71	327·06	230·29	408·82	287·86	51
12	81·71	57·64	163·42	115·28	245·14	172·92	326·85	230·57	408·57	288·21	48
15	81·66	57·71	163·32	115·42	244·99	173·13	326·65	230·86	408·32	288·57	45
18	81·61	57·78	163·22	115·57	244·84	173·35	326·45	231·14	408·07	288·92	42
21	81·56	57·86	163·12	115·71	244·69	173·56	326·25	231·43	407·82	289·28	39
24	81·51	57·93	163·02	115·85	244·53	173·78	326·05	231·71	407·56	289·64	36
27	81·46	58·00	162·92	115·99	244·38	173·99	325·85	232·00	407·31	290·00	33
30	81·41	58·07	162·82	116·14	244·23	174·21	325·64	232·28	407·05	290·35	30
33	81·36	58·14	162·72	116·28	244·08	174·42	325·44	232·57	406·80	290·71	27
36	81·31	58·21	162·62	116·42	243·93	174·63	325·24	232·85	406·55	291·06	24
39	81·26	58·28	162·52	116·56	243·78	174·84	325·04	233·13	406·30	291·42	21
42	81·21	58·35	162·41	116·70	243·62	175·06	324·83	233·41	406·04	291·77	18
45	81·16	58·42	162·31	116·84	243·47	175·27	324·63	233·70	405·79	292·12	15
48	81·10	58·49	162·21	116·99	243·31	175·48	324·42	233·98	405·53	292·47	12
51	81·05	58·57	162·11	117·13	243·16	175·69	324·22	234·27	405·28	292·83	9
54	81·00	58·64	162·00	117·27	243·01	175·91	324·01	234·55	405·02	293·18	6
57	80·95	58·71	161·90	117·41	242·86	176·12	323·81	234·83	404·76	293·54	3
60	80·90	58·78	161·80	117·55	242·70	176·33	323·60	234·11	404·50	293·89	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

54 Deg.

35 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	491·49	344·14	573·40	401·50	655·32	458·86	737·23	516·21	819·15	573·57	60
3	491·19	344·57	573·05	402·00	654·92	459·43	736·78	516·85	818·65	574·28	57
6	490·88	345·00	572·70	402·50	654·51	460·00	736·33	517·50	818·14	575·00	54
9	490·58	345·43	572·35	403·00	654·11	460·57	735·88	518·14	817·64	575·71	51
12	490·28	345·85	572·00	403·50	653·71	461·14	735·42	518·78	817·14	576·43	48
15	489·98	346·28	571·65	404·00	653·31	461·71	734·97	519·42	816·64	577·14	45
18	489·68	346·71	571·29	404·50	652·90	462·28	734·52	520·07	816·13	577·85	42
21	489·38	347·14	570·94	405·00	652·50	462·85	734·07	520·71	815·63	578·56	39
24	489·07	347·57	570·58	405·50	652·10	463·42	733·61	521·35	815·12	579·28	36
27	488·77	348·00	570·23	406·00	651·70	464·00	733·16	522·00	814·62	580·00	33
30	488·46	348·42	569·88	406·49	651·29	464·57	732·70	522·64	814·11	580·71	30
33	488·16	348·85	569·53	406·99	650·89	465·13	732·25	523·27	813·61	581·42	27
36	487·86	349·27	569·17	407·49	650·48	465·70	731·79	523·91	813·10	582·13	24
39	487·56	349·70	568·81	407·98	650·07	466·26	731·33	524·55	812·59	582·84	21
42	487·25	350·12	568·45	408·48	649·66	466·83	730·87	525·18	812·08	583·54	18
45	486·94	350·55	568·10	408·97	649·26	467·39	730·41	525·82	811·57	584·25	15
48	486·63	350·97	567·74	409·47	648·85	467·96	729·95	526·46	811·06	584·95	12
51	486·33	351·40	567·38	409·96	648·44	468·52	729·49	527·10	810·55	585·66	9
54	486·02	351·82	567·03	410·46	648·03	469·09	729·03	527·73	810·04	586·37	6
57	485·71	352·25	566·67	410·95	647·62	469·65	728·57	528·37	809·53	587·07	3
60	485·40	352·67	566·31	411·45	647·21	470·22	728·11	529·00	809·01	587·78	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

54 Deg.

36 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	80·90	58·78	161·80	117·55	242·70	176·33	323·60	235·11	404·50	293·89	60
3	80·85	58·85	161·70	117·69	242·55	176·54	323·40	235·39	404·25	294·24	57
6	80·80	58·92	161·59	117·83	242·39	176·75	323·19	235·67	403·99	294·59	54
9	80·74	58·99	161·49	117·97	242·24	176·96	322·99	235·95	403·74	294·94	51
12	80·69	59·06	161·39	118·12	242·08	177·18	322·78	236·24	403·48	295·30	48
15	80·64	59·13	161·29	118·26	241·93	177·39	322·58	236·52	403·22	295·65	45
18	80·59	59·20	161·18	118·40	241·77	177·60	322·37	236·80	402·96	296·00	42
21	80·54	59·27	161·08	118·54	241·62	177·81	322·16	237·08	402·70	296·35	39
24	80·49	59·34	160·97	118·68	241·46	178·02	321·95	237·36	402·44	296·70	36
27	80·43	59·41	160·87	118·82	241·31	178·23	321·75	237·64	402·18	297·05	33
30	80·38	59·48	160·77	118·96	241·15	178·44	321·54	237·92	401·92	297·41	30
33	80·33	59·55	160·67	119·10	241·00	178·65	321·33	238·20	401·66	297·76	27
36	80·28	59·62	160·56	119·24	240·84	178·86	321·12	238·48	401·40	298·11	24
39	80·23	59·69	160·46	119·38	240·69	179·07	320·92	238·76	401·14	298·46	21
42	80·18	59·76	160·35	119·52	240·53	179·28	320·71	239·04	400·88	298·81	18
45	80·12	59·83	160·25	119·66	240·37	179·49	320·50	239·32	400·62	299·16	15
48	80·07	59·90	160·14	119·80	240·21	179·70	320·29	239·60	400·36	299·51	12
51	80·02	59·97	160·04	119·94	240·06	179·91	320·08	239·88	400·10	299·86	9
54	79·97	60·04	159·93	120·08	239·90	180·12	319·87	240·16	399·84	300·21	6
57	79·92	60·11	159·83	120·22	239·75	180·33	319·66	240·44	399·58	300·56	3
60	79·86	60·18	159·72	120·36	239·59	180·54	319·45	240·72	399·31	300·90	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

53 Deg.

36 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	485·40	352·67	566·31	411·45	647·21	470·22	728·11	529·00	809·01	587·78	60
3	485·10	353·09	565·95	411·94	646·80	470·79	727·65	529·64	808·50	588·49	57
6	484·79	353·51	565·59	412·43	646·39	471·35	727·19	530·27	807·98	589·19	54
9	484·48	353·93	565·23	412·92	645·98	471·91	726·73	530·90	807·47	589·90	51
12	484·17	354·36	564·87	413·42	645·57	472·48	726·26	531·54	806·95	590·60	48
15	483·86	354·78	564·51	413·91	645·16	473·04	725·80	532·17	806·44	591·31	45
18	483·55	355·20	564·15	414·40	644·74	473·60	725·33	532·81	805·92	592·01	42
21	483·24	355·62	563·79	414·89	644·33	474·17	724·87	533·44	805·41	592·72	39
24	482·93	356·05	563·42	415·39	643·91	474·73	724·40	534·07	804·89	593·42	36
27	482·62	356·47	563·06	415·88	643·50	475·29	723·94	534·70	804·37	594·12	33
30	482·31	356·89	562·70	416·37	643·08	475·85	723·47	535·34	803·85	594·82	30
33	482·00	357·31	562·34	416·86	642·67	476·41	723·00	535·97	803·33	595·52	27
36	481·69	357·73	561·97	417·35	642·25	476·98	722·53	536·60	802·81	596·22	24
39	481·38	358·15	561·61	417·84	641·84	477·54	722·06	537·23	802·29	596·92	21
42	481·06	358·57	561·24	418·33	641·42	478·10	721·59	537·85	801·77	597·62	18
45	480·75	358·99	560·88	418·82	641·00	478·66	721·12	538·48	801·25	598·32	15
48	480·43	359·41	560·51	419·31	640·58	479·21	720·65	539·11	800·73	599·02	12
51	480·12	359·83	560·15	419·80	640·16	479·77	720·18	539·74	800·21	599·72	9
54	479·81	360·25	559·78	420·29	639·74	480·33	719·71	540·37	799·68	600·42	6
57	479·50	360·67	559·41	420·78	639·32	480·89	719·24	541·00	799·16	601·12	3
60	479·18	361·09	559·04	421·27	638·90	481·45	718·77	541·63	798·63	601·81	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Minutes.
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

53 Deg.

37 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	79·86	60·18	159·72	120·36	239·59	180·54	319·45	240·72	399·31	300·90	60
3	79·81	60·25	159·62	120·50	239·43	180·75	319·24	241·00	399·05	301·25	57
6	79·75	60·32	159·51	120·64	239·27	180·96	319·03	241·28	398·79	301·60	54
9	79·70	60·39	159·41	120·78	239·11	181·17	318·82	241·56	398·53	301·95	51
12	79·65	60·46	159·30	120·92	238·95	181·38	318·61	241·84	398·26	302·30	48
15	79·60	60·53	159·20	121·06	238·80	181·58	318·40	242·12	398·00	302·65	45
18	79·55	60·60	159·09	121·19	238·64	181·79	318·19	242·39	397·73	302·99	42
21	79·49	60·67	158·99	121·33	238·48	182·00	317·98	242·67	397·47	303·34	39
24	79·44	60·74	158·88	121·47	238·32	182·21	317·76	242·94	397·20	303·69	36
27	79·39	60·81	158·78	121·61	238·16	182·42	317·55	243·22	396·94	304·03	33
30	79·33	60·88	158·67	121·75	238·00	182·62	317·34	243·50	396·67	304·38	30
33	79·28	60·95	158·56	121·89	237·84	182·83	317·13	243·78	396·41	304·73	27
36	79·23	61·01	158·45	122·02	237·68	183·04	316·91	244·05	396·14	305·07	24
39	79·18	61·08	158·35	122·16	237·52	183·25	316·70	244·33	395·88	305·42	21
42	79·12	61·15	158·24	122·30	237·36	183·46	316·49	244·61	395·61	305·76	18
45	79·07	61·22	158·14	122·44	237·20	183·67	316·28	244·89	395·35	306·11	15
48	79·01	61·29	158·03	122·58	237·04	183·87	316·06	245·16	395·08	306·45	12
51	78·96	61·36	157·92	122·72	236·88	184·08	315·85	245·44	394·81	306·80	9
54	78·91	61·43	157·81	122·85	236·72	184·28	315·63	245·71	394·54	307·14	6
57	78·85	61·50	157·71	122·99	236·56	184·49	315·42	245·99	394·27	307·49	3
60	78·80	61·57	157·60	123·13	236·40	184·70	315·20	246·26	394·00	307·83	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

52 Deg.

37 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	479·18	361·09	559·04	421·27	638·90	481·45	718·77	541·63	798·63	601·81	60
3	478·86	361·51	558·67	421·76	638·48	482·01	718·30	542·26	798·11	602·51	57
6	478·55	361·92	558·30	422·24	638·06	482·57	717·82	542·89	797·58	603·21	54
9	478·23	362·34	557·94	422·73	637·64	483·12	717·35	543·52	797·06	603·91	51
12	477·91	362·76	557·57	423·22	637·22	483·68	716·87	544·14	796·53	604·60	48
15	477·60	363·18	557·20	423·71	636·80	484·23	716·40	544·77	796·00	605·30	45
18	477·28	363·59	556·83	424·19	636·37	484·79	715·92	545·39	795·47	605·99	42
21	476·96	364·01	556·46	424·67	635·95	485·35	715·45	546·02	794·94	606·68	39
24	476·64	364·42	556·09	425·16	635·53	485·90	714·97	546·64	794·41	607·37	36
27	476·33	364·84	555·72	425·64	635·11	486·46	714·49	547·26	793·88	608·06	33
30	476·01	365·25	555·34	426·13	634·68	487·01	714·01	547·88	793·35	608·76	30
33	475·69	365·67	554·97	426·62	634·26	487·57	713·54	548·50	792·82	609·45	27
36	475·37	366·08	554·60	427·10	633·83	488·12	713·06	549·13	792·29	610·14	24
39	475·05	366·50	554·23	427·59	633·40	488·67	712·58	549·75	791·76	610·83	21
42	474·73	366·91	553·85	428·07	632·97	489·22	712·10	550·37	791·22	611·52	18
45	474·41	367·33	553·48	428·55	632·55	489·77	711·62	550·99	790·69	612·21	15
48	474·09	367·74	553·10	429·03	632·12	490·32	711·14	551·62	790·15	612·90	12
51	473·77	368·15	552·73	429·52	631·69	490·87	710·66	552·24	789·62	613·59	9
54	473·45	368·57	552·35	430·00	631·26	491·42	710·18	552·85	789·08	614·28	6
57	473·13	368·99	551·98	430·48	630·83	491·98	709·69	553·47	788·55	614·97	3
60	472·80	369·40	551·60	430·96	630·40	492·53	709·21	554·09	788·01	615·66	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

52 Deg.

38 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	78 80	61·57	157·60	123·13	236·40	184·70	315·20	246·26	394·00	307·83	60
3	78·75	61·63	157·49	123·27	236·24	184·91	314·99	246·53	393·74	308·17	57
6	78·69	61·70	157·38	123·40	236·07	185·11	314·77	246·81	393·47	308·51	54
9	78·64	61·77	157·28	123·54	235·91	185·32	314·56	247·08	393·20	308·85	51
12	78·58	61·84	157·17	123·68	235·75	185·52	314·34	247·36	392·93	309·20	48
15	78·53	61·91	157·06	123·82	235·59	185·73	314·13	247·63	392·66	309·54	45
18	78·48	61·98	156·95	123·95	235·43	185·93	313·91	247·91	392·39	309·89	42
21	78·42	62·05	156·84	124·09	235·27	186·14	313·69	248·18	392·12	310·23	39
24	78·37	62·11	156·73	124·23	235·10	186·34	313·47	248·46	391·85	310·57	36
27	78·31	62·18	156·63	124·36	234·94	186·55	313·26	248·73	391·58	310·91	33
30	78·26	62·25	156·52	124·50	234·78	186·75	313·04	249·00	391·30	311·25	30
33	78·21	62·32	156·41	124·64	234·62	186·96	312·82	249·28	391·03	311·59	27
36	78·15	62·39	156·30	124·77	234·45	187·16	312·60	249·55	390·76	311·94	24
39	78·10	62·46	156·19	124·91	234·29	187·37	312·39	249·83	390·49	312·28	21
42	78·04	62·52	156·08	125·04	234·13	187·57	312·17	250·10	390·21	312·62	18
45	77·99	62·59	155·97	125·18	233·97	187·78	311·95	250·37	389·94	312·96	15
48	77·93	62·66	155·86	125·32	233·80	187·98	311·73	250·64	389·67	313·30	12
51	77·88	62·73	155·75	125·46	233·64	188·19	311·51	250·92	389·40	313·64	9
54	77·82	62·79	155·64	125·59	233·47	188·39	311·29	251·19	389·12	313·98	6
57	77·77	62·86	155·53	125·73	233·31	188·59	311·07	251·43	388·85	314·32	3
60	77·71	62·93	155·42	125·86	233·14	188·80	310·85	251·73	388·57	314·66	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

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38 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	472·80	369·40	551·60	430·96	630·40	492·53	709·21	554·09	788·01	615·66	60
3	472·48	369·81	551·23	431·44	629·97	493·08	708·73	554·71	787·47	616·35	57
6	472·16	370·22	550·85	431·92	629·54	493·62	708·24	555·33	786·93	617·03	54
9	471·84	370·63	550·48	432·40	629·11	494·17	707·76	555·95	786·39	617·72	51
12	471·51	371·04	550·10	432·88	628·68	494·72	707·27	556·56	785·85	618·41	48
15	471·19	371·45	549·72	433·36	628·25	495·27	706·79	557·18	785·31	619·10	45
18	470·86	371·86	549·34	433·84	627·82	495·82	706·30	557·80	784·77	619·78	42
21	470·54	372·27	548·96	434·32	627·39	496·37	705·81	558·42	784·23	620·46	39
24	470·21	372·68	548·58	434·80	626·95	496·92	705·32	559·03	783·69	621·14	36
27	469·89	373·09	548·20	435·28	626·52	497·47	704·83	559·65	783·15	621·82	33
30	469·56	373·50	547·82	435·75	626·08	498·01	704·34	560·26	782·60	622·51	30
33	469·24	373·91	547·44	436·23	625·65	498·56	703·85	560·88	782·06	623·19	27
36	468·91	374·32	547·06	436·71	625·21	499·10	703·36	561·49	781·52	623·88	24
39	468·59	374·73	546·68	437·19	624·78	499·65	702·87	562·11	780·98	624·56	21
42	468·26	375·14	546·30	437·67	624·34	500·19	702·38	562·72	780·43	625·24	18
45	467·93	375·55	545·92	438·14	623·91	500·74	701·89	563·33	779·88	625·92	15
48	467·60	375·96	545·53	438·62	623·47	501·28	701·40	563·94	779·33	626·60	12
51	467·27	376·37	545·15	439·10	623·03	501·83	700·91	564·55	778·79	627·28	9
54	466·94	376·77	544·77	439·57	622·59	502·37	700·42	565·16	778·24	627·96	6
57	466·61	377·18	544·39	440·05	622·15	502·91	699·93	565·77	777·69	628·64	3
60	466·28	377·59	544·00	440·52	621·71	503·45	699·43	566·38	777·14	629·32	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

51 Deg.

39 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	77·71	62·93	155·42	125·86	233·14	188·80	310·85	251·73	388·57	314·66	60
3	77·66	63·00	155·31	126·00	232·98	189·00	310·63	252·00	388·30	315·00	57
6	77·60	63·07	155·20	126·13	232·81	189·20	310·41	252·27	388·02	315·34	54
9	77·55	63·13	155·09	126·27	232·65	189·41	310·19	252·54	387·75	315·67	51
12	77·49	63·20	154·98	126·40	232·48	189·61	309·97	252·81	387·47	316·01	48
15	77·44	63·27	154·87	126·54	232·32	189·81	309·75	253·08	387·20	316·35	45
18	77·38	63·34	154·76	126·67	232·15	190·01	309·53	253·35	386·92	316·69	42
21	77·33	63·40	154·65	126·81	231·99	190·21	309·31	253·62	386·64	317·03	39
24	77·27	63·47	154·54	126·94	231·82	190·42	309·09	253·89	386·36	317·36	36
27	77·22	63·54	154·43	127·08	231·65	190·62	308·87	254·16	386·09	317·70	33
30	77·16	63·61	154·32	127·21	231·48	190·82	308·65	254·43	385·81	318·03	30
33	77·11	63·67	154·21	127·35	231·32	191·02	308·43	254·70	385·54	318·37	27
36	77·05	63·74	154·10	127·48	231·15	191·22	308·20	254·97	385·25	318·71	24
39	77·00	63·81	153·99	127·62	230·99	191·42	307·98	255·24	384·98	319·04	21
42	76·94	63·88	153·88	127·75	230·82	191·63	307·76	255·50	384·70	319·38	18
45	76·88	63·94	153·77	127·89	230·65	191·83	307·54	255·77	384·42	319·72	15
48	76·83	64·01	153·65	128·02	230·48	192·03	307·31	256·04	384·14	320·05	12
51	76·77	64·08	153·54	128·16	230·32	192·23	307·09	256·31	383·86	320·39	9
54	76·72	64·14	153·43	128·29	230·15	192·43	306·86	256·58	383·58	320·73	6
57	76·66	64·21	153·32	128·42	229·98	192·63	306·64	256·85	383·30	321·06	3
60	76·60	64·28	153·21	128·55	229·81	192·83	306·41	257·11	383·02	321·39	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

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39 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	466·28	377·59	544·00	440·52	621·71	503·45	699·43	566·38	777·14	629·32	60
3	465·95	378·00	543·62	441·00	621·27	504·00	698·94	567·00	776·59	630·00	57
6	465·62	378·40	543·23	441·47	620·83	504·54	698·44	567·61	776·04	630·67	54
9	465·29	378·81	542·85	441·95	620·39	505·09	697·95	568·21	775·49	631·35	51
12	464·96	379·21	542·46	442·42	619·95	505·63	697·45	568·82	774·94	632·03	48
15	464·63	379·61	542·08	442·90	619·51	506·17	696·95	569·43	774·39	632·70	45
18	464·30	380·02	541·69	443·37	619·07	506·71	696·45	570·04	773·84	633·37	42
21	463·97	380·42	541·30	443·84	618·63	507·25	695·96	570·65	773·29	634·05	39
24	463·64	380·83	540·91	444·31	618·18	507·79	695·46	571·26	772·73	634·73	36
27	463·31	381·23	540·53	444·78	617·74	508·33	694·96	571·87	772·18	635·40	33
30	462·97	381·64	540·14	445·25	617·30	508·86	694·46	572·47	771·62	636·07	30
33	462·64	382·04	539·75	445·72	616·86	509·40	693·96	573·07	771·07	636·75	27
36	462·30	382·45	539·36	446·19	616·41	509·94	693·46	573·68	770·51	637·42	24
39	461·97	382·85	538·97	446·66	615·97	510·48	692·96	574·28	769·96	638·09	21
42	461·64	383·26	538·58	447·13	615·52	511·02	692·46	574·89	769·40	638·76	18
45	461·30	383·66	538·19	447·60	615·07	511·56	691·96	575·49	768·84	639·43	15
48	460·96	384·07	537·80	448·07	614·62	512·07	691·46	576·10	768·28	640·10	12
51	460·63	384·47	537·41	448·54	614·18	512·62	690·96	576·70	767·72	640·77	9
54	460·29	384·87	537·01	449·01	613·73	513·16	690·45	577·30	767·16	641·44	6
57	459·96	385·27	536·62	449·48	613·28	513·69	689·95	577·90	766·60	642·11	3
60	459·62	385·67	536·23	449·95	612·83	514·23	689·44	578·51	766·04	642·78	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

50 Deg.

40 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	76·60	64·28	153·21	128·55	229·81	192·83	306·41	257·11	383·02	321·39	60
3	76·55	64·34	153·10	128·68	229·64	193·03	306·19	257·37	382·74	321·72	57
6	76·49	64·41	152·98	128·82	229·47	193·23	305·96	257·64	382·46	322·06	54
9	76·44	64·48	152·87	128·95	229·30	193·43	305·74	257·91	382·18	322·39	51
12	76·38	64·54	152·75	129·09	229·13	193·63	305·51	258·18	381·89	322·73	48
15	76·32	64·61	152·64	129·22	228·97	193·83	305·29	258·44	381·61	323·06	45
18	76·27	64·68	152·53	129·35	228·80	194·03	305·06	258·71	381·33	323·39	42
21	76·21	64·75	152·42	129·48	228·63	194·23	304·84	258·97	381·05	323·72	39
24	76·15	64·81	152·30	129·62	228·46	194·43	304·61	259·24	380·77	324·06	36
27	76·10	64·88	152·19	129·75	228·29	194·63	304·39	259·50	380·49	324·39	33
30	76·04	64·95	152·08	129·88	228·12	194·83	304·16	259·77	380·20	324·72	30
33	75·98	65·01	151·97	130·01	227·95	195·03	303·94	260·04	379·92	325·05	27
36	75·93	65·08	151·85	130·15	227·78	195·23	303·71	260·30	379·63	325·38	24
39	75·87	65·14	151·74	130·28	227·61	195·43	303·48	260·57	379·35	325·71	21
42	75·81	65·21	151·62	130·42	227·44	195·63	303·25	260·84	379·06	326·04	18
45	75·76	65·28	151·51	130·55	227·27	195·83	303·03	261·10	378·78	326·37	15
48	75·70	65·34	151·40	130·68	227·09	196·02	302·80	261·37	378·49	326·71	12
51	75·64	65·41	151·29	130·81	226·92	196·22	302·57	261·63	378·21	327·04	9
54	75·58	65·47	151·17	130·94	226·75	196·42	302·34	261·89	377·92	327·37	6
57	75·53	65·54	151·06	131·08	226·58	196·62	302·11	262·16	377·64	327·70	3
60	75·47	65·61	150·94	131·21	226·41	196·81	301·88	262·42	377·35	328·03	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

49 Deg.

40 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base	
0	459·62	385·67	536·23	449·95	612·83	514·23	689·44	578·51	766·04	642·78	60
3	459·29	386·07	535·84	450·42	612·38	514·76	688·93	579·11	765·48	643·45	57
6	458·95	386·47	535·44	450·88	611·93	515·30	688·42	579·71	764·92	644·12	54
9	458·61	386·87	535·05	451·35	611·48	515·83	687·92	580·31	764·36	644·79	51
12	458·27	387·27	534·65	451·82	611·03	516·36	687·41	580·91	763·79	645·45	48
15	457·94	387·67	534·26	452·29	610·58	516·89	686·91	581·51	763·23	646·12	45
18	457·60	388·07	533·86	452·75	610·13	517·43	686·40	582·11	762·66	646·79	42
21	457·26	388·47	533·47	453·22	609·68	517·96	685·89	582·71	762·10	647·46	39
24	456·92	388·87	533·07	453·68	609·23	518·49	685·38	583·31	761·53	648·12	36
27	456·58	389·27	532·68	454·15	608·78	519·02	684·87	583·91	760·97	648·79	33
30	456·24	389·67	532·28	454·61	608·32	519·56	684·36	584·50	760·40	649·45	30
33	455·90	390·06	531·89	455·08	607·87	520·09	683·85	585·10	759·84	650·11	27
36	455·56	390·46	531·49	455·54	607·42	520·62	683·34	585·70	759·27	650·77	24
39	455·22	390·86	531·10	456·01	606·97	521·15	682·83	586·29	758·70	651·43	21
42	454·88	391·26	530·70	456·47	606·51	521·68	682·32	586·89	758·13	652·10	18
45	454·54	391·66	530·30	456·93	606·06	522·21	681·81	587·48	757·56	652·76	15
48	454·19	392·05	529·90	457·39	605·60	522·73	681·30	588·08	756·99	653·42	12
51	453·85	392·45	529·50	457·85	605·15	523·26	680·79	588·67	756·42	654·08	9
54	453·51	392·84	529·10	458·32	604·69	523·79	680·27	589·27	755·85	654·74	6
57	453·17	393·24	528·70	458·78	604·23	524·32	679·76	589·86	755·28	655·40	3
60	452·82	393·63	528·30	459·24	603·77	524·84	679·24	590·45	754·71	656·06	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

49 Deg.

41 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	75·47	65·61	150·94	131·21	226·41	196·81	301·88	262·42	377·35	328·03	60
3	75·41	65·67	150·83	131·34	226·24	197·01	301·65	262·68	377·07	328·36	57
6	75·36	65·74	150·71	131·47	226·06	197·21	301·42	262·95	376·78	328·69	54
9	75·30	65·80	150·60	131·60	225·89	197·41	301·19	263·21	376·49	329·01	51
12	75·24	65·87	150·48	131·73	225·72	197·60	300·96	263·47	376·20	329·34	48
15	75·19	65·93	150·37	131·86	225·55	197·80	300·73	263·73	375·92	329·67	45
18	75·13	66·00	150·25	132·00	225·38	198·00	300·50	264·00	375·63	330·00	42
21	75·07	66·07	150·14	132·13	225·21	198·20	300·27	264·26	375·34	330·33	39
24	75·01	66·13	150·02	132·26	225·03	198·39	300·04	264·52	375·05	330·65	36
27	74·95	66·20	149·91	132·39	224·86	198·59	299·81	264·78	374·77	330·98	33
30	74·89	66·26	149·79	132·52	224·68	198·78	299·58	265·04	374·48	331·31	30
33	74·84	66·33	149·68	132·65	224·51	198·98	299·35	265·30	374·19	331·64	27
36	74·78	66·39	149·56	132·78	224·34	199·17	299·12	265·57	373·90	331·96	24
39	74·72	66·46	149·44	132·91	224·17	199·37	298·89	265·83	373·61	332·29	21
42	74·66	66·52	149·32	133·04	223·99	199·56	298·65	266·09	373·32	332·61	18
45	74·60	66·59	149·21	133·17	223·82	199·76	298·42	266·35	373·03	332·94	15
48	74·55	66·65	149·09	133·30	223·64	199·95	298·19	266·61	372·74	333·26	12
51	74·49	66·72	148·98	133·43	223·47	200·15	297·96	266·87	372·45	333·59	9
54	74·43	66·78	148·86	133·56	223·29	200·34	297·72	267·13	372·15	333·91	6
57	74·37	66·85	148·74	133·69	223·12	200·54	297·49	267·39	371·86	334·24	3
60	74·31	66·91	148·62	133·82	222·94	200·73	297·26	267·65	371·57	334·56	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

48 Deg.

41 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	452·82	393·63	528·30	459·24	603·77	524·84	679·24	590·45	754·71	656·06	60
3	452·48	394·02	527·90	459·70	603·31	525·37	678·72	591·04	754·14	656·72	57
6	452·13	394·42	527·49	460·16	602·85	525·90	678·20	591·64	753·56	657·37	54
9	451·79	394·81	527·09	460·62	602·39	526·43	677·69	592·23	752·99	658·03	51
12	451·44	395·21	526·69	461·08	601·93	526·95	677·17	592·82	752·41	658·68	48
15	451·10	395·60	526·29	461·54	601·47	527·48	676·65	593·41	751·84	659·34	45
18	450·75	396·00	525·88	462·00	601·01	528·00	676·13	594·00	751·26	660·00	42
21	450·41	396·39	525·48	462·46	600·55	528·52	675·62	594·59	750·69	660·66	39
24	450·06	396·79	525·08	462·91	600·09	529·05	675·10	595·18	750·11	661·31	36
27	449·72	397·18	524·68	463·37	599·63	529·57	674·58	595·77	749·53	661·97	33
30	449·37	397·57	524·27	463·83	599·16	530·09	674·06	596·36	748·95	662·62	30
33	449·03	397·96	523·87	464·29	598·70	530·62	673·54	596·94	748·37	663·27	27
36	448·68	398·35	523·46	464·74	598·23	531·14	673·02	597·53	747·79	663·92	24
39	448·33	398·74	523·06	465·20	597·77	531·66	672·50	598·12	747·21	664·57	21
42	447·98	399·13	522·65	465·66	597·31	532·18	671·97	598·70	746·63	665·23	18
45	447·63	399·52	522·24	466·12	596·85	532·70	671·45	599·29	746·05	665·88	15
48	447·28	399·92	521·83	466·57	596·38	533·22	670·92	599·88	745·47	666·53	12
51	446·93	400·31	521·42	467·03	595·92	533·74	670·40	600·47	744·89	667·18	9
54	446·58	400·70	521·01	467·48	595·45	534·26	669·88	601·05	744·31	667·83	6
57	446·23	401·09	520·61	467·94	594·98	534·78	669·36	601·63	743·73	668·48	3
60	445·88	401·48	520·20	468·39	594·51	535·30	668·83	602·21	743·14	669·13	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

48 Deg.

42 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	74·31	66·91	148·62	133·82	222·94	200·73	297·26	267·65	371·57	334·56	60
3	74·26	66·98	148·51	133·95	222·77	200·92	297·03	267·91	371·28	334·88	57
6	74·20	67·04	148·39	134·08	222·59	201·12	296·79	268·17	370·98	335·21	54
9	74·14	67·11	148·28	134·21	222·42	201·31	296·56	268·43	370·69	335·53	51
12	74·08	67·17	148·16	134·34	222·24	201·51	296·32	268·68	370·40	335·86	48
15	74·02	67·24	148·04	134·47	222·06	201·70	296·09	268·94	370·11	336·18	45
18	73·96	67·30	147·92	134·60	221·88	201·90	295·85	269·20	369·81	336·50	42
21	73·90	67·37	147·81	134·73	221·71	202·09	295·62	269·46	369·52	336·82	39
24	73·84	67·43	147·69	134·86	221·53	202·29	295·38	269·72	369·23	337·15	36
27	73·79	67·49	147·57	134·99	221·36	202·48	295·15	269·98	368·94	337·47	33
30	73·73	67·56	147·45	135·11	221·18	202·67	294·91	270·23	368·64	337·79	30
33	73·67	67·62	147·34	135·24	221·00	202·86	294·68	270·49	368·35	338·11	27
36	73·61	67·69	147·22	135·37	220·83	203·06	294·44	270·75	368·05	338·43	24
39	73·55	67·75	147·10	135·50	220·65	203·25	294·20	271·01	367·75	338·75	21
42	73·49	67·82	146·98	135·63	220·47	203·44	293·96	271·26	367·46	339·08	18
45	73·43	67·88	146·86	135·76	220·30	203·63	293·73	271·52	367·16	339·40	15
48	73·37	67·94	146·74	135·88	220·12	203·83	293·49	271·77	366·86	339·72	12
51	73·31	68·01	146·63	136·01	219·94	204·02	293·25	272·03	366·57	340·04	9
54	73·25	68·07	146·51	136·14	219·76	204·21	293·01	272·28	366·27	340·36	6
57	73·19	68·14	146·39	136·27	219·58	204·40	292·78	272·54	365·97	340·68	3
60	73·13	68·20	146·27	136·40	219·40	204·60	292·54	272·80	365·67	341·00	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		

47 Deg.

42 Deg.

Minutes.	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	445·88	401·48	520·20	468·39	594·51	535·30	668·83	602·21	743·14	669·13	60
3	445·53	401·87	519·79	468·84	594·05	535·82	668·31	602·80	742·56	669·78	57
6	445·18	402·25	519·38	469·29	593·58	536·34	667·78	603·38	741·97	670·42	54
9	444·83	402·64	518·97	469·75	593·11	536·86	667·25	603·96	741·37	671·07	51
12	444·48	403·03	518·56	470·20	592·64	537·37	666·72	604·55	740·80	671·72	48
15	444·13	403·42	518·15	470·65	592·17	537·89	666·20	605·13	740·22	672·37	45
18	443·78	403·80	517·74	471·10	591·70	538·41	665·67	605·71	739·63	673·01	42
21	443·43	404·19	517·33	471·55	591·23	538·93	665·14	606·29	739·04	673·66	39
24	443·07	404·58	516·92	472·01	590·76	539·44	664·61	606·87	738·45	674·30	36
27	442·72	404·97	516·51	472·46	590·29	539·96	664·08	607·45	737·86	674·95	33
30	442·36	405·35	516·09	472·91	589·82	540·47	663·55	608·03	737·27	675·59	30
33	442·01	405·74	515·68	473·36	589·35	540·99	663·02	608·61	736·68	676·24	27
36	441·65	406·12	515·27	473·81	588·87	541·50	662·49	609·19	736·09	676·88	24
39	441·30	406·51	514·86	474·26	588·40	542·02	661·95	609·77	735·50	677·52	21
42	440·94	406·89	514·44	474·71	587·93	542·53	661·42	610·34	734·91	678·16	18
45	440·59	407·28	514·03	475·16	587·46	543·04	660·89	610·92	734·32	678·80	15
48	440·23	407·66	513·61	475·60	586·98	543·55	660·35	611·49	733·73	679·44	12
51	439·88	408·05	513·20	476·05	586·51	544·06	659·82	612·07	733·14	680·08	9
54	439·52	408·43	512·78	476·50	586·03	544·57	659·28	612·65	732·54	680·72	6
57	439·17	408·82	512·36	476·95	585·56	545·08	658·75	613·22	731·95	681·36	3
60	438·81	409·20	511·94	477·40	585·08	545·60	658·21	613·80	731·35	682·00	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
	Dist. 600.		Dist. 700.		Dist. 800.		Dist. 900.		Dist. 1000.		

47 Deg.

43 Deg.

Minutes.	Dist. 100.		Dist. 200.		Dist. 300.		Dist. 400.		Dist. 500.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	73·13	68·20	146·27	136·40	219·40	204·60	292·54	272·80	365·67	341·00	60
3	73·08	68·26	146·15	136·53	219·22	204·79	292·30	273·05	365·37	341·32	57
6	73·02	68·33	146·03	136·65	219·04	204·98	292·06	273·30	365·07	341·63	54
9	72·96	68·39	145·91	136·78	218·87	205·17	291·82	273·56	364·78	341·95	51
12	72·89	68·45	145·79	136·90	218·69	205·36	291·58	273·81	364·48	342·27	48
15	72·84	68·52	145·67	137·03	218·51	205·55	291·34	274·07	364·18	342·59	45
18	72·78	68·58	145·55	137·16	218·33	205·74	291·10	274·32	363·88	342·91	42
21	72·72	68·64	145·43	137·29	218·15	205·93	290·86	274·58	363·58	343·22	39
24	72·66	68·71	145·31	137·41	217·97	206·12	290·62	274·83	363·28	343·54	36
27	72·60	68·77	145·19	137·54	217·79	206·31	290·38	275·09	362·98	343·86	33
30	72·54	68·83	145·07	137·67	217·61	206·50	290·14	275·34	362·68	344·18	30
33	72·48	68·90	144·95	137·80	217·43	206·69	289·90	275·60	362·38	344·49	27
36	72·42	68·96	144·83	137·92	217·25	206·88	289·66	275·85	362·08	344·81	24
39	72·36	69·02	144·71	138·05	217·07	207·07	289·42	276·10	361·78	345·12	21
42	72·30	69·09	144·59	138·17	216·89	207·26	289·18	276·35	361·48	345·44	18
45	72·24	69·15	144·47	138·30	216·71	207·45	288·94	276·60	361·18	345·75	15
48	72·18	69·21	144·35	138·42	216·52	207·64	288·70	276·85	360·88	346·07	12
51	72·12	69·28	144·23	138·55	216·34	207·83	288·46	277·10	360·58	346·39	9
54	72·06	69·34	144·11	138·68	216·16	208·02	288·22	277·36	360·27	346·70	6
57	72·00	69·40	143·99	138·81	215·98	208·21	287·98	277·61	359·97	347·02	3
60	71·93	69·47	143·86	138·93	215·80	208·40	287·73	277·86	359·67	347·33	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
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	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	438·81	409·20	511·94	477·40	585·08	545·60	658·21	613·80	731·35	682·00	60
3	438·45	409·58	511·53	477·85	584·61	546·11	657·68	614·37	730·76	682·64	57
6	438·09	409·96	511·11	478·29	584·13	546·61	657·14	614·94	730·16	683·27	54
9	437·74	410·34	510·69	478·74	583·65	547·12	656·61	615·51	729·56	683·91	51
12	437·38	410·72	510·27	479·18	583·17	547·63	656·07	616·09	728·96	684·54	48
15	437·02	411·10	509·86	479·63	582·69	548·14	655·53	616·66	728·37	685·18	45
18	436·66	411·49	509·44	480·07	582·21	548·65	654·99	617·23	727·77	685·82	42
21	436·30	411·87	509·02	480·52	581·73	549·16	654·45	617·80	727·17	686·45	39
24	435·94	412·25	508·60	480·96	581·25	549·67	653·91	618·38	726·57	687·08	36
27	435·58	412·63	508·18	481·41	580·77	550·18	653·37	618·95	725·97	687·71	33
30	435·22	413·01	507·76	481·85	580·29	550·68	652·83	619·52	725·37	688·35	30
33	434·86	413·39	507·34	482·29	579·81	551·19	652·29	620·09	724·77	688·98	27
36	434·50	413·77	506·92	482·73	579·33	551·69	651·75	620·66	724·17	689·62	24
39	434·14	414·15	506·50	483·17	578·85	552·20	651·21	621·23	723·57	690·25	21
42	433·78	414·53	506·07	483·61	578·37	552·70	650·67	621·79	722·96	690·88	18
45	433·42	414·90	505·65	484·05	577·89	553·21	650·13	622·36	722·36	691·51	15
48	433·05	415·28	505·23	484·49	577·40	553·71	649·58	622·93	721·75	692·14	12
51	432·69	415·66	504·81	484·93	576·92	554·22	649·04	623·49	721·15	692·77	9
54	432·33	416·04	504·38	485·38	576·44	554·72	648·49	624·06	720·55	693·40	6
57	431·97	416·42	503·96	485·82	575·96	555·22	647·95	624·63	719·95	694·03	3
60	431·60	416·79	503·53	486·26	575·47	555·72	647·40	625·19	719·34	694·66	0
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0	71·93	69·47	143·86	138·93	215·80	208·40	287·73	277·86	359·67	347·33	60
3	71·87	69·53	143·74	139·05	215·62	208·59	287·49	278·11	359·37	347·64	57
6	71·81	69·59	143·62	139·18	215·43	208·77	287·25	278·36	359·06	347·95	54
9	71·75	69·65	143·50	139·30	215·25	208·96	287·00	278·61	358·76	348·27	51
12	71·69	69·72	143·38	139·43	215·07	209·15	286·76	278·86	358·45	348·58	48
15	71·63	69·78	143·26	139·55	214·89	209·34	286·52	279·11	358·15	348·90	45
18	71·57	69·84	143·14	139·68	214·70	209·52	286·27	279·36	357·84	349·21	42
21	71·51	69·90	143·02	139·80	214·52	209·71	286·03	279·61	357·54	349·52	39
24	71·45	69·97	142·89	139·93	214·34	209·89	285·78	279·86	357·23	349·83	36
27	71·39	70·03	142·77	140·05	214·16	210·08	285·54	280·11	356·93	350·14	33
30	71·33	70·09	142·65	140·18	213·97	210·27	285·30	280·36	356·62	350·45	30
33	71·26	70·15	142·53	140·30	213·79	210·46	285·06	280·61	356·32	350·76	27
36	71·20	70·21	142·40	140·43	213·60	210·64	284·81	280·86	356·01	351·07	24
39	71·14	70·28	142·28	140·55	213·42	210·83	284·57	281·11	355·71	351·38	21
42	71·08	70·34	142·16	140·68	213·24	211·01	284·32	281·35	355·40	351·69	18
45	71·02	70·40	142·04	140·80	213·06	211·20	284·07	281·60	355·09	352·00	15
48	70·96	70·46	141·91	140·92	212·87	211·39	283·82	281·85	354·78	352·31	12
51	70·90	70·52	141·79	141·05	212·69	211·58	283·58	282·10	354·48	352·62	9
54	70·83	70·59	141·66	141·17	212·50	211·76	283·33	282·34	354·17	352·93	6
57	70·77	70·65	141·54	141·30	212·32	211·95	283·09	282·59	353·86	353·24	3
60	70·71	70·71	141·42	141·42	212·13	212·13	282·84	282·84	353·55	353·55	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
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	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	
0	431·60	416·79	503·53	486·26	575·47	555·72	647·40	625·19	719·34	694·66	60
3	431·24	417·16	503·11	486·70	574·99	556·22	646·86	625·75	718·73	695·29	57
6	430·87	417·54	502·68	487·13	574·50	556·72	646·31	626·32	718·12	695·91	54
9	430·51	417·91	502·26	487·57	574·02	557·22	645·77	626·88	717·52	696·53	51
12	430·14	418·29	501·83	488·01	573·53	557·73	645·22	627·44	716·91	697·16	48
15	429·78	418·66	501·41	488·45	573·04	558·23	644·67	628·00	716·30	697·79	45
18	429·41	419·04	500·98	488·89	572·55	558·73	644·12	628·57	715·69	698·41	42
21	429·05	419·42	500·56	489·33	572·06	559·23	643·57	629·13	715·08	699·03	39
24	428·68	419·79	500·13	489·76	571·57	559·73	643·02	629·69	714·47	699·66	36
27	428·32	420·17	499·70	490·20	571·09	560·23	642·47	630·25	713·86	700·29	33
30	427·95	420·54	499·27	490·63	570·60	560·72	641·92	630·81	713·25	700·91	30
33	427·58	420·92	498·84	491·07	570·11	561·22	641·37	631·37	712·64	701·53	27
36	427·21	421·29	498·41	491·50	569·62	561·72	640·82	631·93	712·02	702·15	24
39	426·85	421·67	497·98	491·94	569·13	562·22	640·27	632·49	711·41	702·77	21
42	426·48	422·04	497·55	492·37	568·64	562·71	639·72	633·05	710·80	703·39	18
45	426·11	422·41	497·12	492·81	568·15	563·21	639·17	633·61	710·19	704·01	15
48	425·74	422·78	496·69	493·24	567·65	563·70	638·61	634·17	709·57	704·63	12
51	425·37	423·15	496·26	493·68	567·16	564·20	638·06	634·73	708·96	705·25	9
54	425·00	423·52	495·83	494·11	566·67	564·69	637·50	635·28	708·34	705·87	6
57	424·63	423·89	495·40	494·54	566·17	565·19	636·95	635·84	707·72	706·48	3
60	424·26	424·26	494·97	494·97	565·68	565·68	636·39	636·39	707·10	707·10	0
	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Base.	Per.	Minutes.
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THE END.

THEORY

OF

COMPOUND INTEREST

AND

ANNUITIES.

THEORY

OF

COMPOUND INTEREST

AND

ANNUITIES

WITH

A SERIES OF LOGARITHMIC TABLES

BY

FÉDOR THOMAN,

OF THE SOCIÉTÉ CRÉDIT MOBILIER OF PARIS.

SECOND EDITION, CAREFULLY REVISED AND CORRECTED.

LONDON:

LOCKWOOD AND CO., 7, STATIONERS' HALL COURT,

LUDGATE HILL.

1872.

Cambridge:
PRINTED BY C. J. CLAY, M.A.
AT THE UNIVERSITY PRESS.

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FOR THE MORE DIFFICULT COMPUTATIONS OF

INTEREST, DISCOUNT, ANNUITIES, &c.

BY FEDOR THOMAN.

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THE EDITOR'S PREFACE.

THE benefits arising from the more extended practice of Logarithms are too obvious to require any special notice. All numerical calculations, however long and difficult, owing to logarithmic tables, are rendered easy and expeditious; moreover, many chances of error from tedious numerical operations and incorrect numerical tables are removed. It is chiefly with questions of Compound Interest and Annuities, which require the nicest and most accurate solutions, that the great superiority of logarithmic computation is clearly shewn. Besides, there are many inquiries connected with pecuniary investments which can be satisfactorily disposed of only by the aid of logarithms.

Numerical tables have been for some time in the hands of the public, and have rendered many valuable services, but owing to their limits they are only applicable to elementary transactions. Whenever the questions, although of daily commercial occurrence, are more intricate, numerical tables are of little use, as common arithmetic is inadequate to solve them. Consequently most authors have not failed to commend the application of logarithms; and the arduous labour which is implied in the computation of logarithmic tables appears to have been the only reason for preventing their being generally adopted.

To such authors as De Moivre, Smart, Simpson, Price, Milne, Morgan, Baily, and particularly to the latter, we owe most of the improvements in the branch of Mathematics bearing upon Compound Interest and Annuities; but although every one of them has specified the use of logarithms as the best and readiest mode of solution, none has embraced the whole series of logarithmic investigations in connexion with the subject.

The great experience acquired in these matters by the author of the present work, who for some years has been concerned in advising one of the largest foreign financial companies, and his long acquired skill in laborious computations, have induced him to lay out the information collected by former writers of importance with regard to logarithms, to compute the present logarithmic tables, and thereby to accomplish an object which might be to this country useful and important in the highest degree. The prodigious financial business now carried on either by individuals, companies or governments all over the world, has rendered the information which M. Thoman conveys in his Theory particularly necessary at the present time. At first his Introduction was intended to be merely a Key to the following Tables, but the large number and variety of cases involved in computation, and likewise the financial importance of certain problems which had not yet been financially contemplated, have carried the author into a very wide field of inquiry, and engaged him in writing this new and complete mathematical theory.

The concise method followed in developing this theory has helped a good deal to convert the doctrine of annuities into a familiar, regular and uniform system. Besides, as the present work is meant for practical purposes, M. Thoman has carefully represented the theorems and rules by the most intelligible and elegant formulæ which are particularly accommodated to logarithmic calculations, and offer the greatest facilities for solving complicated and abstruse problems.

M. Thoman's theory, as it is, may be divided into two distinct series, the first of which is relative to single, and the second to periodical payments or investments.

Thus the first series comprises:

I. Amount of any sum improved at Compound Interest at the end of a given time.

II. Present value of any sum due at a given time.

The second series includes the new theory of Annuities, which is naturally divided into three distinct varieties, according as the dividend runs either *since*, *before*, or *after* the purchase time.

The Annuities of the first variety are termed *immediate;*

those of the second *deferred*, or *reversions*. With regard to the annuities of the third variety, for which the *entrance* is neither immediate nor deferred, but *past*, they have been termed by the author *annuities with past entrance*.

In the chapter on Annuities the difference has been fully established between *sinking fund* and the sum to be added annually to the interest in order to obtain the gradual redemption of the principal. In fact, the word *fund* is applied to principals or stocks constituting our National Debt. Thus Richard Price, *On Reversionary Payments*, (National Debt, I. page 299) states that "the British Funds have been all formed into the following divisions:

1, the Aggregate Fund;
2, the South Sea Fund;
3, the General Fund,
4, the Sinking Fund, to which was given the name of Sinking Fund because appropriated to the purpose of sinking the public debts."

The Sinking Fund is therefore one of our public funds, and the new word *amortizement*, which is used in all continental languages, has been made to denote the annuity which is to be paid by the sinking fund in order to redeem a given principal.

The Third Part of the Introduction contains the theory and working system of *Obligations*, a special kind of bonds reimbursable by lot, constituting constant annuities, and quoted at the Exchange as the other public funds.

In France only, the principal represented by the Obligations which are to be reimbursed within less than 100 years, and issued by Railway Companies, commercial firms, townships, &c., has risen to above 150 millions.

In Germany, Austria, Italy, Spain, Switzerland, Russia, and the United States of America, the number of Obligations is likewise comparatively very great.

These bonds, which differ entirely from ordinary English debentures, deserved a special chapter.

In Part V. which treats of the *annuities with past entrance*, the author has carefully insisted upon the difference between the amount of annuities in arrear and the amount

of annual investments, which had been hitherto confusedly treated.

The various results obtained according to the periods varying for the payment of the interest and annuities have been accurately shewn throughout the work. When the periods for paying the interest and annuity are not equal to each other the formula previously known was so ill adapted to numerical computations that even logarithms with 7 places of decimals could only secure 4 or 5 true figures in the result: owing to this difficulty the difference between the periods of payment for the annuity and interest used to be neglected, and the solution was consequently more or less erroneous.

M. Thoman has caused this source of error to disappear by giving in a small table (Table IV.) the logarithms with seven decimals computed by means of converging series, which must be either added to, or subtracted from, the logarithm of the annuity in order to obtain a solution rigorously true.

Upon the whole, the Theory now offered to the public is not only new in many of its parts, but complete as far as mathematical inquiry is concerned and illustrated, with references to the Tables, by examples which, however numerous, are perfectly distinct from each other, and developed in the most convenient form. These examples, chosen amidst a vast variety of applications, are rendered the more valuable from having occurred in practice, and their correct solutions have required no small amount of labour. The Editor is in hopes that they will be found to contain some valuable information, especially for the students already acquainted with Mr. Hipsley's rudimentary treatise*.

All theoretical improvements would be of little consequence were there no ready means of carrying them out practically; for that reason the Tables have been calculated. Such Tables containing the quantities which enter into the algebraical formulæ were a natural addition to the Theory, for by their assistance accurate results are obtainable instead of approximations, which in many cases were such as to be altogether unfit for use.

* Vols. 85 and 85* of Mr Weale's Rudimentary Series.

Our original Tables contain nearly 11,000 logarithms with either 7 or 10 places of decimals, applicable to 48 rates of interest from $\frac{1}{2}$ to 12 per cent., between 1 and 100 years. As the rates proceed by eighths from $\frac{1}{2}$ to 6 per cent., the first differences are very nearly constant, and consequently the interpolations for each intermediate rate are very simply performed. In order to attain the highest degree of correctness in these Tables, the logarithms, numerous as they are, have been calculated and proved, when required, for 10 and 13 decimals by means of two and three different processes.

The first column of each page gives, by simply adding or subtracting the logarithms:

1. The amount of £1 at the end of any number of years.
2. The reverse, viz. the present value of £1 due at a given time.

The second column gives:

3. The annuity which £1 will purchase.
4. The reverse, viz. the present value of £1 per annum.

Both columns combined give directly:

5. The amount of an annuity in arrear.
6. The reverse, viz. the annuity in arrear which amounts to £1—the amortizement.
7. The amount of annual investments.
8. The reverse.
9. The present value of a deferred annuity.
10. The reverse.

The second and third Tables are complementary; the second for fractional years, and the third for such rates as are not included in Table I.

Table IV. shews the logarithms of annuities when the interest and annuity are not paid within the same intervals of time.

These Four Tables, which have been successfully and extensively applied for several years by their Author, are made to comprehend the whole system of calculations bearing upon the important subject of financial investments, and, no doubt, they will supersede in time all the existing numerical tables. One single fact may shew the inefficiency of the latter compared with logarithmic tables such as the present. In order to give directly

the same solutions as the logarithmic Tables I. and II combined, numerical tables for deferred annuities ought to contain 5,760,000 different numbers!

And, lastly, if the present Tables exceed in typographical neatness and accuracy other tables hitherto published, the Editor feels it his duty to ascribe the merit to the Superintendent of the Cambridge University Press.

NOTE

By the Editor of the Second Edition,

WITH REFERENCE TO THE WORDS AMORTIZEMENT, &c.

In some parts of the Introduction there has been a great confusion of the words "annuity and interest" with what should have been expressed as *interest and amortizement*. The word "annuity" has been used in two distinct senses,—first, in its proper sense, where it indicates the *whole amount* annually derived from the investment; and, secondly, it is used to express the *simple interest* on the capital invested: also the word "interest" has been used in a great many instances where the word *amortizement* should have been employed, as it indicates, in the cases referred to, the quantity s as used in the formulæ on pages 25, 34, etc, and not the quantity indicated by t, or the simple interest on the capital invested. This improper interchange of words is very perplexing, but with the aid of the following remarks and the introduction of a reference to them in the sections and principal passages affected thereby, it is hoped the principal difficulties arising therefrom will be removed.

The parts of the work (Introduction) in which this improper use of words has principally taken place, are from page 49 Sect. 35, to page 68 Sect. 41, and from page 101 Sect. 58, to page 120 Sect. 70; and such pages as 58, 62, 101 and 110, are conspicuous instances of this.

There will arise but little difficulty to the competent computor in determining the instances in which the word *annuity* is used in its proper and distinctive sense,—indicating the whole annual amount arising from the investment, as in those instances it generally stands apart from the word "interest," and its signification is clearly intelligible from the context;

but where it has been misused it stands, in the majority of instances, in conjunction with the word *interest*, and reads either "annuity and interest," or conversely.

In all such cases the word *interest* should be substituted for *annuity*, and the symbol *Vt*, as stated in Sect. 22, page 25, taken as expressive of the quantity indicated; and for *interest* should be substituted the word *amortizement*, in all those cases in the before-mentioned section where it is connected with the word "annuity," and in the other instances where it is plainly used in the same sense, and as representing the quantity *s* in the formulæ and definitions, or the annual sinking fund to be invested to replace the capital at the end of the term,—always bearing in mind that the amortizement is the *convertible* quantity.

This will be made still more clear if we cite an instance or two. On page 50, at the head of Sect. 37, *for* "Before we deal with this formula, under a general point of view, let us first examine the particular case in which the annuity is paid *half-yearly*, whereas the interest is convertible *yearly*;" etc. *read the latter part thus*,—"let us first examine the particular case in which the interest is paid *half-yearly*, whereas the amortizement is convertible *yearly*," etc. etc.

On page 52, following the heading of Sect. 38, *for* "When the annuity and interest are payable at the same time, ν times per annum, the general formula becomes," *read* "When the interest and amortizement are payable," etc. etc. On page 62, 8th line from the top, *for* "Then if the annuity is paid 2, 4, 6 or 12 times *more frequently* than the interest," etc. etc., *read* "Then if the interest is paid 2, 4, 6 or 12 times *more frequently* than the amortizement," etc. etc. And on the top of page 101, *for* "We have hitherto supposed the interest and annuity to be payable once a year," etc. etc., *read* "We have hitherto supposed the interest and amortizement to be payable once a year," etc. etc., and so on in the other cases, thus shewing that where "annuity" is expressed the *simple interest*, or consumable part of the annuity, is to be understood; and for "interest" the word *amortizement* must be substituted,—indicating the unconsumable, or recuperative, portion of the annuity.

It is hoped and believed that by the aid of these explanations, and corrected examples, the embarrassments which would

otherwise arise from this part of the subject will be completely removed.

There is another part of the subject to which it seems desirable and proper to make especial reference here, i. e. the use of the symbols A and a in the original edition of this work.

It seems to have been the intention of the author, as shewn at page 34 of the Introduction, that the symbol A should be employed to express the whole annual amount arising out of the investment, and consequently including two distinct quantities, viz. the simple interest, and the amortizement of the purchase money; and the symbol a to express the annuity which £1 will purchase for a term of years; but the A was immediately afterwards dropped, and the a became used to express both these—consequently quite different—quantities, thus introducing confusion, since the a is often used in the same examples in both significations.

The most complete removal of this serious blemish of the original book would have been effected by restoring in this issue the A to its proper position throughout the Introduction; but in consequence of the type being cast in metal plates such a very large number of alterations (considerably upwards of one thousand) became impracticable without involving the destruction of a large number of the plates, which cost the publishers did not feel disposed to incur; consequently it became necessary to meet the difficulty in some other way. After mature consideration it appeared to the editor of this edition practicable to meet the case by so altering the symbols that the quantity which *should* have been represented in the original issue by A (as defined at page 34 of the Introduction) should now in this be represented throughout by a simply, thus involving much fewer alterations to the plates; and that the annuity which £1 will purchase for a term of years—being the Tabular quantity—and which was expressed in the first issue sometimes by a and at other times by a^n, or by (say) a^{75} specifically, should now be expressed throughout by a^n distinctly, or by a with the *numeral* index for which n stands.

There is thus introduced a distinction between the symbols for these two dissimilar quantities, and by the observance of this in studying or working the examples the difficulties of the former volume in this respect will be entirely removed.

INTRODUCTION.

Στρεψ.—Τοῦτο δ' ἔσθ' ὁ τόκος τί θηρίον;
Ἀμυν.—Τί δ' ἄλλο γ' ἢ κατὰ μῆνα καὶ καθ' ἡμέραν
πλέον πλέον τἀργύριον ἀεὶ γίγνεται,
ὑπορρέοντος τοῦ χρόνου;

ΑΡΙΣΤΟΦΑΝΟΥΣ ΝΕΦΕΛΑΙ.

DEFINITIONS.

Interest is the allowance paid for the use (usura) of any sum of money called *Principal* (capital), or the profit made by the borrower.

In order to determine this profit, which generally depends upon the conditions made by the borrower to the lender, a term of comparison is necessary, which may be used for calculating the interest of the sum lent, whatever it may be; for that reason, the allowance to be made for the disposal of £100 during one year, has been called the *Rate of Interest per cent.*

This rate of interest varies considerably according to the financial and political situation of a country, and the nature of its transactions, but in most states the laws have given an extreme limit to this rate, which limit has been called *Legal Interest*, and any interest exceeding it is called *Usury**.

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* *Note.* The earliest enactment respecting money interest, mentioned in English history, is an Act passed under the reign of Richard I. (1197) forbidding Christians to take interest for their money.

Under the reign of Henry II. loans were readily obtainable in England at an annual charge of less than 20 per cent., under Henry III. the rate of interest was 50 per cent.

In 1545 (37 Henry VIII. c. 9) and in 1571 (13 Elizabeth, cap. 8) no person was allowed to take above £10 for the forbearance of £100 during one year: or the offender would have to pay treble value.

The legal rate continued at 10 per cent. until 1624 (21 James I. c. 17), when it was reduced to 8 per cent. Then in 1651, by an ordinance of the Rump Parliament, confirmed in 1660 by Charles II., the legal interest was reduced to 6 per cent.

At last, in 1714 (12 Anne, c. 16) no person is to take for a loan of money above £5 for the forbearance of £100 for one year, and bonds, contracts, &c., made at a greater rate of interest are to be void, and the offender to forfeit treble value.

In 1586 a law was passed in Scotland which fixed the legal rate at 10 per cent.

At the time of Nehemiah (B.C. 445) the Jews had to pay 1 per cent. monthly.

When the principal remains several years in the hands of the borrower, he must pay such an interest to the lender, at certain periods, settled beforehand between the parties; otherwise he must keep it himself as a new principal, upon which an interest is due likewise; this is called *Compound Interest.*

The interest which becomes due and remains in the hands of the borrower, without bearing any interest, is called *Simple Interest*, and is ever proportional to the time.

An annuity is any periodical rent or income payable at the end of every year, or during equal intervals of time. It is termed *certain* when for a fixed number of years, and *Life Annuity* when it depends upon the survivorship of one or more lives.

When the interval between the time of the purchase of an annuity is equal to the interval between any two consecutive payments, the annuity is called *immediate.*

When the interval between the time of the purchase of an annuity is *greater* than the interval between two payments immediately consecutive, that is, when the annuity is not to be entered upon immediately, it is called a *Deferred Annuity*, or an *Annuity with deferred entrance.*

A reversion or *Reversionary Annuity* is an annuity already in existence, which at the end of a given term, or after the death of the present possessor, is reversible to another possessor.

When the interest between the time of the purchase of an annuity is *less* than the interval between two immediately consecutive payments, or when the annuity has been entered since a certain period of time, it may be called an *Annuity with past entrance.*

In Athens, no restriction was ever imposed upon the rate of interest, the rate having been declared free by a law attributed to Solon himself. According to Grote (*History of Greece*) the interest was between 10 and 36 per cent., commonly between 12 and 18 per cent.

The rate of interest (unciarum fœnus) at Rome is not exactly known. In the time of Sylla (B.C. 82) the centesima, or 1 per cent. for a month, was recognised as the usual and legal rate of interest. Brutus, owing to a Senatusconsultum, lent money in Cyprus at 48 per cent. Cicero, when he entered upon the government of Cilicia, issued an edict fixing the rate of interest at 1 per cent. for a month, with interest upon interest at the end of every year. Under Tiberius the rate at Rome was 6 per cent., under Alexander Severus (A.C. 230) the legal rate was reduced to 4 per cent., and later to $3\frac{1}{3}$ per cent.

In France an edict of Philip Augustus (1272) limited the legal rate to 48 per cent. Philip IV. in 1311 fixed at 20 per cent. the interest which might be legally exacted in the fairs of Champagne.

Charles V. (in 1545) fixed the rate of interest in the Netherlands at 12 per cent.

FIRST PART.

AMOUNT OF ANY SUM AT COMPOUND INTEREST AT THE END OF A GIVEN TIME.

SECT. 1. *Increase of the principal at compound interest at the end of a whole number of years.*

t, the *interest* on £1 for one year, or the one hundredth part of the rate of interest per cent.;

$r = 1 + t$, the *sum* of £1, increased by its interest, for one year, or the ratio of the yearly increase of the principal;

n, the whole *number* of years for which it was lent;

p, the *principal*,

M, the *amount* of this principal at the end of the given time.

Then, £1 increases in one year to r pounds, and since any other sum will increase in one year, according to the same ratio, these £r will become r^2, and likewise, r^3 will become £r^3 at the end of one year.

Whence, r is the amount of £1 in 1 year.
r^2 ,, £1 in 2 years.
r^3 ,, £1 in 3 years.

And generally r^n will be the amount of £1 in n years.

And if this expression be multiplied by any principal p, it will become

$$M = pr^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which is the amount of this principal at the end of n years.

Thus, the amount of the given principal at the end of any given number of years, is equal to the amount of £1 in one year, raised to the power denoted by the number of years, and multiplied by the principal.

SECT. 2. *Advantages afforded by the use of logarithms.*

When the rate of interest is an integer, as 4 per cent., for instance, the ratio of the increase is made up of two significant figures, one of which is the unit, viz. 1,04; but if the rate is fractional, or if the number of years is considerable, computation is very tedious, and liable to errors. In such cases, and especially in the following

problems, the use of logarithms is highly beneficial, and almost indispensable. In fact, by help of logarithms, all intricate multiplications are reduced to one single and very simple operation. Most mathematicians who have dealt with compound interest have commended the use of logarithms as the safest, easiest, and readiest method of solving the various questions met with.

SECT. 3. *Formulæ and questions of compound interest.*

From formula (1) $M=pr^n$, the following may be deduced for finding any one of the four quantities, the three others being given:

thus, for the principal $p=\frac{M}{r^n}$ .....(2).

for the rate of interest $r=\sqrt[n]{\frac{M}{p}}$ ..... (3).

This diminished by unity gives t, which multiplied by 100 is the rate per cent.

for the number of years $n=\frac{\log M-\log p}{\log r}$...(4).

The use and application of these formulæ will be fully explained by the following examples:

Ex. 1. (Amount, form 1) What is the amount of £527. 15*s.* put out to compound interest, at $4\frac{3}{4}$ per cent. for 34 years?

$$\begin{array}{lll} & \log 527{,}75 = 2{,}72242\ 82 \\ \text{(Tab. 29)}^* & \log r^{34} & = 0{,}68523\ 71 \\ & \log M & = 3{,}40766\ 53 \end{array}$$

that is, 2556,615, or £2556. 12*s.* $3\frac{1}{2}$*d.*†

Ex. 2. (Amount) Let it be required to find the amount of £418 put out to interest at $4\frac{5}{8}$ per cent. at the end of 157 years.

$$\begin{array}{ll} \log 418 = 2{,}62117\ 63 \\ \text{(Tab. 28)} \quad \log r^{100} = 1{,}96354\ 71 \\ \log r^{57} = 1{,}11922\ 18 \\ \log M = 5{,}70394\ 52 \end{array}$$

$M=505760{,}8$, or £505760. 16*s.*

Ex. 3. (Principal, form 2) What principal, put out to interest, at $5\frac{1}{4}$ per cent. will amount to £4817 in 93 years?

$$\begin{array}{lll} \log 4817 = & 3{,}68277\ 66 \\ \text{(Tab. 33)} \quad \log r^{93} = & 2{,}06655\ 57 \\ \log p = & 1{,}61612\ 09 \end{array}$$

$p = 41{,}31625$, or £41. 6*s.* 4*d.*

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* The word Table, or Tab., means that the logarithm is found without any calculation, in the tables given at the end of this work, and at the page indicated.

† See the preceding Introduction, or the decimal tables in Hipsley's *Equational Arithmetic.*

Ex. 4. (Rate per cent. form 3) Suppose a person returns £5000 for the loan of £1335 during 30 years, what is the rate of interest?

$$\begin{aligned}\log 5000 &= 3,69897\ 00\\ \log 1335 &= 3,12548\ 13\\ \log r^{30} &= 0,57348\ 87\\ \log r &= 0,01911\ 63\\ r &= 1,045\end{aligned}$$

thus, the rate of interest is $4\frac{1}{2}$ per cent. very nearly.

Ex. 5. (Time, form 4). How many years ought £3000 to be put out at 4 per cent., compound interest, in order to amount to £102358?

$$\begin{aligned}\log M &= 5,01012\ 18\\ \log p &= 3,47712\ 13\\ \log M - \log p &= 1,53300\ 05\end{aligned}$$

$$n = \frac{1,53300\ 05}{0,01703\ 334} = 90 \text{ years.}$$

Ex. 6. (Time) Two sums are simultaneously laid out at compound interest, the first $= £38000$ at $4\frac{1}{2}$, and the second $= £99398$ at $3\frac{1}{2}$ per cent. Required to determine the time during which they will be raised to the same amount.

Statement:

$$38000 \times 1,045^n = 99398 \times 1,035^n$$

$$\therefore \left(\frac{1045}{1035}\right)^n = \frac{99398}{38000}$$

$$\begin{aligned}\log 99398 &= 4,99737\ 76\\ \log 38000 &= 4,57978\ 36\\ &\quad 0,41759\ 40\end{aligned}$$

(Tab. 57) $\log 1,045 = 0,01911\ 629$

(Tab. 57) $\log 1,035 = 0,01494\ 035$

$$0,00041\ 7594$$

$$\text{therefore } n = \frac{0,41759\ 40}{0,00417\ 594} = 100 \text{ years.}$$

SECT. 4. *Time required for doubling a principal.*

Ex. 7. How long will it take any principal to double itself at 3 per cent. per annum, compound interest?

We have (form 4) $M = 2p$; consequently,

$$n = \frac{\log 2p - \log p}{\log r} = \frac{\log 2}{\log r},$$

$$n = \frac{\log 2}{\log 1,03} = \frac{0,30103\ 00}{0,01283\ 72}$$

$n = 23,45$, or 23 years and 164 days.

If it were asked, to find only the approximate time required, in order to double the principal at any rate of interest, it could be easily done by dividing the number 70 by the rate per cent.

Thus in the foregoing example:

$$n = \frac{70}{3} = 23{,}333.$$

Again, if it were required to find the time approximately within 3 days at the most, we could divide the constant number 69,315 by the rate of interest, and add to the quotient half per cent. of said number, for all rates between 0 and 10 per cent.

Thus at 3 per cent.

$$\begin{array}{r} 23{,}105 \\ \underline{{,}346} \\ 23{,}451 \end{array}$$

years, as above.

SECT. 5. *Amount of any principal at compound interest, the interest being payable yearly, half-yearly, quarterly, &c.*

Hitherto, we have always assumed the interest to be converted into principal at the end of every year; let us now consider the case in which the conversion is to be made k times every year, at k equal intervals and for k payments, each payment being the k^{th} part of the annual interest.

The method of proceeding will still be the same as in the case of whole years, provided the number of payments n denote always the whole number of periods in the given time, and t denote the interest of £1 for each period.

Let us see first, what a principal of £100 becomes at the expiration of 1 year, at the annual rate of 4 per cent., payable in equal portions, and within equal intervals of time.

If the interest is payable yearly, the principal will amount at the end of 1 year to £1,04.

If the interest is payable out half-yearly, and by halves, that is, at 2 per cent. for 6 months, we find at the expiration of the first half-year, each pound will have become £1,02, which in the second half-year becomes $1{,}02^2$, consequently the amount of the principal will be

$$100 \times 1{,}02^2 = £104{,}04.$$

If the interest is payable quarterly, that is, at 1 per cent. for one quarter, the amount of the principal at the end of the year will be

$$100 \times 1{,}01^4 = £104{,}060401.$$

If the interest is payable monthly, at $\frac{1}{3}$ per cent. for the month, the principal would amount at the expiration of the year to

$$100 \times \left(1 + \frac{1}{300}\right)^{12} = £104{,}074155.$$

If, finally, the interest is instantaneously convertible, for we may assume intervals becoming less and less, and therefore indefinitely small, the amount would at the expiration of the year amount to its maximum £104,081077.

Hence, the nominal rate, being 4 per cent. per annum, the true rate will be, for half-yearly payments, 4,04
quarterly 4,060401
monthly 4,074155
instantaneous 4,081077

Thus, we see that the amount of the principal increases with the number of intervals, and reaches its maximum when the interest is convertible instantaneously; consequently, the advantages are so much greater, the more considerable the number of payments during the year is.

SECT. 6. *General formula, applied to interest, convertible, half-yearly, quarterly, &c.*

If the interest convertible into principal is paid k times in one year, each payment being the k^{th} part of the annual interset, the amount of £1 at the end of the year will be $\left(1+\frac{t}{k}\right)^k$, (form 1)

Thus, if the nominal rate is t, the *true* rate will be

$$\left(1+\frac{t}{k}\right)^k-1.$$

And consequently a principal p will become at the end of n years,

$$M=p\left(1+\frac{t}{k}\right)^{kn} \quad \ldots\ldots\ldots\ldots(5)$$

according as k is equal to 1, 2, 4, 12, ... or infinite,

that is, according as the payment is yearly, half-yearly, monthly, or instantaneously, the amount will be

for half-yearly payments $M=p.\left(1+\frac{t}{2}\right)^{2n}$,

quarterly ..... $=p.\left(1+\frac{t}{4}\right)^{4n}$,

monthly ..... $=p.\left(1+\frac{t}{12}\right)^{12n}$,

instantaneously ..... $=p.e^{tn}$,

e being the basis of the system of hyperbolic logarithms* $=2,7182818$.

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* When the expression $\left(1+\frac{1}{k}\right)^k$ is expanded upon the principle of the binomial theorem, we have,

$$\left(1+\frac{1}{k}\right)^k=1+1+\left(1-\frac{1}{k}\right)\frac{1}{2}+\left(1-\frac{1}{k}\right).\left(1-\frac{2}{k}\right).\frac{1}{2.3}+\ldots$$

SECT. 7. *Examples of interest payable half-yearly, quarterly, &c.*

Ex. 8. What is the amount of £398. 11s. put out to compound interest at 6 per cent., for 7 years, and convertible quarterly?

This amount will be just the same as that of the same principal put out for 28 years, at 1½ per cent., therefore:

$$\log 398{,}55 = 2{,}60048\ 28$$
(Tab. 3) $$\log r^{28} = 0{,}18104\ 92$$

$$\log M = 2{,}78153\ 20$$
$$M = 604{,}6889 \text{ or } £604.\ 13s.\ 9\tfrac{1}{2}d.$$

Ex. 9. What sum will amount to £1000 in 23 years, at 4½ per cent. interest, and convertible half-yearly?

$$\log 1000 = 3$$
$$\log r^{46} = 0{,}44451$$

$$\log M = 2{,}55549$$
$$M = £359{,}325$$

Ex. 10. What is the amount of £1 at the end of 87 years, when put out to compound interest, at 9 per cent., the interest being convertible yearly, half-yearly, quarterly, and instantaneously?

Yearly, 9 per cent.

$$\log p = 0.$$
(Tab. 46) $$\log r^{87} = 3{,}2561053$$

$$M = £1803{,}455$$

Half-yearly, 4½ per cent.

(Tab. 27) $$\log r^{100} = 1{,}91162\ 90$$
$$\log r^{74} = 1{,}41460\ 55$$

$$\log M = 3{,}32623\ 45$$
$$M = £2119{,}505$$

Quarterly, 2¼ per cent.

(Tab. 9) $$\log r^{300} = 2{,}89899\ 50$$
$$\log r^{48} = 0{,}46383\ 92$$

$$\log M = 3{,}36283\ 42$$
$$M = £2305{,}867.$$

Instantaneously.

$$\log e = 0{,}43429\ 448$$
$$\log M = \frac{87 \times 9}{100} \log e = 3{,}40052\ 58$$
$$M = £2514{,}929.$$

If k is indefinitely great, that is, if $k = \infty$, the formula will become

$$\left(1 + \frac{1}{\infty}\right)^{\infty} = 1 + 1 + \frac{1}{2} + \frac{1}{2.3} + \frac{1}{2.3.4} + \ldots$$

or in decimals = ... 2,71828 18284 59045.

This number, the common logarithm of which is 0,43429 44819, is denoted in analysis by the letter e.

Were it required to determine the value of $M = \left(1 + \frac{t}{k}\right)^{kn}$ on the hypothesis $k = \infty$, we should have by substituting $k = mt$, m being likewise $= \infty$,

$$M = \left\{\left(1 + \frac{1}{m}\right)^{m}\right\}^{tn} = e^{tn}.$$

SECT. 8. *Amount of any principal, for fractions of years.*

It has already been shown, that the amount of the principal varies and increases, when the interest, instead of being payable yearly, is convertible into yearly fractional periods, and reciprocally, that the interest of any principal, put out to compound interest, during a fractional portion of the year, is always less than the same portion of the yearly interest.

Thus, if £2 were to be received at compound interest for £100 at the end of half a year, it will be readily perceived (§ 5) that the rate of interest would not be 4 per cent. per annum, but $4\frac{1}{25}$ per cent., and reciprocally, if the rate of interest is 4 per cent. per annum, the interest for half a year would be *less* than 2 per cent.

Now, let the rate of interest be such that the amount of £1 at the expiration of one year will be $(1+t)$, and let x denote the interest of £1 for $\frac{1}{k}$th year, the amount of that £1 at the end of the first period of conversion will be $(1+x)$,

at the end of the second $(1+x)^2$,

..... third $(1+x)^3$,

..... n^{th} $(1+x)^n$,

finally, ... k^{th} period, that is, at the end of the year, will be $(1+x)^k$,

but as this amount is required to be $(1+t)$,

we shall have $(1+x)^k=(1+t)$,

whence $(1+x)=(1+t)^{\frac{1}{k}}$ ...(1)

and therefore the amount of the principal at the end of the $\frac{n}{k}$th year will be

$$=(1+t)^{\frac{n}{k}},$$

and the interest payable at the end of this period will be

$$\left\{(1+t)^{\frac{n}{k}}-1\right\}.$$

If in (1) $k=2$ we shall have $1+x=(1+t)^{\frac{1}{2}}=\sqrt{1+t}$.

Consequently, *the amount of £1 during half a year can be found by extracting the square root of the amount of £1 during one year, at the given rate of interest. When found, this root will represent the amount put out at the same rate for half-a-year.*

For instance, the square root of 1,04 is 1,01980 39027, and represents the amount of £1 at 4 per cent. in half a year,

this amount 1,01980 39027 being multiplied by
1,01980 39027, that is, by itself, the product

will be 1,04000 00000, which is the amount in one year.

Likewise, if it be required to determine the amount of £1 in $\frac{1}{3}$ year, or 4 months, we should have $\sqrt[3]{1+t}$, that is, the cubic root of

$1+t$, which in the case of 4 per cent. interest per annum would be $=1{,}01315\ 940$.

Thus, the interest of £1000 at 4 per cent. compound interest will be for half a year.....£19,804
and for 4 months £13,159.

Ex. 11. What is the amount and interest of £20000 put out to compound interest, at 5 per cent., at the end of 6 months?

$\log p = 4{,}30103\ 00$
(Tab. 52) $\log r^{\frac{1}{2}} = 0{,}01059\ 46$

$\log M = 4{,}31162\ 46$
$M = £20493{,}901$
and the interest will be $= £493.\ 18s.$

Ex. 12. Required to find the amount of £1358. 13s. at $4\frac{3}{4}$ per cent., at the end of 17 years, 5 months.

$\log 1358{,}65 = 3{,}13310\ 76$
$\log r^{17}_{\frac{5}{12}} \quad \ldots = 0{,}31614\ 03$
(Tab. 52) $\log r^{\frac{5}{12}} \quad \ldots = 0{,}00774\ 85$

$\log M \quad \ldots = 3{,}45699\ 64$
$M \quad \ldots = £2864{,}154.$

Ex. 13. What principal will amount in 23 years and 85 days to £3800, at $3\frac{1}{2}$ per cent.?

(Tab. 17) $\log r^{23} = 0{,}31947$
$\frac{1}{365} \log r^{85} = 0{,}00323$

$0{,}32270$
$\log 3800 = 3{,}57978$

$\log p = 3{,}25708$
Present value, $p = £1807{,}50.$

Ex. 14. Required the interest of £4000 for 3 months, at the rate of $4\frac{1}{2}$ per cent.

$\log 4000 = 3{,}60206\ 00$
$\log r^{\frac{1}{4}} \quad = \quad 477\ 91$

$\log M = 3{,}60683\ 91$
Amount $= £4044{,}260$
Interest $= £44{,}260.$

Ex. 15. How long will it take £635, bearing interest at the rate of $3\frac{1}{2}$ per cent. per annum, and convertible half-yearly, to amount to £1200?

$\log 1200 = \quad 3{,}07918$
$\log 635 \ = \quad 2{,}80277$

$0{,}27641$

consequently $n = \frac{0{,}27641}{0{,}0075344}$

$n = 36{,}686$ half years,
or $n = 18$ years and 125 days.

If we wish to divide the numerator of n by its denominator, by means of logarithms, we may from the logarithm of the numerator subtract the logarithm of the logarithm of the ratio r. (Tab. 57)

Thus, we should have in the foregoing example

$$\log 0,27640\ 75 = 9,44155$$
(Tab. 57) $$\log \log r = 7,87705$$
$$\log n = 1,56450$$
$$n = 36,686 \text{ half years, as above.}$$

SECOND PART.

PRESENT VALUE OF ANY SUM DUE AT A FUTURE TIME.

SECT. 9. The *present value* of a sum due at a future time is the *principal* which will amount to that sum in the given time.

Thus, £100 becomes £105 at the end of one year, at 5 per cent. interest, consequently, the present value of £105 payable in one year, is £100.

Discount is the *allowance* made for the payment of a sum before it becomes due, that is, the *difference* between the sum due at a future time and its present value.

Therefore (form 2) the present value of £1, due at the expiration of n years is $\frac{1}{r^n}$, which being multiplied by the amount of the sum due $= M$, will give $p = \frac{M}{r^n}$ (form 2) for the present value of the sum M due after n years, and therefore the discount will be

$$D = M - \frac{M}{r^n}. \text{ (form 6)}$$

Ex. 16. Let it be required to determine the present value and discount of £3600 payable at the end of seven years, at 6 per cent.

$$\log 3600 = 3,55630\ 25$$
(Tab. 39) $$\log r^7 = 0,17714\ 11$$
$$\log p = 3,37916\ 14$$

the present value $p = £2394,206$, this sum subtracted from £3600 will give for the discount $D = £1205,794$.

Ex. 17. What is the present value of £385. 11*s.* due in 43 days, at 8 per cent., compound interest?

$$\log 385,55 = 2,58608\ 07$$
(Tab. 45) $$\tfrac{1}{365} \log r^{43} = 0,00393\ 76$$
$$\log p = 2,58214\ 31$$

consequently, $p = £382,070$ present value,
and discount, $D = £3,480$

Ex. 18. Required the present value of £44,260, due in three months, at 4½ per cent.

$$\log 44{,}260 = 1{,}64601$$
(Tab. 51) $$\log r^{\frac{1}{4}} = 478$$
$$\log p = 1{,}64123$$
$$p = £43{,}775$$

Ex. 19. Present value of a sum payable *in case* of survivorship, in a given time.

It is required to find the present value of £750, due after 17 years, and *held on a single life* of 26 years, according to the *Northampton Table of Mortality*, the rate of interest being 4 per cent.

Statement: £750 due at the end of 17 years,

$$\text{Present value} = £\frac{750}{r^{17}}.$$

According to the Northampton Table of Mortality, out of 11650 new-born infants, 4685 survive 26 years, and 3404 survive 43 years.

For paying £1 to each survivor, aged 43 years, the 4685 persons, aged 26 years, ought to pay the present value of £3404 due at the end of 17 years, so that each ought to pay

$$V = \frac{3404}{4685} \times \frac{750}{r^{17}}$$
$$\log 750 = 2{,}87506\ 13$$
(Tab. 23)* $$C^t \log r^{17} = 9{,}71043\ 32$$
$$\log 3404 = 3{,}53198\ 96$$
$$C^t \log 4685 = 6{,}32929\ 04$$
$$\log V = 2{,}44677\ 45$$
$$V = £279{,}753$$

which is the present value of £750.

SECT. 10. *Relation between Discount and Interest.*

We have found that the discount of a sum M payable at the end of n years is $D = M\left(1 - \frac{1}{r^n}\right)$.

Likewise, a principal M will give at the expiration of n years an interest $I = M(r^n - 1)$.

Therefore $I = Dr^n$, and $D = \frac{I}{r^n}$.

---

* C^t denotes the *Arithmetical complement of a logarithm*, i. e. the difference between a logarithm and the number 10.

Arithmetical complements are used for saving subtractions, thus, in the present case, instead of subtracting $\log r^{17}$ from $\log 750$, and $\log 4685$ from $\log 3404$, we write down the complements of $\log r^{17}$ and $\log 4685$; then we add the four logarithms together, and from the total characteristic we subtract as many tens as there are complements or divisions, that is, 2×10 or 20.

Otherwise, *the interest of a capital is equal to the amount of the discount of that capital and interest for an equal period of time.*

Reciprocally, *Discount is equal to the present value of Interest.*

Ex. 20. What is the discount of £4000, payable in 3 months, at $4\frac{1}{2}$ per cent.?

$$\begin{aligned} &\log 4000 = 3,60206\ 00 \\ (\text{Tab. } 51)\quad &\log r^{\frac{1}{4}} = \quad 477\ 91 \\ &\log p = 3,59728\ 09 \\ &p = £3956,2244 \\ &D = £43,7756. \end{aligned}$$

We have already found (Ex. 14) that the interest of £4000 for 3 months is £44,260.

The present value of this last sum, payable at 3 months' date, is (Ex. 18) £43,7756.

Hence, it may be seen, that the discount of £4000 for 3 months is equal to the present value of the interest of that principal for the same period of time.

Sect. 11. When the interest is convertible half-yearly, quarterly, &c. the present value of a principal is found by means of the following formula:

$$p = \frac{M}{\left(1+\frac{t}{k}\right)^{kn}} \qquad \ldots\ldots(7)$$

where k represents the number of payments yearly. (form 2)

However, if $k > 1$, the value of $\left(1+\frac{t}{k}\right)^{k}$ is greater than $(1+t)$, and the value of the denominator $\left(1+\frac{t}{k}\right)^{kn}$ will increase with k, although it will be always less than e^{nt}.

Therefore the greater number of intervals for converting the interest into capital, the less the present value, and the greater the discount.

Ex. 21. What is the present value, and the discount of £3600 due after 7 years, the interest being 6 per cent., convertible half-yearly, quarterly, and instantaneously (see Ex. 16).

Half-yearly.

$$\begin{aligned} &\log 3600 = 3,55630\ 25 \\ (\text{Tab. } 15)\quad &\log r^{14} = 0,17972\ 11 \\ &\log p = 3,37658\ 14 \end{aligned}$$

Present value, $p = £2380,024$

Discount $D = £1219,976$

Quarterly.

$$\begin{aligned} &\log 3600 = 3,55630\ 25 \\ (\text{Tab. } 3)\quad &\log r^{28} = 0,18104\ 92 \\ &\log p = 3,37525\ 33 \end{aligned}$$

Present value, $p = £2372,7517$

Discount, $D = £1227,2483$

Instantaneously.

$\log 3600 = 3,55630\ 25$

$\frac{48}{100} \log e = 0,18240\ 37$

$\log p = 3,37389\ 88$

Present value, $p = £2365,368.$

Discount, $D = £1234,632.$

Therefore, the present value of £3600 due at the end of 7 years, at the nominal rate of 6 per cent., will be:

The interest being convertible yearly : £2394,206

" half-yearly : £2380,024

" quarterly : £2372,757

" instantaneously : £2365,368, which is the minimum attainable.

SECT. 12. *Remarkable relation between Discount and Annuities.*

The value of the discount of the sum M payable at the end of n years is according to Sect. 9 (form 6) $D = M\left(1 - \frac{1}{r^n}\right)$.

But in the following chapter it will be shewn that the present value of £1 per annum for n years is

$$V = \frac{1}{i}\left(1 - \frac{1}{r^n}\right) \ldots\ldots (8),$$

whence, $D = VMt$.

Also Mt denotes the yearly interest of the sum M.

Therefore, the discount of $£M$ payable in n years is equal to the present value of an annuity for n years, the annual income being equal to the annual interest of $£M$.

EX. 22. What is the discount of £3600, payable in seven years, the rate of interest being 6 per cent. per annum?

Statement:

The interest of £3600 is £216 per annum, consequently the discount is equal to the annuity of £216 per annum, for seven years.

$\log 216 = 2,33445\ 38$

(Tab. 39) $\log a^{n*} = 9,25318\ 05$

$\log D = 3,08127\ 33$

$D = £1205,794$ as above $(= 3600 - 2394,206)$.

EX. 23. What is the discount of £3600 due at the end of seven years, the rate of interest being $1\frac{1}{2}$ per cent. quarterly?

The interest of £3600 is £54 quarterly, consequently the discount is equal to an annuity of £54 per annum for 28 years,

$\log 54 = 1,73239\ 38$

(Tab. 3) $\log a^n = 8,64346\ 33$

$\log D = 3,08893\ 05$

$D = £1227,243$ as above.

---

* a^n being the annuity purchased by £1 for a given number of years.

SECT. 13. *Equation of Payments.*

When a sum is to be paid in two or several portions at various periods, it is often required to convert the whole of these fractional payments into a single one. Payments ought then to be equated, i.e. the time at which the whole debt is to be paid up ought to be determined.

It is very simple to solve this problem.

Let S denote the whole debt,

V the present value of the single payment, or of all the instalments, and x the true equated time;

the value of x will consequently be expressed by the formula (4)

$$x = \frac{\log S - \log V}{\log r}.$$

Ex. 24. £1890 are to be paid in three instalments, viz. £1294 in seven months, £735 in two years, and £861 in $5\frac{1}{2}$ years; at what time, reckoning compound interest at $5\frac{3}{8}$ per cent, ought the whole to be discharged in one single payment of £2890?

The present value of the three instalments is = £2562,584,

consequently, $\log S = 3{,}46090$

$\log V = 3{,}40868$

$\log \frac{S}{V} = 0{,}05222$

$\log^2 = 8{,}71784$ [$\log^2$ means log log. See p. 9.]

(Tab. 57) $\log^2 r = 8{,}35675$

$\log x = 0{,}36109$

$x = 2{,}2966$, or 2 years and 108 days.

When the two instalments are equal to each other, if the *true* equated time is required, we observe that it will always be *less* than the mean time of both payments.

Suppose the first payment is to be made after n years, and the second after $(n+2d)$ years, and x to be the equated time;

we shall have $\frac{2}{r^x} = \frac{1}{r^n} + \frac{1}{r^{n+2d}}$,

whence $r^x = \frac{2r^{n+d}}{r^d + r^{-d}}$;

but $(n+d)$ is the mean of both instalments, and $(r^d + r^{-d})$ is always* greater than 2, therefore $r^x < r^{n+d}$,

and $x < n+d$.

---

* As the square of a real number, positive or negative, is always positive, the square of $(r^d - 1)$ is always greater than 0, hence we have

$$(r^d - 1)^2 > 0,$$

or, developing $r^{2d} - 2r^d + 1 > 0$,

and dividing by r^d, we get $r^d - 2 + r^{-d} > 0$,

therefore $r^d + r^{-d} > 2$.

Ex. 25. Suppose £1000 are to be paid in two equal instalments, the *first* in seven years, and the *second* in 43 years, at what time, reckoning the interest at 4½ per cent., ought the whole debt to be discharged in one single payment of £1000?

The present value of both payments of £500 each is = £442,7445, consequently

$$\log S = 3,$$
$$\log V = 2,64615$$
$$\log \frac{S}{V} = 0,35385$$
$$\log^3 = 9,54882$$
(Tab. 57) $$\log^3 r = 8,28140$$
$$\log x = 1,26742$$
$$x = 18,510 \text{ years}$$

Therefore the time is 18 years, 186 days.

Thus, the first sum will be paid 11 years and 186 days later, and the second sum will be paid 24 years and 179 days beforehand. As a proof of this being correct, it will be found that the interest for 11 years and 186 days of the first £500 is equal to the discount of £500 payable at the end of 24 years and 179 days, reckoning the whole compound interest at 4½ per cent.

Sect. 14. *Usual Discounts.*

Commercial Discount.

In formula (2) $p = (1+t)^{-n}$, which represents the present value of £1 payable at the end of n years. If we expand $(1+t)^{-n}$ by means of the binomial theorem into an infinite series, ascending with the powers of t, we shall obtain

$$p = 1 - nt + \frac{n(n+1)}{2} t^2 \ldots\ldots\ldots$$

If we suppress in this series all the terms, excepting the first two, we shall have $p = 1 - nt$, for the present value of £1 at the end of n years, and consequently $D = nt$, for the whole discount, that is, the commercial discount.

However, as nt denotes the simple interest during n years, the commercial discount is merely the interest of the whole sum or debt, calculated from the present time, till it becomes due*.

Ex. 26. Let it be required to find the commercial discount of £250 at 6 per cent., interest, for one month.

$$D = 250 \times \frac{1}{12} \times \frac{6}{100} = £1\tfrac{1}{4}, \text{ or } £1.\ 5s.$$

---

* Among bankers and merchants it is usual to add to the time at which the money becomes due, three days, called days of grace.

In order to simplify the calculation, 360 days only are usually reckoned for the year. If the principal is very small, and the time it becomes due very near, the present value of the principal thus computed differs but very little from its true present value. But, as $(1+t)^{-n}$ is always greater than $(1-nt)$, the present value is always too small, and the error increases with the rate of interest and the number of years. Hence, the present value could be negative, and the required discount might be larger than the principal. This extreme result shows at once that the formula $D=nt$ is objectionable.

Thus, if we apply it to the rate 5 per cent., n being $=20$ years, we should find $D=1$, that is, £1 payable in 20 years would have no present value at all.

SECT. 15. *Discount at Simple Interest.*

If again we consider the formula $p=\frac{1}{(1+t)^n}$ and exp nd the denominator into a series by the binomial theorem, we find

$$p=\frac{1}{1+nt+\frac{1}{2}n(n-1)t^2+\&c.}.$$

Then, if we omit all terms, except the first two of the series, we shall have $p=\frac{1}{1+nt}$,

$$\text{whence } D=\frac{nt}{1+nt},$$

which affords another method of valuing discount with a sufficient approximation.

Ex. 27. Let it be required to determine the discount of £250 at 6 per cent. interest for one month.

£1 becomes £1,005 at the end of one month, consequently the present value of £250 will be $=\frac{250}{1,005}=$£248,756, and the discount will be £1,244, or £1. 4*s*. 10$\frac{1}{2}$*d*.

Otherwise: $D=250\times\frac{,005}{1,005}=\frac{1,25}{1,005}=1,244$,

that is, the discount is £1,244.

However, as $(1+t)^n$ is greater than $(1+nt)$ when $n>1$ and $(1+t)^n$ is smaller than $(1+nt)$ when $n<1$, the present value determined by this last method is either too small or too great, according as the principal is due before or after the expired first year.

For better appreciating the difference between the true discount and the two usual discounts, the reader will find in the subjoined Table this difference fully exemplified for £100, reckoning the interest to be at 6 per cent.

Discount at 6 per cent. of £100.			
Time of Expiration	True Discount.	Commercial Discount.	Discount at Simple Interest.
1 month	0,484	0,5	0,498
2 months	0,966	1	0,990
3 ...	1,446	1,5	1,478
4 ...	1,924	2	1,961
6 ...	2,871	3	2,913
8 ...	3,810	4	3,846
1 year	5,660	6	5,660
2 years	11,000	12	10,714
3 ...	16,033	18	15,254
4 ...	20,791	24	19,355
10 ...	44,161	60	37,500
15 ...	58,273	90	47,368

THIRD PART.

PRESENT VALUE OF IMMEDIATE ANNUITIES.

SECT. 16. *Present value of £1 per annum for* n *years, with immediate entrance.*

The last chapter shows the method for determinating the present value of a *single sum* due at a given time; consequently, when it is required to determine the present value of *several payments* due at different times, there is nothing else required but to find the present value of each separate payment, and the sum of all these different values will be the present value of the whole payment.

Thus are found the *present value* and *amount of all kind of annuities* certain or for life, terminal or perpetual, constant or variable, immediate or deferred.

Let therefore

t = the *annual rate* of interest,

$r = (1+t) = £1$ increased by its interest for one year,

$\phi(n)$, or simply ϕ = the *present value* of £1 per annum, for n years,

a = *any annuity* or annual rent,

a^n = the annuity which £1 will purchase for n years,

and V = the *present value* of £1 per annum, for n years, the first annuity being assumed to be payable at the end of the first year.

Let us first calculate the present value of n annuities of £1; the *first annuity being* payable at the end of the first year, the second

annuity at the end of the second year, and the last at the end of the n^{th} year.

The present value of

£1 payable at the end of one year is $=\frac{1}{r}$,

..... two years $=\frac{1}{r^2}$,

..... three years $=\frac{1}{r^3}$,

..... n years $=\frac{1}{r^n}$,

and as the sum of all these values will express the present value of the n annuities of £1, we shall obtain

$$\phi=\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}\ldots+\frac{1}{r^n}.$$

Therefore the present value of any annuity is equal to the sum of a series in geometrical progression, the ratio of which is $\frac{1}{r}$.

Multiplying all the terms of this equation by r, we shall have

$$r\phi=1+\frac{1}{r}+\frac{1}{r^2}+\ldots+\frac{1}{r^{n-1}},$$

and subtracting the first equation from this,

$$t\phi=1-\frac{1}{r^n},$$

whence $\phi=\frac{1-r^{-n}}{t}$, which is the general expression of the present value of £1 per annum, for n years.

As ϕ denotes the present value of £1 per annum, it is clear that the present value of any other annuity of £a for the same time is equal to such annuity multiplied by ϕ, that is,

$$V=a\phi=\frac{a}{t}(1-r^{-n}).$$

The ratio between the present value of whatever annuity, and its annual amount, or the value of the annuity divided by the annuity itself, being called the *number of years' purchase*, we shall obtain reciprocally:

$$a=\frac{V}{\phi}=\frac{Vt}{1-r^{-n}},$$

that is, the *annuity* which a given sum or principal V will purchase for a given number n of years will be equal to this principal divided by ϕ, the present value of £1 per annum, for n years.

If the principal $V=£1$, and if a^n denotes the annuity which £1 will purchase for n years, the last equation becomes

$$a^n=\frac{1}{\phi}=\frac{i}{1-r^{-n}},$$

therefore $a^n\phi=1$,

and $\log a^n+\log\phi=0$*,

that is, the logarithm of the annuity which £1 will purchase for n years is equal to the arithmetical complement of the logarithm of the present value of £1 per annum for n years, and *vice versâ*.

Thus, for instance, the annuity being £1 for nine years, and at 4½ per cent., we have

$$\log\phi=0,86146\ 21$$

(Tab. 27) $\log a^n=9,13853\ 79.$

The tables 1 to 48 shew the logarithm of a^n for several rates of interest, and for every number of years, from 1 to 100.

SECT. 17. *Limits of Annuities.*

When in the equation $V=\frac{a}{i}(1-r^{-n})$ the value of n, or the number of years increases, the value of r^{-n} diminishes; consequently, the present value of the annuity increases likewise, but remains still less than the present value of the perpetuity.

Thus, for instance, the present value of £1 at 5 per cent. is

for	62 years	= £19,02883	40
	109 ...	= 19,90196	16
	156 ...	= 19,99010	31
	203 ...	= 19,99900	09
	251 ...	= 19,99990	39
	298 ...	= 19,99999	03
	345 ...	= 19,99999	90
	392 ...	= 19,99999	99,

and the present value of the perpetuity $=£20$.

When in the equation $a=\frac{Vi}{1-r^{-n}}$, n likewise increases, the denominator of the fraction *increases*, and the annual rent to be paid diminishes, but remains *still greater* than the perpetual annuity.

Thus, for instance, *the annuity* which £100 will purchase at 5 per cent. is,

for	36 years	= £6,04344	57
	80 ..	= 5,10296	24
	127 ...	= 5,01020	50
	174 ...	= 5,00102	83
	221 ...	= 5,00010	38
	268 ...	= 5,00001	05
	316 ...	= 5,00000	10
	363 ...	= 5,00000	01
and for perpetuity		= 5.	

* In the tables $\log a^n$ is increased by 10 to exclude negative logarithms.

SECT. 18. *Relation between terminal and perpetual annuities.*

It has been found that $V=\frac{a}{t}(1-r^{-n})=\frac{a}{t}-\frac{a}{tr^{n}}$,

but $\frac{a}{t}$ denotes the present value of the perpetuity £a, consequently *the present value of the annuity £a for* n *years is equal to the present value of the perpetuity of £a per annum, minus this same perpetuity discounted for* n *years, both at the same rate.*

If $n=1$, and $a=1$, the formula above becomes more simply:

$$\phi=\frac{1}{t}\left(1-\frac{1}{r}\right)=\frac{1}{r}.$$

If $n=2$, it becomes: $\phi^2=\frac{1}{t}\left(1-\frac{1}{r^2}\right)=\frac{r+1}{r^2}$.

Thus, for instance, two annuities of £1 at 4¾ per cent. will give

$\log(r+1)=0,31122\ 39104$
(Tab. 57) $2\log r=0,04030\ 80632$
$\log\ \phi^2=0,27091\ 58472$
(Tab. 29) $\log a^n=9,72908\ 41528$, the complement of the above, since $a^n=\frac{1}{\phi}$.

EX. 28. What is the present value of a lease of £1 per annum, for 62 years, at 5 per cent.?

$\log r^{62}=1,31373\ 65$
$\log t\ \ =8,69897\ 00$
$\log tr^{62}=0,01270\ 65$
$C^t=9,98729\ 35$
$\frac{r^{-62}}{t}=0,97116\ 60$

Perpetuity = 20.

Present value of the annuity = £19,0288340, or nearly 19 years' purchase of the clear annual rent.

SECT. 19. *Calculation of the present value and annuity by means of the formula* 8.

EX. 29. What sum will be required, for the purchase of an annuity of £840 to continue 37 years, interest 4¾ per cent.?

$\log r^{-37}=9,25430\ 08$
$r^{-37}=0,17959\ 77$
$1-r^{-37}=0,82040\ 23$
$\log(1-r^{-37})=9,91402\ 69$
$\log t=8,67669\ 36$
$\log\phi=1,23733\ 33$
$\log a=2,92427\ 93$
$\log V=4,16161\ 26$
$V=$ £14508,17

Ex. 30. Required the present value of an £86 annuity, to continue 123 years, allowing $3\frac{1}{2}$ per cent.

(Tab. 22)
$$\log r^{100} = 1,65110\ 37$$
$$\log r^{23} = 0,37975\ 38$$
$$\log r^{123} = 2,03085\ 75$$
$$C^l = 7,96914\ 25$$
$$r^{-123} = 0,00931\ 41$$
$$1 - r^{-123} = 0,99068\ 59$$
$$\log (1 - r^{-123}) = 9,99593\ 60$$
$$\log t = 8,58827\ 17$$
$$\log \phi = 1,40766\ 43$$
$$\log a = 1,93449\ 85$$
$$\log V = 3,34216\ 28$$
$V = £2198,684$, present value of £86 per annum, for 123 years.

Ex. 31. A sum of £84,000 is borrowed at $4\frac{3}{4}$ per cent., which is to be reimbursed in 37 annuities. What amount is to be paid per annum?

$$\log V = 4,92427\ 93$$
(Tab. 29) $$\log \phi = 1,23733\ 33$$
$$\log a = 3,68694\ 60$$
$$a = £4863,468.$$

The annual interest of £84,000 at $4\frac{3}{4}$ per cent. being $= £3990$, it would be required to pay £873. 9*s*. $4\frac{1}{3}d$. more, in order to reimburse the capital.

Ex. 32. What annuity, or yearly income, to continue 64 years, can be purchased for £100,000 at $5\frac{1}{4}$ per cent.?

$$\log r = 0,02222\ 21045$$
(Tab. 33) $$\log r^{64} = 1,42221\ 46885$$
$$C^l = 8,57778\ 53115$$
$$r^{-64} = 0,03782\ 55552$$
$$1 - r^{-64} = 0,96217\ 44448$$
$$\log (1 - r^{-64}) = 9,98325\ 38179$$
$$C^l = 0,01674\ 61821$$
(Tab. 54) $$\log t = 8,72015\ 93034$$
$$\log V = 5$$
$$\log a = 3,73690\ 54855$$
$$a = £5456,391020.$$

Thus, every question, relating to the subject, reduces itself to multiplying or dividing the given number by ϕ, the present value of £1 per annum, in conformity with the foregoing formulæ:

$$V = a\phi,$$

$$a = \frac{V}{\phi},$$

according as the present value of an annuity, or the annuity itself, which a given sum will purchase for a given number of years, is looked for.

It is chiefly the latter which occurs, especially whenever it is required to borrow large sums, reimbursable after a long period of time, and by means of *constant annuities*, of which each pays the interest of the year, and at the same time pays off a portion of the principal.

This mode of reimbursement of a principal is of great benefit to the borrower, as it affords him the means of paying off every year, without any sudden or extraordinary effort, and causes the fatal time for the reimbursement of the whole, when new sacrifices are almost necessary, to vanish away.

An annually appointed rent can be made to reimburse several different sums, according to the rate stipulated for; thus, for instance, £117,23 are to be paid for £1000, borrowed at the rate of 3 per cent.; but this same annuity is again necessary for reimbursing £905,22 at 5 per cent. It is therefore just the same to borrow £905,22 at 5 per cent., or to reimburse £1000 at 3 per cent., without any prejudice to the contractors, as in either case £117,23 are to be paid annually for the same number of years.

This method of transmuting reimbursable principals and rates is often resorted to, especially in the case of bonds reimbursable by lottery, and for which the remaining capital affords what is called the "premium of reimbursement."

SECT. 20. *Logarithmic Tables for computing Annuities.*

As in most calculations, the use of logarithms is extremely advantageous, and often of absolute necessity, I have shown in Tables 1 to 48 the logarithms of $\left(\frac{1}{\phi}\right)$, i. e. the logarithms of the annuity which £1 will purchase for a given number of years, at 48 different rates of interest, and for every year from 1 to 100 years.

Consequently, as $a = \frac{V}{\phi}$,

and $V = a\phi$,

if it is required to compute the annuity which a given sum will purchase for a given time, the tabular logarithm of $\left(\frac{1}{\phi}\right)$ which corresponds to the given rate and number of years, is to be added to the logarithm of the principal.

If, on the contrary, it is required to find the *present value* of any annuity, either the tabular logarithm is subtracted from the loga-

rithm of the annuity, or the arithmetical complement of the tabular logarithm is added to the logarithm of the annuity.

Ex. 33. What annuity, or yearly income, to continue 20 years, may be purchased for £1000, at 3½ per cent.?

$$\log 1000 = 3$$
(Tab. 19) $$\log a^n = \underline{8,84733\ 25}$$
$$\log a = 1,84733\ 25$$
$$a = £70,36108,$$ i. e. the annuity will be £70. 7*s*. 2¾*d*.

Ex. 34. What is the *title to an estate* worth for 21 years, the clear annual produce of the estate being £135, and the rate of interest 5 per cent.?

$$\log 135 = 2,13033\ 38$$
(Tab. 31) $$\log a^n = \underline{8,89207\ 30}$$
$$\log V = 3,23826\ 08$$
$$V = £1730,855$$

thus, the required value is £1730. 17*s*. 1½*d*.

Ex. 35. A £5600 principal has been converted into 28 annuities, at 3 per cent.; the owner of these annuities wishes to sell them at the end of the second year; how much is to be paid, in order to invest one's money at 5½ per cent.?

$$\log 5600 = 3,74818\ 80$$
(Tab. 15) $$\log a^n = \underline{8,72667\ 20}$$
$$\log a = 2,47486\ 00$$
(Tab. 32) $$\log a^n = \underline{8,84796\ 48}$$
$$\log V = 3,62689\ 52$$
$$V = £4235,408,$$ required value.

Thus, for a £5600 principal, at 3 per cent., there would be 28 annuities of £298,442 to be paid, and 26 annuities of that amount are worth at 5½ per cent. £4235. 8*s*. 2*d*.

SECT. 21. *Interpolation of the Tables.*

Whenever it is required to calculate an annuity for a rate which is not contained in the Tables, it can be done very exactly, in most cases, by means of interpolation.

Generally, if *proportional parts* were only taken, the logarithm of the annuity would be *too small*, because the differences with regard to logarithms are constantly decreasing, when the rate or number of years increases.

When, however, for the same number of years, the logarithms of these annuities are compared, the rate being successively increased by ½ per cent., we find that the differences between the successive tabular logarithms proceed pretty nearly according to an arithmetical progression, therefore for an increment less than ½ per cent. we may consider the tabular logarithms to vary, according to an arithmetical progression.

The general formula for interpolating equations of the second degree becomes in the case of equidistant arguments*

$$y_k = y + k \cdot \Delta y + \frac{k \cdot (k-1)}{2} \cdot \Delta^2 y,$$

y denoting the resultant corresponding to the highest tabular rate contained in the given rate, and Δy, Δy^2 the first and second differences, and k being the relation between the differences of the tabular arguments ($\frac{1}{8}$ per cent.) and the difference of the highest tabular rate.

Ex. 36. Required the annuity which £10000 will purchase in 23 years, at $4\frac{7}{16}$ per cent.

The given rate being $4\frac{7}{16} = 4\frac{3\frac{1}{2}}{8}$, $k = \frac{1}{2}$, so we shall find in the Table for 23 years, and by means of a subtraction:

	y	Δy	$\Delta^2 y$
$4\frac{3}{8}$ per cent.	8,44405 12		
$4\frac{1}{2}$	8,49931 18	526 06	−320
$4\frac{5}{8}$	8,54504 04	522 86	

$$\text{therefore, } \log a = 8,84405.12 + \tfrac{1}{2} \times 526\ 06 + \frac{\tfrac{1}{2} \cdot \tfrac{1}{2}}{2} \cdot 320$$

$$8,84405\ 12$$
$$263\ 03$$
$$40$$

i. e. $\log a^n = 8,84668\ 55$, rigorously true,

$\log 10000 = 4$

$\log a = 2,84668\ 55$

$a = £702,5634.$

Ex. 37. Required the present value of £100 per annum for 51 years and at £4. 17*s.* per £100.

$$\text{the rate} = 4\tfrac{17}{20} = 4\frac{6\frac{4}{5}}{8}$$

therefore $k = \frac{4}{5}$

$4\frac{3}{4}$ per cent.	8,71743 01	+	−
$4\frac{7}{8}$	72615 06	872 05	1153
5	73475 58	860 52	

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* All numerical tables consist of two series of corresponding values, each value in either series having a value corresponding to it in the other; the first series forms commonly an arithmetical progression.

The number with which a numerical table is *entered*, that is, the given number for which the corresponding number in the table is sought, is called the *argument;* and the number so taken is called the *tabular result*.

therefore, $\log a^n = 8,71743\ 01 + \frac{4}{5} \times 872\ 05 + \frac{\frac{4}{5} \cdot \frac{1}{5}}{2} \times 1153$

$= 8,71743\ 01$
$697\ 64$
92

$\log a^n = 8,72441\ 57$
$\log 100 = 2$

$\log a = 3,27558\ 43$
$a = £1886,185$

Ex. 38. Required the annuity which £7563 will purchase i years at $4\frac{3}{5}$ per cent.

rate $4\frac{3}{5} = 4\ \frac{4\frac{4}{5}}{8}$

therefore $k = \frac{4}{5}$

		+	−
$4\frac{1}{2}$ per cent.	70184 69		
$4\frac{3}{5}$	71071 30	886 61	1156
$4\frac{3}{4}$	71946 35	875 05	

$\log a^n = 8,70184\ 69 \times \frac{4}{5} \times 886\ 61 + \frac{\frac{4}{5} \cdot \frac{1}{5}}{2} \times 1156$

$= 8,70184\ 69$
$709\ 29$
92

$\log a^n = 8,70894\ 90$
$\log V = 3,87898\ 11$

$\log a = 2,58793\ 01$
$a = £387,1954$

Ex. 39. Required the present value of £345. 14s. per ani for 48 years, and at $4\frac{1}{3}$ per cent.

rate $4\frac{1}{3} = 4\ \frac{2\frac{2}{3}}{8}$

$k = \frac{2}{3}$

		+	−
$4\frac{1}{3}$ per cent.	69168 82		
$4\frac{2}{3}$	70048 49	879 67	1085
$4\frac{1}{2}$	70917 31	868 82	

consequently $\log a^n = 8,69168\ 82$
$586\ 45$
$1\ 21$

$\log a^n = 8,69756\ 48$
$\log 345,7 = 2,53869\ 94$

$\log V = 3,84113\ 46$
$V = £6936,410$

Let us notice, that in the four preceding examples, the l rithms obtained by means of proportional parts would be too s by 40, 92, 92 and 120.

SECT. 22. *Amortizement or Sinking Fund.*

From the equation (8) $V=\frac{a}{i}\left(1-\frac{1}{r^n}\right)$

we deduce directly $a=Vi+\frac{a}{r^n}$ ..... (9)

but Vi represents the annual interest of the principal V; therefore the annuity which a given sum will purchase for n years may be considered as being composed of two distinct parts: the first of which would be chiefly devoted to the payment of the annual interest, and the second, which I denote by s, would be kept for the gradual redemption of the principal.

It is evident that the annuity can only pay off the principal and interest if it exceeds the yearly interest. The difference $\left(\frac{a}{r^n}\right)$ between the annuity and annual interest is termed *amortizement.* In loans contracted by Government the reimbursement of the principal is carried out by means of a special fund, to which the name of Sinking-fund has been given, because it is appropriated to the purpose of sinking the public debt*.

The annuity can therefore be considered as including two very distinct operations:

1st. The payment at the end of every year of the simple and annual interest of the borrowed principal;

2ndly. The gradual reimbursement of the principal.

Hence, by deducting from the amount of an annuity the sum to be paid annually for the simple interest of the received principal, the overplus invested successively at compound interest and at the fixed rate must give a sum equal to the principal which is to be paid off, according to the agreement made.

If s denotes the annual sum appropriated to the sinking fund, or the annual amortizement, the amount of which will produce the principal, we have

$$a=Vi+s$$

and by (10) $s=\frac{a}{r^n}=\frac{Vi}{r^n-1}$,

but a denotes the annuity, and $\frac{a}{r^n}$ denotes the present value of this annuity at the expiration of n years; therefore, the amortizement, or annual sinking-fund, is equal to the present value of the last annuity.

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* See R. Price: *Observations on Reversionary Payments*, 1. p. 269.

Ex. 40. A loan of £4½ millions is contracted at 3½ per cent., reimbursable by means of 73 annuities. What sum is to be paid annually, besides the interest, for the purpose of redeeming this loan in a given time?

$$\begin{array}{rll} & \log 4{,}500{,}000 = & 6{,}65321\ 25 \\ \text{(Tab. 19)} & \log a^n = & 8{,}58082\ 92 \\ \text{(Tab. 19)} & C^t \log r^n = & 8{,}90935\ 45 \\ & \log s = & 4{,}14339\ 62 \\ & s = & £13912{,}21 \end{array}$$

Consequently, in order to extinguish gradually £4,500,000 debt, it is necessary to pay annually, besides the interest = £157.500, a sum = £13912,21.

Ex. 41. A person purchasing for £500 the lease of an unexpired term of 60 years on certain premises, requires to know the sum he must annually put by at 3 per cent., in order to replace his capital at the termination of the lease.

$$\begin{array}{rll} & \log 500 = & 2{,}69897 \\ (15) & \log a^n = & 8{,}55790 \\ (15)\ C^t & \log r^n = & 9{,}22977 \\ & \log s = & 0{,}48664 \\ & s = & 3{,}0665 \end{array}$$

consequently the amortizement is = £3. 1*s.* 4*d.*

The calculation of amortizement furnishes us with excellent means for confirming and determinating more scrupulously the value of an annuity.

The annual interest of a principal being always easily obtained with desirable accuracy, nothing else is required but to calculate the *amortizement* and add it to the interest, in order to find the value of the annuity. However, the amortizement being (r^n) times smaller than the annuity, the error on the last figure of the annuity will be (r^n) times less than the error caused by direct calculation.

Ex. 42. In order to reimburse in 83 years a principal of £1,000,000 borrowed at the rate of 4⅞ per cent., what sum is to be paid annually for amortizement?

$\log 1{,}000{,}000 + \log a^n = 4{,}69641\ 23$

$a = £49.706,\ 40$ which is the annuity required.

$\log 1{,}000{,}000 + \log a^n = 4{,}69641\ 23$

(Tab. 30) $\log r^n = 1{,}71577\ 38$

$\log s = 2{,}98063\ 85$

$s = £956{,}3976$ amortizement,

£48750 annual interest,

£49706,3976, annuity.

As the rate of interest increases, the present value of £1 per annum

$$\phi = \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \ldots\ldots\ldots + \frac{1}{r^n},$$

diminishes.

Therefore, as the rate of interest increases, the annuity which a given sum will purchase for a given number of years, increases likewise.

The interest increases proportionally.

But the amortizement decreases proportionally.

For example, in order to reimburse £100,000 in 27 years, we must pay according to the following table:

Rate per cent.	Annual Interest.	Amortizement.	Amount of the Annuity.
1	£ 1000	£3245	£ 4245
2	2000	2829	4829
3	3000	2456	5456
4	4000	2124	6124
5	5000	1829	6829
6	6000	1570	7570
7	7000	1343	8343
8	8000	1145	9145
9	9000	973	9973
10	10000	826	10826

SECT. 23. In order to exhibit conveniently the progress of the gradual redemption of a loan, and the relations between the principal, annuity and amortizement, let us examine, first, the special case of a principal £100, invested at 5 per cent. interest, and reimbursable by 4 annuities.

(Tab. 31)
$$\begin{aligned} \log 100 &= 2 \\ \log a^n &= 9{,}45027 \\ \log a &= 1{,}45027 \\ a &= £28{,}201. \end{aligned}$$

Consequently, in exchange for a £100 principal, 4 annuities of £28,20 will be paid, of which £5 is for the annual interest, and £23,20 for the amortizement.

The exchange of a principal against annuities, and the extinction of that principal, may be considered under four different heads:

I.

Simple settlement of accounts at the end of every year.

The annuity being known, the operation to be performed at the end of every year may be looked upon as a simple account between creditor and debtor, and this account may be settled without any question of compound interest ever arising.

Principal on the 1st year	£100	
Interest	5	
Due at the end of the 1st year	105	
First annuity	28	20
Principal on the 2nd year	76	80
Interest	3	84
Due at the end of the 2nd year	80	64
Second annuity	28	20
Principal on the 3rd year	52	44
Interest	2	62
Due at the end of the 3rd year	55	06
Third annuity	28	20
Principal on the 4th year	26	86
Interest	1	34
Due at the end of the 4th year	28	20
Fourth annuity	28	20
Balance at the end of the 4th year	0	0

Instead of laying down the interest and annuity in the account, we might have merely shown the sum paid on account at the end of every year, for the purpose of redeeming the loan, or the surplus of the annuity over the interest. As owing to the partly made reimbursement, the principal diminishes annually, the interest diminishes likewise, and consequently the amortizement or difference between the constant annuity and interest yearly increases.

Principal of the 1st year	£100	
Surplus of the constant annuity over the interest of this sum	23	20
Principal of the 2nd year	76	80
Surplus of the constant annuity	24	36
Principal of the 3rd year	52	44
Surplus of the constant annuity	25	58
Principal of the 4th year	26	86
Surplus of the constant annuity	26	86
Remnant at the end of the 4th year	0	0

II.

The Amortizement (Annual Sinking Fund) to be set apart, and accumulated at compound interest, in order to make up again the principal at the end of the four years.

We have found that £28 20 are to be paid per annum, £5 of which are appropriated to the annual interest, and £23 20 to the gradual redemption of the principal. Let us suppose actually that the debtor pays only the annual interest, until the settled time, and yearly invests at the same rate the £23 20 destined to the amortizement.

The result will be

Amount at end of 1st year	£ 23 20
Invested at beginning of 2nd year	23 20
One year's interest	1 16
Amortizement for 2nd year	23 20
Amount at end of 2nd year	£ 47 56
Invested at beginning of 3rd year	47 56
One year's interest on same	2 38
Amortizement for 3rd year	23 20
Amount at end of 3rd year	£ 73 14
Invested at beginning of 4th year	73 14
One year's interest on same	3 66
Amortizement for 4th year	23 20
Amount at end of 4th year	£ 100 00,

by means of which the principal will be reimbursed.

III.

Discount of every particular annuity.

Every particular annuity may be again made to confirm the calculation. Thus, the present value of the

1st	annuity of	£28 20	payable in	1 year	is = £26 86
2nd	.....	28 20	.....	2 years	= 25 58
3rd	.....	28 20	.....	3	= 24 36
4th	.....	28 20	.....	4	= 23 20

and adding successively these values, we have for the present value of the 1st annuity = £ 26 86

of the first	2	annuities =	52 44
.....	3	..... =	76 80
.....	4	..... =	100

It may be noticed, that the present value of the last annuity is equal to the amortizement or annual sinking fund, and that the successive values of the annuities are equal to the invested numbers which represent the principal of every year.

IV.

Annuity to be set apart, and accumulated at compound interest, in order to provide at the end of the given number of years a sum equal to the amount of the principal at the end of the same number of years.

Let us now suppose that there is no settlement of accounts between creditor and debtor before the time of the integral reimbursement, and that the debtor at the expiration of every year invests the annuity, and let us value the amount of the principal and annuities.

Amount of the principal,

Principal of the 1st year	£100
Interest	5
Principal of the 2nd year	105
Interest	5 25
Principal of the 3rd year	110 25
Interest	5 51
Principal of the 4th year	115 76
Interest	5 79
Amount of the principal at the end of the 4th year	£121 55

Amount of the annuity,

First annuity	£ 28 20
Interest	1 41
Second annuity	28 20
Amount at the end of the 2nd year	57 81
Interest	2 89
Third annuity	28 20
Amount at the end of the 3rd year	88 90
Interest	4 45
Fourth and last annuity	28 20
Amount of the annuity of £28 20 for 4 years	£121 55

Therefore, at the end of the 4th year the amount of the principal is £121 55; the amount of the annuity is also £121 55, and the results are again concordant.

If we take another example, a principal of £100,000 at 4 per cent. to be reimbursed by equal annuities in 16 years, we find directly,

(Tab. 23)
$$\log 100{,}000 = 5$$
$$\log a^n = 8{,}93358\ 85$$
$$\log a = 3{,}93358\ 85$$
$$a = £8582,$$

consequently,

£4000 for the interest,
4582 for the sinking fund,

or, £8582, amount of the annuity which will have to be paid annually, during 16 years.

The following Table shows the different methods of redemption of the principal, as in the foregoing example:

Redemption of a loan of £100,000 at 4 per cent. against 16 annuities of £8582.

	I. Settlement of Accounts at the end of every year.				II. Sinking Fund.	III. Discount of Annuities.		IV. Amount at the end of every year of the	
	Principal at the beginning of the year. 1.	Annuity of £8582. Annual Interest of the Principal. 2.	Annuity of £8582. Amortizement. 3.	Principal left at the end of the year. 4.	Amount of the Amortizement at the end of the year. 5.	Present value of each Annuity. 6.	Present value of all Annuities paid at the end of the year. 7.	Principal. 8.	Annuity. 9.
1	£100,000.	£4000.	£4582.	£95,418.	£4582.	£8,251.92	£8,251.92	£104,000.	£8582.
2	95,418.	3,816.72	4,765.28	90,692.72	9,347.28	7,934.54	16,186.46	108,160.	17,507.28
3	90,652.72	3,626.11	4,955.89	85,696.83	14,303.17	7,629.37	23,815.83	112,486.40	26,789.57
4	85,696.83	3,427.87	5,154.13	80,542.70	19,457.30	7,335.93	31,151.76	116,985.86	36,443.15
5	80,542.70	3,221.71	5,360.29	75,182.41	24,817.59	7,053.78	38,205.54	121,665.29	46,482.88
6	75,182.41	3,007.30	5,574.70	69,607.71	30,392.29	6,782.48	44,988.02	126,531.90	56,924.19
7	69,607.71	2,784.31	5,797.69	63,810.02	36,189.98	6,521.62	51,509.64	131,593.18	67,783.16
8	63,810.02	2,552.40	6,029.60	57,780.42	42,219.58	6,270.78	57,780.42	136,856.91	79,076.49
9	57,780.42	2,311.22	6,270.78	51,509.64	48,490.36	6,029.60	63,810.02	142,331.18	90,821.55
10	51,509.64	2,060.39	6,521.62	44,988.02	55,011.98	5,797.69	69,607.71	148,024.43	103,136.41
11	44,988.02	1,799.52	6,782.48	38,205.54	61,794.46	5,574.70	75,182.41	153,945.41	115,739.87
12	38,205.54	1,528.22	7,053.78	31,151.76	68,848.24	5,360.29	80,542.70	160,103.22	128,951.46
13	31,151.76	1,246.07	7,335.93	23,815.83	76,184.17	5,154.13	85,696.83	166,507.35	142,691.52
14	23,815.83	952.63	7,629.37	16,186.46	83,813.54	4,955.89	90,692.72	173,167.64	156,981.18
15	16,186.46	647.46	7,934.54	8,251.92	91,748.08	4,765.28	95,418.	180,094.35	171,842.43
16	8,251.92	330.08	8,251.92	0. 0	100,000.	4,582.	100,000.	187,298.13	187,298.13

SECT. 24. In order to generalise the matter, and the different methods of representing the exchange of a principal against annuities, the following table (page 33) has been calculated, which plainly illustrates the mechanism of annuities.

As formerly, let

a = the annual rent,

$s = \frac{a}{r^n}$ the difference between this annuity and interest, or the annual amortizement,

$\phi^n = \frac{1 - r^{-n}}{i}$ the present value of £1 per annum for n years,

the principal to be reimbursed will be $= a \cdot \phi^n$, and we find :

I. *For the settlement of accounts at the end of every year*, that

1st, the *principal* diminishes progressively, and represents at the end of every year a sum equal to the present value of as many annuities as there are years left, until the total extinction of the principal.

2ndly, the *interest* diminishes proportionately to the principal.

3rdly, the *annual amortizement*, or the amount to be paid at the end of every year, besides what is still owing of the principal, increases progressively, and forms a geometrical progression, the ratio of which is equal to r, the produce of £ at the end of 1 year.

II. *For the amount of the sinking fund, at the end of every year*, that

4thly, the amount of the *sinking fund* obtained by the investment of the amortizing sum ar^{-n} increases progressively, and is always equal to the difference between the whole principal and the sum due at the end of every year.

III. *For the discounting of annuities*, that

5thly, the *present values of every annuity* form a decreasing geometrical proportion. This progression is the same as the progression of the amortizement, but inverted; so that the sum paid for the redemption of the principal at the end of any year whatever, is always equal to the present value of the annuity, which has as many years to run before the complete reimbursement of the principal.

6thly, the *present value of all the annuities* to be paid, up to a certain time, forms a series like the series of the principal (no. 1).

IV. *For the amount of the principal, and of the annuity*, that

7thly, the *principal* increases according to a geometrical progression (like 3), and the terms of this series remain proportional to the terms of the series 3.

8thly, The *amount of the annuity* increases according to the same proportions as the amount of the sinking fund (no. 5).

Table of Amortizement for a principal borrowed at the rate t, by means of n Annuities of £a.

I. Settlement of Accounts at the end of every year.					II. Sinking fund.	III. Discount on Annuities.		IV. Amount at the end of every year of the	
	Principal at the beginning of the year. 1.	Annuity £a for Interest. 2.	Amortizement. 3.	Principal left at the end of every year. 4.	Amount of the Amortizement at the end of every year. 5.	Present value of each particular Annuity. 6.	all Annuities paid at the end of every year. 7.	Principal. 8.	Annuities. 9.
1	$a.\phi(n)$	$at.\phi(n)$	$\frac{a}{r^n}$	$a.\phi(n-1)$	$\frac{a}{r^n}$	$\frac{a}{r}$	$\frac{a}{r}$	$ar.\phi(n)$	a
2	$a.\phi(n-1)$	$at.\phi(n-1)$	$\frac{a}{r^{n-1}}$	$a.\phi(n-2)$	$\frac{a}{r^{n-2}}.\phi(2)$	$\frac{a}{r^2}$	$a.\phi(2)$	$ar^2.\phi(n)$	$ar^2.\phi(2)$
3	$a.\phi(n-2)$	$at.\phi(n-2)$	$\frac{a}{r^{n-2}}$	$a.\phi(n-3)$	$\frac{a}{r^{n-3}}.\phi(3)$	$\frac{a}{r^3}$	$a.\phi(3)$	$ar^3.\phi(n)$	$ar^3.\phi(3)$
k	$a.\phi(n-k+1)$	$at.\phi(n-k+1)$	$\frac{a}{r^{n-k+1}}$	$a.\phi(n-k)$	$\frac{a}{r^{n-k}}.\phi(k)$	$\frac{a}{r^k}$	$a.\phi(k)$	$ar^k.\phi(n)$	$ar^k.\phi(k)$
$n-2$	$a.\phi(3)$	$at.\phi(3)$	$\frac{a}{r^{n-2}}$	$a.\phi(3)$	$\frac{a}{r^{n-2}}.\phi(n-2)$	$\frac{a}{r^{n-2}}$	$a.\phi(n-2)$	$ar^{n-2}.\phi(n)$	$ar^{n-2}.\phi(n-2)$
$n-1$	$a.\phi(2)$	$at.\phi(2)$	$\frac{a}{r^2}$	$a.\phi(2)$	$\frac{a}{r}.\phi(n-1)$	$\frac{a}{r^{n-1}}$	$a.\phi(n-1)$	$ar^{n-1}.\phi(n)$	$ar^{n-1}.\phi(n-1)$
n	$a.\phi(1)$	$at.\phi(1)$	$\frac{a}{r}$	$a.\phi(1)$	$a.\phi(n)$	$\frac{a}{r^n}$	$a.\phi(n)$	$ar^n.\phi(n)$	$ar^n.\phi(n)$

SECT. 25. *Annuities for which the rate of interest of the principal is different from the rate of interest of the sinking fund.*

We have hitherto supposed that the rate of extinction of the principal was the same as the rate of the loan, but the two rates often differ.

It may happen that the creditor, unwilling to admit successive and partial reimbursements, claims the integral reimbursement of the principal at a given time; the debtor, in order to pay off by means of annual payments, is bound to invest the overplus of his annuity elsewhere and on different terms.

Again, as partial reimbursements being profitable to the debtor may be prejudicial to the creditor, for the latter receives in small portions the principal which he has lent entire and may run the risk of keeping dead such principal which otherwise he could have invested with security, during the period the loan has been contracted for, he may agree on partial reimbursement, but claim at the same time a lesser rate for these payments, in order to counterbalance the waste he might experience.

In either case, the payment of the annual interest, and reimbursement of the capital, include two special accounts almost independent of each other: the payment of the interest, and the annual investment of a fixed sum which, at the end of the time settled for the reimbursement, must provide a sum equal to the borrowed principal. It is quite evident that the rates in both operations may be very different.

Let V = the principal lent for n years, or the present value of the annuity,

t' = the rate of interest of this principal,

t = the rate of extinction, or the rate at which the debtor may invest the surplus of his annuity,

a^n = the annuity which £1 will purchase for n years at this rate, and

a = the annuity paid by the debtor during n years for interest and amortizement.

The annuity a will include:

1st, the annual interest of the principal V at the rate $t' = Vt'$,

2ndly, the annual sum (s) necessary to provide this principal in n years, and at the rate of t per cent., which will be by form (9)... $s = \frac{Va^n}{r^n}$. Consequently,

$$a = Vt' + \frac{Va^n}{r^n},$$

and reciprocally, $$V = \frac{a}{t' + \frac{a^n}{r^n}} \quad \ldots\ldots (10).$$

Ex. 43. What annuity must be paid during 29 years, for a principal of £740, when the rate of investment of the principal is $4\frac{1}{2}$ per cent., and the rate of extinction $3\frac{1}{4}$ per cent.?

$$\begin{aligned} \log 740 &= 2,86923 \\ \log a^n &= 8,73051 \\ C^t \log r^n &= \underline{9,59719} \\ \log s &= 1,19693 \end{aligned}$$

$s = £15,737$ to be paid annually, in order to extinguish the principal.

Indeed, £15,737 invested at the end of every year at $3\frac{1}{4}$ per cent. give at the end of 29 years a sum of £740; besides, the interest of £740 at $4\frac{1}{2}$ per cent. is £33,300; therefore, the annuity to be paid is £49,037 per annum.

Ex. 44. A person wishes to hire for 10 years an estate, the clear income of which is £10000. The money could be borrowed at 5 per cent. in order to pay at once the 10 years' lease, or the savings could be invested at 3 per cent. How much is this person to pay for this 10 years' lease?

$$\begin{aligned} \log a^n &= 9,06904 \\ \log r^n &= 0,12837 \\ \log a^n r^{-n} &= 8,94067 \\ a^n r^{-n} &= 0,08723 \\ t' &= 0,05 \\ t' + a^n r^{-n} &= 0,13723 \\ \log &= 9,13745 \\ \log a &= 4 \\ \log V &= 4,86255 \\ V &= £72870, \text{ sum to be paid.} \end{aligned}$$

In order to value the annual sum, the amount of which at the end of 10 years and at 3 per cent. is £72870, we have

$$\begin{aligned} \log 72870 &= 4,86255 \\ \log a^n r^{-n} &= 8,94067 \\ \text{whence, } \log s &= 3,80322 \end{aligned}$$

and $s = £6356\ 5$, i. e. £6356 5 invested at the end of every year at 3 per cent., provide at the end of 10 years £72870; the principal £72.870 is therefore equal to 10 annuities of

£3643 5 for the interest at 5 per cent.
£6356 5 for the extinction of the capital at 3 per cent.
£10000

Ex. 45. What sum is to be paid immediately for the 38 years' lease of an estate, the clear income of which is £560, when the rate of interest of the principal is $4\frac{3}{4}$ per cent., and if the surplus may be invested at $3\frac{1}{4}$ per cent.?

$$\begin{aligned}
\log a^n &= 8,66468\ 37\\
\log r^n &= 0,52782\ 23\\
\log a^n r^{-n} &= 8,13686\ 14\\
a^n r^{-n} &= 0,01370\ 445\\
t' &= 0,0475\\
t'+a^n r^{-n} &= 0,06120\ 445\\
\log &= 8,78678\ 30\\
\log 560 &= 2,74819\ 80\\
\log V &= 3,96140\ 50
\end{aligned}$$

$V = £9149,660$, the required value.

But, in order to reimburse this sum at 3½ per cent., £125,391 are to be paid during 38 years.

Consequently,

£434 609 for the interest at 4½ per cent.,
125 391 for the extinction of the principal at 3½ per cent.,
£560 are to be paid per annum.

SECT. 26. *Calculation of the number of annuities.*

Whenever it is required to valuate the number of annuities let

$$V = \frac{a}{i}(1 - r^{-n})$$

be turned into $r^{-n} = \frac{a - Vi}{a}$,

or $r^n = \frac{a}{a - Vi}$,

consequently, by making use of logarithms,

$$n = \frac{\log \frac{a}{a - Vi}}{\log r};$$

but a is equal to the annual income, Vi denotes the annual interest of the principal, and $(a - Vi)$ is equal to the difference between the annuity and annual interest, or the annual sinking fund, therefore

$$n = \frac{\log \frac{a}{s}}{\log r}.$$

Ex. 46. In how many years will a principal of £100,000 be amortized by an annuity of £6505 144, the interest being 5 per cent. per annum?

Principal V = £100,000
Annual interest, at $5\frac{0}{0}$, Vt = 5,000
Annuity, a = 6,505 144
Amortizement, s = 1505 144
$\log a$ = 3,81325 70
$\log s$ = 3,17757 80
$\log \frac{a}{s}$ = 0,63567 90

therefore, $n = \frac{0,63567\ 90}{0,02118\ 93} = 30$ years (Tab. 31).

Ex. 47. For how many years may an annuity of £180 per annum be purchased, the principal being £2846,481, and the interest $4\frac{1}{2}$ per cent.?

Principal V = 2846 481
Annual interest at $4\frac{1}{2}$ per cent. Vt = 128 09164
Annuity a = 180
Annual sinking fund s = 51,90836
$\log a$ = 2,25527 25
$\log s$ = 1,71523 73
$\log \frac{a}{s}$ = 0,54003 52

therefore $n = \frac{0,54003\,52}{0,191163} = 28\frac{1}{4}$,

or by means of Tab. 27, $\log r^{28}$ = 0,53525 61
477 91
Tab. 50, $\log r^{\frac{1}{4}}$ = 477 91, or 28 years, 3 months.

SECT. 27. *Interpretation and payment of annuities when their number is fractional.*

If in the formula $V = \frac{a}{i}\left(1 - \frac{1}{r^n}\right)$ the number of years is a whole one, followed by any fraction whatever, and if we suppose this number to be $n = m + \frac{1}{k}$, m being a whole number and $\frac{1}{k}$ a fraction of the year, we have:

$$V = \frac{a}{i}\left(1 - \frac{1}{r^{m+\frac{1}{k}}}\right) = \frac{a}{i}\left(1 - \frac{1}{r^m} + \frac{1}{r^m} - \frac{1}{r^{m+\frac{1}{k}}}\right),$$

$$\text{or } V = \frac{a}{i}\left(1 - \frac{1}{r^m}\right) + \frac{a}{ir^{m+\frac{1}{k}}}\left(r^{\frac{1}{k}} - 1\right) \ldots\ldots\ldots\ldots (11).$$

However, $\left(r^{\frac{1}{k}} - 1\right)$ is equal to the compound interest of £1 for

$\frac{1}{k}$th year, and $\left(\frac{a}{i}\right)$ denotes the principal which gives £a interest, or the present value of the perpetuity of £a per annum.

Consequently, $\frac{a}{i}(r^{\frac{1}{k}}-1)$ is equal to the compound interest for $\frac{1}{k}$th year, which corresponds to £a per annum.

Therefore: the value V of the annuity £a for a fractional number of years includes first m annuities of £a with immediate entrance, and secondly, at the expiration of the whole time settled upon for the extinction of the annuity, a sum equal to the interest for $\frac{1}{k}$th year.

Thus, in the last Example 47, we have found that a £2846 481 principal, at 4½ per cent., would be amortised, by means of £180 annuities, in 28¼ years; but the principal which at 4½ per cent. gives £180 interest per annum is £4000, and the interest of this principal for 3 months is (Ex. 14) = £44 260.

Therefore, in exchange of a £2846 481 principal, 28 annuities of £180 will be paid, then 3 months afterwards, £44 260.

In order to prove the truth of this calculation:

Present value of 28 annuities of £180 =	£2833 717
Present value of £44 260, payable at the expiration of 28 years and 3 months =	12 764
Adding these two sums =	2846 481 as above.

Or again,

Amount of the principal at the end of 28 years	9762 576
Amount of 28 annuities at the end of same time	9718 800
Profit for the creditor	£ 43 776
which becomes after 3 months	£ 44 260.

We have found (Ex. 20) that the *discount* of £4,000 payable in 3 months is £43 776, and that the interest of this same principal for 3 months is £44 260.

Besides, the formula (11) shows that generally the sum still due after the payment of the last annuity is equal to the *discount* of the annuity £a for the fraction of year left.

Ex. 48. How many annuities of £945 will be necessary to reimburse a principal = £11,249 300 at the rate of 5¼ per cent.?

Principal V =	£11249 300
Annual interest at 5¼ per cent., Vt =	590 588
Annuity a =	945
Annual sinking fund s =	354 412

$\log a = 2,97543$
$\log s = 2,54951$

$\log \frac{a}{s} = 9,42592$

(Tab. 33) $\log r^{19} = 0,42222$

370

(Tab. 51) $\log r^{\frac{1}{6}} = 370$

The time required is therefore 19 years and 2 months.

The simple interest for 2 months, which corresponds with £945 per annum, would be £157½.

$\log 157\ 5 = 2,19728$
$\log P_6 = 930\ 5$
log of the interest $= 2,18797\ 5$
and the interest = £154 161

In order to show the truth of this calculation:

Present value of 19 annuities of £945 = £11191 484
Present value of £154 161 payable at the end of 19 years and 2 months = 57 816
Together £11249 300.

Ex. 49. How many years will it take to reimburse a £5000 principal by paying £300 per annum, the rate of interest being at 3¾ per cent.?

Principal V = £5000
Interest Vi = 175
Annuity a = 300
Amortizement s = 125

$\log \frac{a}{s} = 0,38021$

$\log^2 \frac{a}{s} = 9,58002$

(Tab. 57) $\log^2 r = 8,17436$
$\log n = 1,40506$
n = 25 448 years, (25 years, 163¾ days).

£300 interest per annum correspond with £133,307 compound interest for 163¾ days, therefore £300 per annum will be paid during 25 years; then at the end of 164 days £133,307.

Verification:

Present value of 25 annuities of £300 £4944 454
Present value of £133,307 payable at the end of 25, 448 years 55 546
£5000

SECT. 28. *Application of Table I. to the valuation of years.*

If the number of years is a whole number, or if it is to be valued with an approximation of a fraction of year, it may be found at once in Tables 1 to 48 by a simple subtraction: you find the logarithm of $\frac{a}{V}$ and then look for it in the Table which corresponds to the given rate. Thus, we have (Ex. 46)

$$\begin{aligned} \log a &= 3,81325\ 70 \\ \log V &= 5 \\ \log a^n &= 8,81325\ 70. \end{aligned}$$

For the rate 5 per cent. this logarithm may be found exactly in Tab. 31 as corresponding with 30 years; the annuity is precisely of 30 years as above. (Ex. 47)

$$\begin{aligned} \log 180 &= 2,25527 \\ \log V &= 3,45431 \\ \log a^n &= 8,80096. \end{aligned}$$

For the rate $4\frac{1}{2}$ per cent. this logarithm is found between 28 and 29 years; the annuity is therefore to run for more than 28 years and for less than 29. In order to valuate this fraction of year, we have

for 28 years	8,8029
for 29 ...	8,7953
difference for 1 year	76.

But the difference between the logarithm which has been calculated, and the logarithm which corresponds with 28 years is 19 only; therefore the annuity will be found for $28\frac{19}{76}$ years, as above.

SECT. 29. *On the determination of the rate of interest in annuities.*

Of all questions which occur in the theory of annuities, none is more complicated and more laborious than the calculation of the rate of annuities. In the simplest case, that is, in the calculation of immediate annuities, the valuation of the rate requires the resolution of an algebraic equation, the index of which is higher by one unit than the number of years which results from equation (8) when it is developed, according to the decreasing powers of r:

$$r^{n+1} - \left(1 + \frac{a}{V}\right) r^n + \frac{a}{V} = 0.$$

The valuation of the rate is very important, because it is the rate which acts as a standard for the investment of money; but generally it may be sufficient to estimate the rate of interest with an approximation of $\frac{1}{20}$th per cent.; and this is only in some special cases, which rarely occur, for instance, when parities are to be calculated a nearer approximation is required. Besides, this calculation may be considerably abbreviated, because generally the rate

is included between 1 and 12 per cent. per annum; it is therefore easy to find a near value for the root which by means of consecutive corrections can be calculated with full accuracy.

In most practical cases, our logarithmic tables show directly by means of interpolations a greater accuracy than is required; for those cases which exceed the limits of these tables special formulæ have been given, which give very approximate results. Lastly, a formula for correcting the results will assist in calculating with the greatest precision the rate of interest.

Rate of perpetuities. When n is infinite, i.e. when the annuity is perpetual, form .8 becomes

$$V=\frac{a}{t},$$

$$\text{whence, } t=\frac{a}{V}.$$

And consequently, if it is required to find the rate of a perpetuity with *immediate entrance*, let the annual income be divided by the value of the annuity.

Ex. 50. The 3 per cent. stock being at $98\frac{3}{8}$, what is the rate of interest?

$$t=\frac{3}{98\frac{3}{8}}=0,03049555\ldots \text{or nearly } £3.\ 1s.\ \text{per cent.}$$

SECT. 30. *Calculation of the rate by means of logarithmic tables.*

When the value of $\log\frac{a}{V}$ is found for the given number of years in the tables, the rate is obtained directly; if on the contrary it is not to be found exactly, it is calculated by interpolation.

Ex. 51. An annuity of £12,873 for 16 years is sold for £150,000, what is the rate of interest allowed?

$$\log a = 4,10967\ 98$$
$$\log V = 5,17609\ 13$$
$$\log a^n = 8,93358\ 85.$$

By looking for this logarithm in the tables on the column 16 years, it is found Tab. 23, which shews 4 per cent.; the rate is therefore 4 per cent. exactly.

Ex. 52. Let us determine at what rate of interest an annuity of £176, to continue 31 years, may be purchased for £2680.

$$\log\ 176 = 2,24551$$
$$\log\ 2680 = 3,42813$$
$$\log a^n\ = 8,81738.$$

This logarithm is not found in the column of 31 years, but is therein included between $5\frac{1}{2}$ and $5\frac{1}{4}$ per cent.

Indeed, we find in the tables for 31 years,

at $5\frac{1}{8}$ per cent.	81338
at $5\frac{1}{4}$	81963
Difference for $\frac{1}{8}$	625.

The difference between the logarithm calculated and the logarithm which corresponds with $5\frac{1}{8}$ per cent. is $=400$, therefore the rate is $=5\frac{1}{8}+\frac{400}{625}\cdot\frac{1}{8}=5.205$ per cent.

Ex. 53. Required the rate of interest of an annuity of £27. 10s. for 22 years which £382,038 will purchase for 22 years.

$$\begin{aligned}\log 27{,}5 &= 1{,}43933\\ \log V &= 2{,}58211\\ \log a &= 8{,}85722.\end{aligned}$$

This logarithm for 22 years is included between $4\frac{3}{8}$ and $4\frac{1}{2}$ per cent. In the tables we find for 22 years,

at $4\frac{3}{8}$ per cent.,	85553
$4\frac{1}{2}$	86061
Difference for $\frac{1}{8}$	508.

Difference between the logarithm calculated and the logarithm of $4\frac{3}{8}$ per cent. is $=169$, consequently the rate is $=4\frac{3}{8}+\frac{169}{508}\cdot\frac{1}{8}=4\frac{5}{12}$.

Ex. 54. £7600 are paid for a £380 annuity, to continue 49 years, what is the rate of interest?

$$\begin{aligned}\log 380 &= 2{,}57978\\ \log V &= 3{,}88081\\ \log a &= 8{,}69897.\end{aligned}$$

This rate is included between $4\frac{3}{8}$ and $4\frac{1}{2}$ per cent., for we find in the table for 49 years,

at $4\frac{3}{8}$ per cent.	69782
$4\frac{1}{2}$	70661
Difference for $\frac{1}{8}$	879.

But the difference between the logarithm calculated and the logarithm of $4\frac{3}{8}$ per cent. is $=115$,

$$\text{therefore the rate is} = 4\tfrac{3}{8}+\frac{115}{879}\cdot\frac{1}{8},$$

$$= 4{,}3912 \text{ per cent.}$$

Sect. 31. It would be useless to recur to the interpolation of the second degree by means of formula, page 23; for the table showing the logarithms of the annuities for every $\frac{1}{8}$th per cent. the rate obtained through simple interpolation is always either perfectly true, or *too great* by an excessively small quantity.

This error is so much the smaller, as the number of annuities is small itself, and it increases imperceptibly when the rate decreases. In all foregoing examples the difference between the true rate and the rate calculated shows an error less than one millionth. We find, moreover, (Sect. 34,) a formula by means of which the true rate may be valued with full accuracy.

SECT. 32. Whenever the number of years or the rate is sufficiently *great*, a most simple formula can be obtained, which gives the value of the rate with great correctness.

Thus, when $(1+i)^n$ is infinite, the equation (8)

$$i=\frac{a}{V}-\frac{a}{V}(1+i)^{-n}$$

becomes

$$i=\frac{a}{V};$$

and generally, the rate is always smaller than the relation between the annual income and the present value of the annuity, and the difference between them is so much smaller, as $(1+i)^n$ is greater. By substituting in the above equation $(1+i)$ by $\left(1+\frac{a}{V}\right)$, the result

$$i=\frac{a}{V}-\frac{a}{V}\left(1+\frac{a}{V}\right)^{-n}$$

will be very approximate,

if $(1+i)^n$ is very great.

EX. 55. The present value of £317 for 120 years being £6648 223, the rate is required.

$$\log a = 2,50106$$
$$\log V = 3,82271$$
$$\log \frac{a}{V} = 8,67835$$
$$\frac{a}{V} = 0,04768$$
$$\log \left(1+\frac{a}{V}\right) = 0,02023$$
$$\log \left(1+\frac{a}{V}\right)^n = 2,4276$$
$$\log . \frac{a}{V}\left(1+\frac{a}{V}\right)^{-n} = 6,251$$
$$\frac{a}{V}\left(1+\frac{a}{V}\right)^{-n} = 0,00018;$$

therefore, $i=0,04750$, or $4\frac{3}{4}$ per cent. (true value).

Ex. 56. An annuity of £100 for 78 years is worth £1958. 10s, at what rate?

$\log a = 2$ $\qquad$ $\log\left(1+\frac{a}{V}\right) = 0,02166$

$\log V = 3,29126$ $\qquad$ $\log\left(1+\frac{a}{V}\right)^{n} = 1,6895$

$\log \frac{a}{V} = 8,70874$ $\qquad$ $\log . \frac{a}{V}\left(1+\frac{a}{V}\right)^{-n} = 7,0192$

$\frac{a}{V} = 0,05114$ $\qquad$ $\frac{a}{V}\left(1+\frac{a}{V}\right)^{-n} = 0,00105$;

therefore $t = 0,05009$, or nearly 5 per cent. (the true value is 5 per cent.)

Ex. 57*. A £1500 bond bearing 3 per cent. interest, and to be reimbursed through 50 annuities, is worth £565; what is the rate of interest, when the annual rent and rate of interest are payable only once a year?

In order to solve this question, let us first valuate the annual income, which amortizes £1500 in 50 years, at 3 per cent.

We have $\log 1500 = 3,17609$
(Tab. 15) $\log a^{n} = 8,58956$
$\log a = 1,76565$
$a = £58,298.$

The problem is thereby reduced to the following: At what rate are 50 annuities of £58 298 worth £565?

$\log a = 1,76565$
$\log 565 = 2,75205$

$\log \frac{a}{V} = 9,01360$

$\frac{a}{V} = 0,10318$

$\log \left(1+\frac{a}{V}\right) = 0,04265$

$\log \left(1+\frac{a}{V}\right)^{n} = 2,1325$

$\log . \frac{a}{V}\left(1+\frac{a}{V}\right)^{-n} = 6,8811$

$\frac{a}{V}\left(1+\frac{a}{V}\right)^{-n} = 0,00076$

$t = 0,10242$ (true rate = 10,24 per cent.)

* See Explanatory Note, page xiv.

SECT. 33. *Baily's formula.*

When the number of years is small, Baily's formula shows the rate with great accuracy. From equation (8)

$$V=\frac{a}{t}[1-(1+t)^{-n}]$$

we deduct $1-\frac{Vt}{a}=(1+t)^{-n}$;

for if we expand the quantity $(1+t)^{-n}$ by means of the binomial theorem into a series ascending by the powers of t we shall have

$$1-\frac{Vt}{a}=1-nt+\frac{n(n+1)}{2}t^2-\frac{n(n+1)(n+2)}{2.3}t^3,$$

or, after subtracting the unity and dividing by $(-nt)$

$$\frac{V}{an}=1-\frac{n+1}{2}.t+\frac{(n+1)}{2}\frac{(n+2)}{3}t^2.....(12).$$

Since t is generally but a small fraction of unity we reject all the terms affected with the second, third, and still higher powers of t as very small with regard to the first. We raise both numbers of this last equation to such a power that the coefficient of t may become positive and equal to unity, which ever occurs when the exponent is equal to the reciprocal of the coefficient of t, but of a contrary sign.

Generally, we get by the binomial theorem

$$P=1+at+bt^2,$$

$$P^m=(1+at+bt^2)^m=(1+at)^m+m(1+at)^{m-1}.bt^2,$$

or leaving but the first two powers of t,

$$P^m=1+mat+\frac{m(m-1)}{2}a^2t^2+mbt^2;$$

therefore, if $a=-\frac{n+1}{2}$,

$$b=\frac{(n+1)(n+2}{2.3},$$

and $$m=-\frac{2}{n+1},$$

the equation (12) will become

$$\left(\frac{V}{an}\right)^{-\frac{2}{n+1}}=\left(\frac{an}{V}\right)^{\frac{2}{n+1}}=1+t+\frac{1-n}{12}t^2.$$

Consequently, if we suppose

$$q=\left(\frac{ax}{y}\right)^{\frac{2}{n+2}}-1,$$

we shall obtain

$$q=t-\frac{n-1}{12}t^2.$$

From this equation of the second degree, we get *

$$t=q.\ \frac{1-\frac{n-1}{12}q}{1-\frac{n-1}{12}.2q}.$$

The rate which is found by Baily's formula is always *too great*; but from 1 to 50 years, and for common rates, the error is excessively small.

In order to form an opinion of the accuracy which this remarkable formula affords, we shall subjoin the result which it gives when the true rate is exactly $1\frac{3}{4}$ and 5 per cent.

Annuity			Rate per cent.	
for 3 years	. . .	1 75	. .	5 00002
5 ,,	. . .	1 75001	. .	5 00006
11 ,,	. . .	1 75001	. .	5 00043
21 ,,	. . .	1 75003	. .	5 00195
31 ,,	. . .	1 75007	. .	5 00521
41 ,,	. . .	1 75016	. .	5 01093
51 ,,	. . .	1 75027	. .	5 01991
99 ,,	. . .	1 75175	. .	5 13604

---

* When in the equation of the second degree

$$x+ax^2=q,$$

x is a simple fraction, and a a number sufficiently small, the value of x will show a very small difference with that of q.

Rigorously, however, we have $x=\frac{q}{1+ax}$.

If x be replaced by q, this equation becomes

$$x=q\cdot\frac{1+aq}{1+2aq},$$

and then by substituting this latter value for x,

$$x=q\cdot\frac{1+aq}{1+2aq}.$$

Ex. 58. If an annuity of £290 to continue 11 years is purchased with £2634,391, what rate of interest affords the purchase money?

$$\begin{aligned} \log a &= 2,46240 \\ \log n &= 1,04139 \\ C^t \log V &= 6,57932 \\ \log \frac{an}{V} &= 0,08311 \\ \frac{1}{t} \log \frac{an}{V} &= 0,01385 \end{aligned}$$

$$\begin{aligned} q &= 0,03241 \\ (n-1)\,q &= 0,3241 \\ t = q \cdot \frac{11,6759}{11,3518} & \\ \log q &= 51068 \\ \log 11,676 &= 06729 \\ C^t \log 11,352 &= 94493 \\ \log t &= 52290 \\ t &= 3,333, \text{ or } 3\tfrac{1}{3} \text{ per cent.} \end{aligned}$$

SECT. 34. *Corrective formula.*

We will now give a formula by means of which the rate may be always valuated with the greatest accuracy.

From (8) we have $\frac{Vt}{a} = 1-(1+t)^{-n}$

If actually, t denotes the true rate,

τ an approximate value of t,

and x the difference between the true and approximate rates, so that $t = \tau + x$,

and $\tau = \rho + x$,

we have, $$\frac{V}{a}(\tau+x) = 1-\rho^{-n}+nx\,.\,\rho^{-(n+1)},$$

therefore $$x\left\{\frac{V}{a}-n\rho^{-(n+1)}\right\} = 1-\frac{V\tau}{a}+\rho^{-n},$$

$$x = \frac{1-\frac{V\tau}{a}-\rho^{-n}}{\frac{V}{a}-n\rho^{-(n+1)}}.$$

Ex. 59. Required the rate of interest of an annuity of £176 for 31 years, at £2680. (See Ex. 52.)

We have already found an approximate value of $t = 5,205$ per cent.

therefore $a = 176$
$V = 2680$
$n = 31$
and $\rho = 1,05205$
$\log \rho = 0,02203\ 638070$
$\log \rho^{31} = 0,68312\ 780170$
$\log \rho^{-31} = 9,31687\ 219830$
$\rho^{-31} = 0,20743\ 030136$
$\frac{V\tau}{a} = 0,79257\ 954545$

$\frac{V\tau}{a} + \rho^{-31} = 1,00000\ 984681$
$\log n = 1,49136\ 17$
$\log \rho^{32} = 0,70516\ 42$
$\log n\rho^{-32} = 0,78619\ 75$
$n\rho^{-32} = 6,11219\ 9$
$\frac{V}{a} = 15,22727\ 3$

$\frac{V}{a} - n\rho^{-32} = 9,11507\ 4$

$$\text{or } x = -\frac{0,00000\ 984681}{9,115074} = -0,00000\ 108028$$

and $i = 0,05205 + x = 0,05204\ 891972$,
the required rate is therefore exactly
5,20489 1972 per cent.

Ex. 60. The present value of 49 annuities of £380 is £; what is the rate? (See Ex. 54.)

Let us take an approximate value of i,
$\tau = 4,3912$ per cent.
then, $\rho = 1,043912$
$\log \rho = 0,01866\ 38899\ 22$
$\log \rho^{49} = 0,91453\ 06061\ 8$
$\log \rho^{-49} = 9,08546\ 93938\ 2$
$\rho^{-49} = 0,12175\ 01188\ 4$
$\frac{V\tau}{a} = 0,87824$

$\frac{V\tau}{a} + \rho^{-49} = 0,99999\ 011884$
$\log n = 1,69019\ 61$
$\log \rho^{50} = 0,93319\ 45$
$\log n\rho^{-50} = 0,75700\ 16$
$n\rho^{-50} = 5,71480\ 8$
$\frac{V}{a} = 20$

$\frac{V}{a} - n\rho^{-50} = 14,285192$,

$$\text{or } x = \frac{0{,}00000\ 988116}{14{,}2851\ 92} = 0{,}00000\ 0691706.$$

Consequently, $t = 0{,}04391\ 26917\ 06$ the *true value* of the rate.

SECT. 35*. *On Periodical Rents in general.*

Present value of Annuities, payable yearly, half-yearly, quarterly, &c., when the interest is convertible yearly, half-yearly, quarterly, &c.

We have hitherto considered but those cases in which the annuity and interest are both paid at the expiration of the year; we will now consider the periodical rents in general, i.e. the case in which the annuity and interest are payable at different unconnected times, and at any other interval of time than a whole year. It is well known that generally public rents and others are paid half-yearly, quarterly, or monthly.

The calculations for rents payable on fractions of the year are longer and more minute than those relating to annuities paid once a year; besides, the numerical tables of annuities which have been published require such long calculations, that the rather serious difficulties arising from their use have induced people to limit their calculations to whole years, which give a result more or less inaccurate; whereas with our logarithmic tables these calculations are very simple for the most practical cases. It is chiefly with regard to such questions that the advantages of logarithmic over numerical tables may be duly appreciated.

SECT. 36*. *General Formula.*

Let us first examine the most general case, and determine the present value of an annuity of £a, payable k times in the year, each payment being the $\frac{1}{k}$th of the yearly annuity, when the interest is convertible into principal at ν equal intervals.

Let then t = the *nominal rate* of the interest, ν times convertible per annum;

a = the *annual rent* for n years, payable by equal portions at k equal intervals;

V = the present value of this annuity,

and ρ = £1 increased by its interest at the end of $\frac{1}{k}$th year, i.e. ring the interval between two consecutive payments of the rent.

We shall have t = *nominal rate*,

$\frac{t}{\nu}$ = *true rate* of interest, convertible per $\frac{1}{\nu}$th year,

$1 + \frac{t}{\nu}$ = amount of £1 at the end of $\frac{1}{\nu}$th year,

$\left(1 + \frac{t}{\nu}\right)^{\nu}$ = amount of £1 at the end of a whole year,

* See Explanatory Note, page xiv.

$\rho = \left(1 + \frac{\ell}{\nu}\right)^{\frac{\nu}{k}}$ = amount of £1 at the end of $\frac{1}{k}$th year, or of its period between two successive payments of the annuity.

Consequently the present value of the annuity, or of the n payments of £$\frac{a}{k}$, will be equal to

$$V = \frac{a}{k}\left(\frac{1}{\rho} + \frac{1}{\rho^2} + \frac{1}{\rho^3} + \ldots \frac{1}{\rho^{kn}}\right)$$
$$= \frac{a}{k} \cdot \frac{1 - \rho^{-kn}}{\rho - 1},$$

and substituting for ρ its value, $\left(1 + \frac{\ell}{\nu}\right)^{\frac{\nu}{k}}$, we have

$$V = \frac{a}{k} \cdot \frac{1 - \left(1 + \frac{\ell}{\nu}\right)^{-\nu n}}{\left(1 + \frac{\ell}{\nu}\right)^{\frac{\nu}{k}} - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13).$$

SECT. 37*. Before we deal with this formula, under a general point of view, let us first examine the particular case in which the annuity is paid *half-yearly*, whereas the interest is convertible *yearly*.

In this case, we have $\left\{\begin{matrix} k = 2 \\ \nu = 1 \end{matrix}\right\}$ and the formula (13) becomes

$$V' = \frac{a}{2} \cdot \frac{1 - r^{-n}}{\sqrt{r} - 1}.$$

But we have already found that the present value of £a per annum for n years is $V = \frac{a}{\ell} \cdot (1 - r^{-n})$.

Therefore, the relation between two annuities, the first of which is payable half-yearly, and the second at the end of every year, the number of years, and the annual interest being still the same, and denoted by p_2, we have

$$p_2 = \frac{V'}{V} \frac{\frac{\ell}{2}}{\sqrt{r} - 1};$$

but $(\sqrt{r} - 1)$ is the compound interest of £1 for half a year, and $\frac{\ell}{2}$ is the simple interest of £1 for the same time. We have consequently the remarkable general law which follows:

* See Explanatory Note, page xiv.

The relation between two annuities, one of which is payable half-yearly, and the other yearly, is invariable, whatever may be the number of years, and equal to the relation between the simple interest for a half year and the compound interest for the same period of time*.

Thus, for instance, when the annual interest is 5 per cent., we have for one half year,

Simple interest: $\frac{t}{2} = 0{,}025$.

Compound interest: $\sqrt{r} - 1 = 0{,}02469\ 50765\ 96$,

consequently,

$$\frac{\frac{t}{2}}{\sqrt{r} - 1} = 1{,}01234\ 75383,$$

$$\log = 0{,}005\ 3296.$$

It is by this number that the present value of an annuity is to be multiplied in order to obtain the value of the same annuity, but payable by halves, and by half years.

Thus the present value of an annuity of £1 per annum, payable yearly for 25 years at 5 per cent., is according to the tables equal to 14,09394 4566, whence the present value of the same annuity payable half-yearly, and at the same rate of interest, will be

$$= 14{,}09394\ 4566 \times 1{,}01234\ 75381 = 14{,}26797\ 009.$$

Ex. 61. What is the present value of 29 annuities of £144, payable by halves, and half-yearly, the interest being 5 per cent. per annum?

$$\begin{array}{rl} \log 444 = & 2{,}64738\ 30 \\ \log \phi\ = & 1{,}18015\ 67 \\ \text{(Tab. 63)}\quad \log P_2 = & \underline{\quad 532\ 96} \\ \log V\ = & 3{,}83286\ 93 \end{array}$$

$V = £6805{,}645$, the required value.

Ex. 62. What is the annuity, payable half-yearly, in order to amortize a principal of £6500 at 5 per cent. in 41 years?

$$\begin{array}{rl} \log 6500 = & 3{,}81291\ 34 \\ \log a^{41}\ = & 8{,}76209\ 53 \\ \log P_2\ = & \underline{-\ 532\ 96} \\ \log a\ = & 2{,}56967\ 91 \end{array}$$

$a = £371{,}2608$.

Ex. 63. It is required to convert into a half-yearly rent an annual rent of £560, payable during any number whatever of years; how much is to be paid, the interest being 5 per cent.?

---

* Table 63. P_2 shows the logarithm of this relation for 51 different rates.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] Annuity and interest payable at the same time.

When the annuity and interest are payable at the same time, times per annum, the present worth becomes

$$V = \frac{a}{i}\left(1 - \frac{1}{(1+i)^{n}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14.)$$

[illegible] as 1 and 2 are each equal to 2, 4, 12, ...; that is, as the interest and annuity are each payable yearly, half-yearly, quarterly or monthly, we have

$$V = \frac{a}{i}\left(1 - \frac{1}{(1+i)^{n}}\right),$$

$$V_2 = \frac{a}{i}\left(1 - \frac{1}{\left(1+\frac{i}{2}\right)^{2n}}\right),$$

$$V_4 = \frac{a}{i}\left(1 - \frac{1}{\left(1+\frac{i}{4}\right)^{4n}}\right),$$

$$V_{12} = \frac{a}{i}\left(1 - \frac{1}{\left(1+\frac{i}{12}\right)^{12n}}\right).$$

From Sect. 9, the present value of pn annuities of $\pounds\frac{a}{p}$ and at the annual rate of $\frac{i}{p}$ is

$$= \frac{a}{i}\left(1 - \frac{1}{\left(1+\frac{i}{p}\right)^{pn}}\right).$$

Consequently, whenever the annuity is payable at the same time, like the interest, the present value of the annuity of $\pounds a$ per annum, payable p times in the year, each payment being $\frac{1}{p}$th of the yearly annuity, is generally equal to the present value of pn annuities of $\pounds\left(\frac{a}{p}\right)$, the rate of interest being $\frac{1}{p}$th of the first.

This property is in most cases useful to find directly, in the tables computed for the present value of annuities payable yearly, the value of a half-yearly or quarterly rent.

*Ex. 64**. What is the present value of an annuity of £400 for 30 years, the rate of interest being 6 per cent., when the annuity and the interest are payable monthly?

---

* See Explanatory Note, page xiv.

The general formula becomes $V=\frac{a}{t}\left\{1-\left(1+\frac{t}{12}\right)^{-360}\right\}$,

and as $\log\left(1+\frac{t}{12}\right)=\log 1{,}005 \quad =0\ 00216\ 60618$

$\log 1{,}005^{360} =0\ 77978\ 22$
$\log 1{,}005^{-360}=9\ 22021\ 78$
$1{,}005^{-360}=0\ 16604\ 20$
$1-1{,}005^{-360}=0\ 83395\ 80.$

Then,

	$\log$	$=9\ 92114\ 42$
	$\log 400$	$=2\ 60206\ 00$
C^t	$\log t$	$=1\ 22184\ 87$
	$\log V$	$=3\ 74505\ 29$

Required value $V=$ £5559,719.

In the same manner we should obtain, for instance, the present value of 25 annuities of £1, payable generally at 4 per cent. per annum = £15,62208.

When the annuity and interest are payable *half-yearly*, the present value of the annuity is equal to the present value of the annuity £½, for 50 years, at the rate of 2 per cent. per annum = £15,71180.

When the annuity and interest are payable *quarterly*, the present value is equal to the present value of the annuity £¼, for 100 years, at the rate of £1 per cent. per annum = £15,75722.

At last, when the annuity and interest are payable *monthly*, the present value of the annuity is equal to the annuity of £$\frac{1}{12}$ for 300 years, at the rate of $\frac{1}{3}$ per cent. per annum = £15,78771.

It may be seen by the simple perusal of the formulæ contained in this Section, that the present value of an annuity increases with the number of annual payments; thus, we have found

$$V=15{,}62208$$
$$V_2=15{,}71180$$
$$V_4=15{,}75722$$
$$V_{12}=15{,}78771.$$

The present value of the annuity would likewise be greater, if the payments were to take place daily, &c.; and these increasing values come nearer and nearer to an *extreme limit*, which is the present value of an annuity, when payments take place at infinitely small periods of time. This limit we will now determine.

If ν is infinite, the expression $\left(1+\frac{t}{\nu}\right)^{\nu}$ becomes equal to ϵ^t, ϵ denoting the basis of *hyperbolic logarithms*, and formula 14 becomes

$$V=\frac{a}{t}\left(1-\epsilon^{-nt}\right).$$

Thus, if we were to ask the present value of £1 for 25 years, and at 4 per cent., when annuity and interest are both payable *instantaneously*, that is, at periods of time infinitely small, and by equal portions, proportional to the time, we should find

$$nt = 1,$$

$$\frac{1}{e} = 0{,}36787\ 94411\ 71,$$

$$1 - \frac{1}{e} = 0{,}63212\ 05588\ 29,$$

and consequently, $V = \frac{1}{0{,}04}\left(1 - \frac{1}{e}\right) = £15{,}80301\ 39707$, which is the extreme limit of the value of the annuity. (See the Table, page 60.)

If the annuity is *perpetual*, all the formulæ of the present value of the annuity become equal to each other, and to

$$V = \frac{a}{t}.$$

Therefore the present value of a perpetuity, whether payable *yearly*, *half-yearly*, *quarterly*, or at any interval of time, remains invariable, provided the annuity and interest are paid at the same time.

As $\left(1 + \frac{t}{2}\right)^2$ or $\left(1 + t + \frac{t^2}{4}\right)$ is always greater than $(1 + t)$, the present value of an annuity, when interest and annuity are payable *half-yearly*, is greater than the value of the same annuity when interest and annuity are payable *yearly*.

The *difference*, however, between two corresponding values for the same rate varies with the number of years; first, it *increases* from nothing (when $n = 0$) to a certain value of n for which the difference is a *Maximum*; then it *decreases* as n increases until at last, when n is infinite, it vanishes entirely.

Thus, if we compare the present value of £1 per annum, at 4 per cent., and if we set down the annuity and interest when both are payable yearly and half-yearly, we find:

Years	Annuity and Interest payable*		
	yearly	half-yearly	difference
1	£ 0,962	£ 0,971	9
2	1,886	1,904	18
3	4,452	4,491	39
10	8,111	8,176	65
20	13,590	13,678	88
25	15,622	15,712	90
30	17,292	17,380	88
51	21,617	21,683	66
125	24,814	24,823	9
250	24,999	24,999	0

* In order to find out in a general way the number of years for which $(V_2 - V)$, i.e. the difference between two corresponding values, is a *Maximum*, let its differential be equal to nothing when n is the variable: the general expression for the maximum or greatest difference becomes then:

$$n = \frac{\log^2 . \left(1 + \frac{t}{2}\right)^2 - \log^2 r}{\log . \left(1 + \frac{t}{2}\right)^2 - \log r} .$$

This being expanded according to a series ascending with the powers of t, is converted into the following very simple and easily applicable expression:

$$n = \frac{1}{t} + \frac{3}{8} - \frac{t}{16} + \ldots$$

The third term being less than 2 days, for all practical rates may be left aside, as the first two terms are quite sufficient to secure the greatest accuracy.

For example, the difference between the present values of £1 per annum, at 4 per cent., payable half-yearly or yearly, is greatest when

$$n = 25\tfrac{3}{8} - \frac{0{,}04}{16} = 25{,}372 \text{ years, rigorously true.}$$

It is often required in practice to know the *relation* $\frac{V_2}{V}$, that is, the relation between the present values of an annuity according to the payment of the interest and annual rent taking place half-yearly or yearly.

* See Explanatory Note, page xiv.

It is quite evident that this relation

$$\frac{V_2}{V} = \frac{\phi\left(\frac{t}{2}\right)}{\phi(t)} = \frac{1-\left(1+\frac{t}{2}\right)^{-2n}}{1-(1+t)^{-n}}$$

varies with the rate of interest, or with the number of years.

Whenever the *number of years* is given and constant, this relation is first equal to unity when $t=0$; then it keeps on increasing to a certain value of t to be determined by the equation

$$(1+t)^{n+1} = \frac{t}{2} + \left(1+\frac{t}{2}\right)^{2n+1},$$

for which the relation is a *Maximum*; then from this value of t it keeps on decreasing and approaching indefinitely towards unity, its extreme limit.

* For instance, the relation between £1 per annum for 25 years the interest and annuity being payable yearly or half-yearly, is

at 2 per cent. = 1,0038,
6 ..... = 1,0064,
10 ..... = 1,0056.

When, on the contrary, the *rate of interest* is given and constant, the relation continually diminishes, according as the number of years increases, and it diminishes faster proportionally as the rate of interest is higher.

The relations with regard to the different rates, when $n=1$, form an increasing series (between 0 per cent. and 200 per cent.); then after a sufficient number of years, they form a decreasing series. There is consequently between both these limits a number n for which the relations of two different rates are always equal; whence we may deduce the following remarkable theorem:

If any two rates of interest τ and t (the former being the greater) are given, the relations between the yearly and half-yearly annuities are always equal for a certain number of years; thus

$$\frac{1-\left(1+\frac{\tau}{2}\right)^{-2n}}{1-(1+\tau)^{-n}} = \frac{1-\left(1+\frac{t}{2}\right)^{-2n}}{1-(1+t)^{-n}},$$

$$\text{or } \frac{\phi\left(\frac{\tau}{2}\right)}{\phi(\tau)} = \frac{\phi\left(\frac{t}{2}\right)}{\phi(t)}.$$

For a smaller number of years it becomes

$$\frac{\phi\left(\frac{\tau}{2}\right)}{\phi(\tau)} > \frac{\phi\left(\frac{t}{2}\right)}{\phi(t)},$$

* See Explanatory Note, page xiv.

and for a longer period than this number of years

$$\frac{\phi\left(\frac{\tau}{2}\right)}{\phi(\tau)} < \frac{\phi\left(\frac{t}{2}\right)}{\phi(t)}.$$

With a view of showing the nature of the *relations* between the present value of an annuity payable yearly and the same annuity payable half-yearly, I have subjoined the following table, which exhibits the logarithms at 4 places of these relations for several years and rates of interest.

Years.	Rates of Interest per cent.										
	1	2	3	3½	4	4½	5	5½	6	8	10
	0,0										
1	107	212	315	366	415	465	514	562	610	796	974
15	100	184	254	284	311	335	356	375	392	437	455
20	98	175	234	258	278	296	310	321	330	345	335
25	95	166	216	234	249	260	268	273	276	269	243
30	93	157	198	212	221	228	231	231	230	208	173
35	90	149	182	191	197	199	198	195	190	159	122
40	88	140	167	172	174	173	169	163	156	120	79
45	86	133	152	155	154	150	143	136	127	90	59
60	79	112	115	111	104	95	86	77	67	37	18
80	71	87	78	69	60	50	42	34	27	10	4
120	56	52	33	25	18	14	9	6	4	1	0
∞	0	0	0	0	0	0	0	0	0	0	0

SECT. 39*. Substituting in form. 13 for ν and k the numbers 1, 2, 4 and 12, we have deduced the following table, which gives for the most ordinary cases the present value of an annuity of £1, which is payable *yearly*, *half-yearly*, *quarterly*, or *monthly*, supposing for each of these cases the interest to be convertible into principal, yearly, half-yearly, quarterly, or monthly.

If we assume the successive intervals between the payments of the annuity or of the interest to become infinite, and infinitely small, we obtain the extreme limits of the present value of the annuity for every possible case.

Thus, when k is infinite, that is, when the payments of the *annuity* are supposed to be made instantaneously, the expression

$$k\left\{\left(1+\frac{t}{\nu}\right)^{\frac{\nu}{k}}-1\right\}$$

* See Explanatory Note, page xiv.

becomes equal to the hyperbolic logarithm of

$$\left(1+\frac{t}{r}\right)^{r}.$$

When r is infinite, that is, when the payments of the *interest* are supposed to be made instantaneously, the expression

$$\left(1+\frac{t}{r}\right)^{-rn} \text{ becomes equal to } e^{-nt},$$

and the expression $\left(1+\frac{t}{r}\right)^{\frac{r}{k}}$ becomes equal to $e^{\frac{t}{k}}$.

At last, if $r=k=\infty$, that is, if the payments of the *interest* and *annuity* are supposed to be made instantaneously, and both at the same time, the expression

$$k\left\{\left(1+\frac{t}{r}\right)^{\frac{r}{k}}-1\right\}$$

becomes equal to t.

These expressions complete the following table.

In order to make still easier the application of this theory, from the table page 59 has been calculated another table which shows the present value of £1 per annum for 25 years at 4 per cent. in conformity with the 25 different combinations.

These two tables will show at once that

(1) the value of the annuity *increases* when the number of annual payments of the annuity increases;

(2) the value of the annuity *decreases* when the number of annual conversions of the interest increases;

(3) the value of the annuity *increases* when the number of annual payments of the annuity and interest increases, and the interest and annuity are both paid at the same time.

Present value of an Annuity of £1 per annum for n years.

The Interest being payable* \ The Annuity being payable*	yearly	half-yearly	quarterly	monthly	instantaneously
yearly	$\frac{1}{t}\left(1-v^{-n}\right)$	$\frac{1}{2}\cdot\frac{1-v^{-n}}{v^{\frac{1}{2}}-1}$	$\frac{1}{4}\cdot\frac{1-v^{-n}}{v^{\frac{1}{4}}-1}$	$\frac{1}{12}\cdot\frac{1-v^{-n}}{v^{\frac{1}{12}}-1}$	$\frac{1-v^{-n}}{\log_e v}$
half-yearly	$\frac{1-\left(1+\frac{t}{2}\right)^{-2n}}{\left(1+\frac{t}{2}\right)^{2}-1}$	$\frac{1-\left(1+\frac{t}{2}\right)^{-2n}}{t}$	$\frac{1}{4}\cdot\frac{1-\left(1+\frac{t}{2}\right)^{-2n}}{\left(1+\frac{t}{2}\right)^{\frac{1}{2}}-1}$	$\frac{1}{12}\cdot\frac{1-\left(1+\frac{t}{2}\right)^{-2n}}{\left(1+\frac{t}{2}\right)^{\frac{1}{6}}-1}$	$\frac{1-\left(1+\frac{t}{2}\right)^{-2n}}{2\log_e\left(1+\frac{t}{2}\right)}$
quarterly	$\frac{1-\left(1+\frac{t}{4}\right)^{-4n}}{\left(\frac{1+t}{4}\right)^{4}-1}$	$\frac{1}{2}\cdot\frac{1-\left(1+\frac{t}{4}\right)^{-4n}}{\left(1+\frac{t}{4}\right)^{2}-1}$	$\frac{\left(1+\frac{t}{4}\right)^{-4n}}{1-\frac{1}{t}}$	$\frac{1}{12}\cdot\frac{1-\left(1+\frac{t}{4}\right)^{-4n}}{\left(1+\frac{t}{4}\right)^{\frac{1}{3}}-1}$	$\frac{1-\left(1+\frac{t}{4}\right)^{-4n}}{4\cdot\log_e\left(1+\frac{t}{4}\right)}$
monthly	$\frac{1-\left(1+\frac{t}{12}\right)^{-12n}}{\left(1+\frac{t}{12}\right)^{12}-1}$	$\frac{1}{2}\cdot\frac{1-\left(1+\frac{t}{12}\right)^{-12n}}{\left(1+\frac{t}{12}\right)^{6}-1}$	$\frac{1}{4}\cdot\frac{1-\left(1+\frac{t}{12}\right)^{-12n}}{\left(1+\frac{t}{12}\right)^{3}-1}$	$\frac{\left(1+\frac{t}{12}\right)^{-12n}}{1-\frac{1}{t}}$	$\frac{1-\left(1+\frac{t}{12}\right)^{-12n}}{12\log_e\left(1+\frac{t}{12}\right)}$
instantaneously	$\frac{1-\epsilon^{-nt}}{\epsilon^{t}-1}$	$\frac{1}{2}\cdot\frac{1-\epsilon^{-nt}}{\epsilon^{\frac{t}{2}}-1}$	$\frac{1}{4}\cdot\frac{1-\epsilon^{-nt}}{\epsilon^{\frac{t}{4}}-1}$	$\frac{1}{12}\cdot\frac{1-\epsilon^{-nt}}{\epsilon^{\frac{t}{12}}-1}$	$\frac{1-\epsilon^{-nt}}{t}$

* See Explanatory Note, page xiv.

Present value of an Annuity of £1 per annum for 25 years, at 4 per cent.

The Interest being payable* \ The Annuity being payable*	yearly	half-yearly	quarterly	monthly	instantaneously
yearly	£15.62208	£15.77677	£15.85449	£15.90645	£15.93247
half-yearly	15.55624	15.71180	15.78998	15.84223	15.86840
quarterly	15.52282	15.67883	15.75722	15.80963	15.83588
monthly	15.50035	15.65665	15.73520	15.78771	15.81401
instantaneously	15.48906	15.64551	15.72413	15.77669	15.80301

* See Explanatory Note, page xiv.

SECT. 40*. The numerator in form. 13 does not vary if ν remains invariable, that is, when the periods at which the interest is paid remain the same; consequently when the present value of the annuity is known in the particular case of the periods of the payments of the annuity and interest corresponding with each other, the value of this annuity, payable k times in the year, is found by multiplying it by the invariable factor

$$\frac{\frac{t}{k}}{\left(1+\frac{t}{\nu}\right)^{\frac{\nu}{k}}-1}.$$

It is worthy of remark, that this factor, or constant ratio, is entirely independent of the number of years of the annuity.

Thus, all the calculations of annuities according to the different suppositions are easily brought down to the multiplication of the value of a known annuity by this factor. However, as the investigation of this factor requires often some hard and rather minute calculations, especially when the exponent $\frac{\nu}{k}$ is fractional, and also the use of logarithms with more than 10 decimals; in order to get a result with seven true figures, I have constructed a table (63) which converts these tedious computations into the addition or subtraction of a simple logarithm.

This table shows the logarithms of this factor intended for the most practical cases, and for 51 different rates. Besides, the differences between the functions of two consecutive rates vary so little, that the rate is again applicable through interpolation to every intermediate rate.

Let us denote by P_2, P_4, P_6 and P_{12} the *invariable ratio* for the same rate by which the *annual rent* is to be *multiplied* in order to obtain the value of a *half-yearly*, *quarterly*, *two-monthly*, or *monthly* rent.

We shall have,

$$P_2=\frac{\frac{t}{2}}{r^{\frac{1}{2}}-1}, \qquad P_6=\frac{\frac{t}{6}}{r^{\frac{1}{6}}-1},$$

$$P_4=\frac{\frac{t}{4}}{r^{\frac{1}{4}}-1}, \qquad P_{12}=\frac{\frac{t}{12}}{r^{\frac{1}{12}}-1}.$$

*Let us now denote by Q_2 and Q_4 the *invariable ratio* by which is to be *divided* the value of the half-yearly or quarterly rent when the payments of the annuity and interest do not correspond with each

* See Explanatory Note, page xiv.

other, in order to obtain the value of the *annual rent*. The interest being payable only twice or four times per annum, we have

$$Q_2 = \frac{\left(1+\frac{t}{2}\right)^2 - 1}{t}, \qquad Q_4 = \frac{\left(1+\frac{t}{4}\right)^4 - 1}{t}.$$

Every one of these factors, P and Q, will be greater than 1.

*Therefore, whenever the calculation of similar annuities is required; first, let us determine the value of the annuity, the payments of which correspond with the payments of the interest, like Sect. 22.

*Then, if the annuity is paid 2, 4, 6 or 12 times *more frequently* than the interest, *multiply* the value obtained by the corresponding factor P.

*If the annuity is paid 2 or 4 times *less often* than the interest, *divide* the obtained value by the corresponding factor Q.

Thus, for instance, if we know the value of a £100 annuity for 19 years, at the rate of 4 per cent., payable *yearly*, let us *multiply* it by P_2, P_4, P_6 or P_{12} when the annuity is half-yearly, quarterly, of two months, or monthly.

*If, on the contrary, this rent is *annual*, and the interest is convertible *half-yearly*, let us look first for the present value of the £50 annuity for 38 years at 2 per cent., and divide it by Q_2.

*If the rent is *annual*, and the interest *quarterly*, let us look first for the present value of the £25 annuity, for 76 years, at 1 per cent., and divide it by Q_4. In Table 63 we find $t = 4$ per cent.

$$\log Q_2 = 0{,}00432\ 14$$
$$\log Q_4 = 0{,}00650\ 89.$$

If the values of P and Q were required for a rate which is not included in Table 63, they could be obtained by means of simple interpolations.

For instance, the value of Q_2 is required for $t = 3\frac{1}{2}\frac{7}{0}$ per cent.

This rate, which is $= 3\frac{6}{8}\frac{4}{5}$, is included between $3\frac{3}{4}$ and $3\frac{7}{8}$, and we find

$$\text{for } 3\tfrac{3}{4},\ \log Q_2 = 4{,}0525$$
$$3\tfrac{7}{8},\ \log Q_2 = 4{,}1870$$
$$\text{Difference} = \quad 1345$$

Consequently for $3\frac{6}{8}\frac{4}{5}$ we have

$$\log Q_2 = 4{,}0525 + \frac{4}{5} \times 1345$$
$$= 4{,}1601.$$

* See Explanatory Note, page xiv.

If the value of P_2 were required for $i = 1\frac{17}{24}$ per cent., this rate $= 1\frac{7\frac{2}{5}}{8}$, and it is contained between $1\frac{1}{2}$ and 2, but we find

$$\begin{array}{rl} \text{for } 1\frac{1}{2} & \log P_2 = 2{,}0216 \\ 2 & = 2{,}1554 \\ \text{Difference} & \underline{\quad 1338}; \end{array}$$

therefore $1\frac{7\frac{2}{5}}{8}$ corresponds to $\log P_2 = 2{,}0216 + \frac{2}{5} \times 1338$

$$= 2{,}0751.$$

Table 63 is therefore easily applicable to every rate: the following examples show still better its immediate use.

Ex. 65*. What is the present value of an annuity of £40, for 30 years, payable quarterly, when the interest is convertible into principal half-yearly, and the annual rate of interest is 4 per cent.?

The present value of the above-named annuity being equal to the present value of an annuity of £20 for 60 years at 2 per cent., multiplied by a constant factor P_2,

$$\begin{array}{rl} \log 20 = & 1{,}30103\ 00 \\ \log \phi = & 1{,}54109\ 07 \\ \log P_2 = & \quad 215\ 54 \\ \log V = & 2{,}84427\ 61 \\ V = & £698{,}6765. \end{array}$$

Ex. 66. Required the present value of an annuity of £1 per annum for 50 years, and at 5 per cent., payable half-yearly; also when the annuity is payable yearly, quarterly, or monthly.

The annuity payable half-yearly is equal to the present value of £½ per annum, for 100 years, at 2½ per cent. per annum,

$$\begin{array}{rl} \log \tfrac{1}{2} = & 9{,}69897\ 00 \\ \log a^* = & 8{,}43635\ 16 \\ \log V = & 1{,}26261\ 84 \\ V = & £18{,}30705. \end{array}$$

In order to get the present value of the annuity payable yearly, quarterly, or monthly, this number must be multiplied by the corresponding constant ratios:

yearly		quarterly		monthly
	1,26261 84		1,26261 84	1,26261 84
$\log Q_2 =$	− 539 50	$\log P_2 =$	+ 268 92	$\log P_6 =$ + 447 90
	1,25722 34		1,26530 76	1,26709 74

therefore the value of the annuity is

$$\begin{array}{rl} \text{when payable yearly} = & £18{,}08104 \\ \text{half-yearly} = & 18{,}30705 \\ \text{quarterly} = & 18{,}42076 \\ \text{monthly} = & 18{,}49684. \end{array}$$

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* See Explanatory Note, page xiv.

Ex. 67. It is required to exchange an annuity of £340 annum, for an unlimited number of years, against a half-ye quarterly, two-monthly, or monthly rent; what amount will it r per annum, at the annual rate of 5 per cent.?

	half-yearly		quarterly
	$\log 340 =$ 53148	=	53148
(Tab. 63)	$\log P_s =$ 533	=	799
	$\log V =$ 52615	=	52349
	$V =$ £335,85	=	£333,80
	two-monthly		monthly
	$\log 340 =$ 53148	=	53148
(Tab. 63)	$\log P =$ 887	=	975
	$\log V =$ 52261	=	52173
	$V =$ £333,13	=	£332,45.

The sum to be paid will be

half-yearly	£167,925
quarterly	83,450
two-monthly	55,521
monthly	27,704,

instead of £340 per annum.

Ex. 68*. What is the present value of an annuity of £584 23 years, payable yearly, when the interest is convertible into p cipal quarterly, the rate of interest being 6 per cent.?

$$\log \frac{584}{4} = 2,16435\ 29$$

(Tab. 3) $\log \phi = 1,69654\ 86$

$\log Q_4 = -\ 975\ 92$

$\log V = 3,85114\ 23$

$V = £7098,104.$

Ex. 69*. What is the annuity which £13600 will purchas 27 years, when the annuity is payable quarterly, and when the terest at 4½ per cent. is convertible half-yearly?

$\log 13,600 = 4,13353\ 89$

$\log a^n = 8,49566\ 10$

$\log P_2 = -\ 228\ 90$

$\log a = 2,62691\ 09$

$a = £423,5561.$

Ex. 70. What is the present value of a *perpetuity* of £78. receivable in equal *monthly* payments, when the interest at the of 4½ per cent. can only be converted into principal twice a year

$\log 78,6 = 1,89542\ 25$

$\log \phi = 1,32330\ 64$

$\log P_6 = 425\ 72$

$\log V = 3,22298\ 61$

$V = £1671,037.$

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* See Explanatory Note, page xiv.

Ex. 71*. It is required to exchange an *annual* rent of £480, payable during an *unlimited* number of years, against a *quarterly* rent, the interest being £3. 17*s.* per cent., payable half-yearly.

As the rate $3\frac{17}{20}$ per cent. is not to be found in Table 63, the values of log P_2 and log Q_2 are to be sought by simple interpolation. We have

$$V = 480 \times \frac{1-\left(1+\frac{t}{2}\right)^{-2n}}{\left(1+\frac{t}{2}\right)^{2}-1} = x \times \frac{1-\left(1+\frac{t}{2}\right)^{-2n}}{\left(1+\frac{t}{2}\right)^{\frac{1}{2}}-1},$$

$$\text{therefore } x = 480 \times \frac{\left(1+\frac{t}{2}\right)^{\frac{1}{2}}-1}{\left(1+\frac{t}{2}\right)^{2}-1} = \frac{480}{P_2 Q_2},$$

$$\begin{aligned} \log 480 &= 2{,}68124\ 12 \\ \log P_2 &= -\ 207\ 51 \\ \log Q_2 &= -\ 416\ 01 \\ \log a &= 2{,}67500\ 60 \\ a &= £473{,}1579. \end{aligned}$$

Consequently £118. 5*s.* 9½*d.* are to be paid quarterly.

Ex. 72*. It is required to convert an annuity of £845 payable *half-yearly* for any number whatever of years, into a *quarterly* rent, the interest being at $3\frac{3}{4}$ per cent., payable *yearly*. What amount is to be paid per annum?

$$\begin{aligned} \log 845 &= 2{,}92685\ 67 \\ +\log P_2 &= \quad 401\ 54 \\ -\log P_4 &= \quad 601\ 86 \\ \log V &= 2{,}92485\ 35 \\ V &= £841{,}1114. \end{aligned}$$

Ex. 73*. An *annuity* of £61. 14*s.* for 40 years is to be converted into a *quarterly* rent, payable for 22 years, the interest being at $3\frac{3}{4}$ per cent., convertible into principal half-yearly. What is to be paid per annum?

Let x denote the annuity, we then have

$$V = 61{,}7 \times \frac{1-\left(1+\frac{t}{2}\right)^{-80}}{1-\left(1+\frac{t}{2}\right)^{2}-1} = \frac{x}{4} \cdot \frac{1-\left(1+\frac{t}{2}\right)^{-44}}{\left(1+\frac{t}{2}\right)^{\frac{1}{2}}-1};$$

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* See Explanatory Note, page xiv.

therefore

$$z = 4 \times 61{,}7 \times \frac{1-\left(1+\frac{i}{2}\right)^{-80}}{1-\left(1+\frac{i}{2}\right)^{-44}} \cdot \frac{\left(1+\frac{i}{2}\right)^{\frac{1}{4}}-1}{\left(1+\frac{i}{2}\right)^{\frac{1}{44}}-1}$$

$$= \frac{617 \cdot \phi^{80}}{\phi^{44} P_2 Q_2},$$

$\log 617$	$= 1{,}79029$
$\log \phi^{80}$	$= 1{,}61560$
$C^l \log \phi^{44}$	$= 8{,}52605$
$\log P_2$	$= -\ 405$
$\log Q_2$	$= -\ 202$
$\log z$	$= 1{,}92587$
z	$= £84{,}308,$

that is, instead of paying £61. 14*s.* per annum for 40 years, £21. 1*s.* 6½*d.* will have to be paid quarterly.

SECT. 41*. *Calculation of the time of annuities when interest and annuity are paid at various periods.*

Table 58 renders the calculation of the time very easy, as it shows directly the relation between the given annuity and the annuity at the same rate of interest, but the payments of which do not agree with the payments of the rate.

In this case, from the logarithm of the annuity is to be subtracted the logarithm of the constant ratio (Sect. 40), and the rate is then to be determined as for the ordinary annuities (Sect. 22).

EX. 74*. In how many years will a principal of £7200 be amortized by an annuity of £504,582, payable quarterly, when the rate is of 5 per cent. per annum, payable half-yearly?

The true rate is 2½ half-yearly, the half-yearly rent is consequently = £252,291,

$\log 252{,}291$	$= 2{,}40190\ 17$
$\log P_2$	$=\ \ 268\ 92$
$\log V$	$- = 3{,}85733\ 25$
$\log a^n$	$= 8{,}54725\ 84$

This logarithm is found in Tab. 11 (2½ per cent.) to correspond with 50 years; therefore the time for the annuity is exactly 50 half-years, or 25 years.

Formula 13 supplies a general formula for calculating the time. In fact we have

$$V = \frac{a}{k} \cdot \frac{1-\left(1+\frac{i}{\nu}\right)^{-n\nu}}{\left(1+\frac{i}{\nu}\right)^{\frac{k}{\nu}}-1},$$

* See Explanatory Note, page xiv.

whence

$$\left(1+\frac{t}{\nu}\right)^{-n\nu}=1-\frac{\nu k}{a}\left\{\left(1+\frac{t}{\nu}\right)^{\frac{\nu}{k}}-1\right\};$$

and if we denote by p_k the constant factor

$$\frac{\left(1+\frac{t}{\nu}\right)^{\frac{\nu}{k}}-1}{\frac{t}{k}},$$

we have

$$\left(1+\frac{t}{\nu}\right)^{-n\nu}=1-\frac{Vtp_k}{a};$$

and using the logarithms

$$n=\frac{-\log\left(1-\frac{Vtp_k}{a}\right)}{\nu\log\left(1+\frac{t}{\nu}\right)}.$$

Ex. 75*. In what time would a principal of £5700 be amortized by a monthly rate of £26, the interest being 4½ per cent. payable yearly?

$$\nu=1,\quad k=12,\quad a=312.$$

Consequently,

$\log V$	= 3,755587	$\frac{Vtp}{a}$	=0,80562
$\log t$	= 8,65321	$1-\frac{Vtp}{a}$	=0,19438
$\log P_{12}$	= − 880	log	=9,28865
$-\log a$	= −2,49415	−log	=0,71135
$\log\frac{Vtp}{a}$	= 9,90613		

$$\text{therefore } n=\frac{0,71135}{0,019116}=37,212,$$

the time required is =37 years, and 2,544 months.

In conclusion:

First, assume as the rate the interest of £1 for each interval in the year during which the *interest* is convertible, and calculate as if each interval represented a whole year. For instance, if interest is half-yearly, take half the annuity and the rate of interest, and double the number of years; if the interest is quarterly, take a fourth part of the annuity and of the rate of interest, then quadruplicate the number of years.

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* See Explanatory Note, page xiv.

Secondly, if the annuity is paid *more frequently* than the interest, multiply its value by the constant factor P, the Table 63 being entered with the simple interest of £1 for every interval between two consecutive payments of *interest*.

If, on the contrary, the annuity is paid *less frequently* than the interest, divide its value by the constant factor Q, the Table 63 being entered with the simple interest for any interval between two consecutive payments of the *annuity*.

Sect. 42. *Equation of Annuities.*

Mean Annuity.

Whenever two equal rents to be continued for different periods of time, the annual payments and rate being the same, are to be converted into one single annuity; the duration of the mean annuity is always expressed by a number of years *less* than the mean duration of the two given rents.

Let us first find x, the number of years during which are to continue two annuities of £a, which are to be equal to two annuities of £a, one for n years, and the other for $(n+2d)$ years: we have

$$\frac{2a}{i}\left(1-\frac{1}{r^x}\right)=\frac{a}{i}\left(1-\frac{1}{r^n}\right)+\frac{a}{i}\left(1-\frac{1}{r^{n+2d}}\right);$$

therefore

$$\frac{2}{r^x}=\frac{1}{r^n}+\frac{1}{r^{n+2d}},$$

or,

$$r^x=\frac{2r^{n+d}}{r^d+r^{-d}}.$$

But this is the formula we have already found (Sect. 13) for equating the payments, consequently the time of the mean annuity is equal to the true equated time of both the equal rents payable at the end of both annuities.

Thus, for instance, we found (Ex. 25) that at the rate of 4½ per cent., two principals of £1000, one payable at the end of 7 years, and the other at the end of 43 years, were equivalent to two payments of £1000, both being made at the end of nearly 18½ years; consequently two annuities of £1000, one for 7 years, the other for 43 years, are equal to two annuities of £1000 for 18½ years. In the present case, instead of 51 annuities there would be but 37, and the first would benefit by 11½ annuities, whereas the other would lose by 24½.

We have already demonstrated (Sect. 13) that the number of years of the mean annuity is always smaller than the half sum of the years during which the proposed annuities are to continue, and this number will be so much smaller as the rate is itself greater.

Thus, assuming two annuities of £1, one for 16 years, the other for 34, both will give 50 annuities of £1 taken together, and equivalent to two annuities of £1 likewise, which are

at ½ per cent.	for	24,907 years
1		24,599 ...
2		24,202 ...
3		23,816 ...
4		23,444 ...
6		22,741 ..
10		21,537 ...

instead of 25 years.

SECT. 43. Reciprocally the annuity corresponding to the *mean duration* of two annuities is ever greater than the *mean annuity* of both annuities. If the first rent contains n annuities, and the second $(n+2d)$ of £1, the difference between the rent of the *mean duration* of the first two is equal to

$$\Delta = \frac{(r^d - 1)^2}{ir^{n+2d}};$$

which quantity is always positive.

Thus, for instance, two annuities of £1 for 30 years are worth more than two annuities of £1, one for 10, the other for 50 years, although there are in both cases 60 annuities of £1.

In fact, at 3 per cent., we have:

Value of £1 per annum for 10 years =	£ 8,530
..... 50 ... =	25,730
Sum =	34,260
Value of £2 per annum for 30 years =	39,201
Difference =	£ 4,941

The foregoing formula, which represents the difference Δ, shows directly that this difference *decreases* when the number of years, n or d, *increases*.

If the rate varies, the value of Δ shows a maximum when

$$2d = \left(n + \frac{r}{i}\right) \cdot (r^d - 1).$$

After this maximum, the value of the difference diminishes, whatever may be the increase or decrease of the rate.

Thus the difference between two annuities of £1, one for 10 years, the other for 50 years, and the annuity £2 for 30 years, is

at ½ per cent. =	£0,926
1 ..... =	2,948
2 ..... =	4,387
3 ..... =	4,941
3½ ..... =	5,012
4 ..... =	4,991

at 5 per cent. = £4,767
6 ..... = 4,408
7 ..... = 3,994
8 ..... = 3,572
10 ..... = 2,792.

If instead of two annuities there were several annuities, the mean of these annuities would be found to be smaller than the annuity of the *mean duration*.

SECT. 44. A general mistake in the estimation of life annuities is to believe that the value of a *life annuity*, held on a single life, is equal to the value of a *certain annuity* corresponding to the mean duration of this single life.

Thus, by finding according to the Northampton Table, that the life expectation of a child 7 years old is 41,03 (Inwood, p. 142), it is supposed that the value of the *life annuity* of £1, payable during 41,03 years, which at the rate of 5 per cent. would represent an amount of £17,5495, is the value of the life annuity; whereas that amount is too considerable, *V* being only £15,166, which corresponds to a certain annuity for 29,105 years.

Moreover, when the value of an annuity on a single life, aged 7 years, is determined with the Northampton Table, at different rates of interest, as well as the number of years of a certain annuity, the latter is less than the mean lifetime, but it varies and decreases when the rate increases.

In the following Table we have shown the value of an annuity of £1, on a single life of 7 years at different rates, according to the Northampton Table, and the year's purchase of a certain annuity which corresponds to this life-rent, the mean duration of life being 41,03 years.

Rate of both rents.	Value of the Life Annuity.	Number of years of the certain Annuity.
3 per cent.	£20,853	33,236 years
4	17,611	31,078 ...
5	15,166	29,105 ...
6	13,275	27,323 ...
7	11,777	25,710 ...
8	10,570	24,275 ...

But what is the meaning of the *mean lifetime*?

The *mean duration* of life, or *expectation*, according to Price, is the age that would result from the equal distribution amongst a certain number of persons of the same age, of the whole future life of these persons; thus, in the present case, it means that 1000

viduals, being each 7 years old, may rely upon having all ther to live 41,030 years more, which they will divide unequally ween themselves. This number of 41,030 years gives place to 30 annuities of £1 divided between 87 unequal and decreasing ments.

Our demonstrations in the above chapter relating to annuities also generally applicable to life annuities. We may therefore ply state, without any subsequent demonstration, that;

(1) The number of years of a certain annuity which corre-onds with a life annuity is always smaller than the mean dura-on or expectation of the same age.

(2) The certain annuity for the mean duration of life is always reater than the life annuity held on the same life.

Thus, the value of an annuity of £1 on a life of 7 years is = £20,853; he value of an annuity of £1 for 41,03 years (mean duration) is = £23,419. Difference = £2,566.

The error at	3	per cent. is =	£2,566
.....	4	.....	2,765
.....	5	.....	2,132
.....	6	.....	1,866
.....	7	.....	1,619
.....	8	.....	1,398.

ON NATIONAL STOCKS.

SECT. 45. The redeemable* annuities established for the pay-ent of the interest and discharge of government loans differ from rdinary annuities by their *interest* being *invariable*, whereas the *ue capital* they represent, or the principal to be reimbursed, is *riable*, or submitted to all the fluctuations of the Exchange market.

Generally, Governments are obliged to redeem such stocks at a igher price than that at which they were originally granted.

The *nominal principal* of every title to an annuity is represented y the *fictitious sum* of £100, and gives an annual interest of 2½, 3, ½ and £5, payable half-yearly. When the *price* of the principal is 100, it is said to be *at par:* it is evident that these stocks cannot e at par all at once, for it is impossible to assume that a £2½ nnual stock and a £5 stock can fetch the same price.

These loans compose all together nominal principals or *stocks*, he whole of which compose the *funded national debt*, which is eimbursable at no fixed period; or rather, not reimbursable at all. The half-yearly payments and annuities on these stocks are called *lividends.*

* These stocks are called redeemable, because Government keeps he right of redeeming them, by paying off the owner of the annuity.

The most important of all these stocks, the *Consols*, or *Consolidated Annuities*, derive their name from the consolidation in the year 1751 of the stocks of various annuities into a joint stock 3 per cent. The *highest* prices which this stock attained were £107 (in 1737), 101¼ (1844), 100⅝ (1845), and 101⅝ (1852); the *lowest* prices of the same were £47½ (in 1797), 47½ (1798), 50¼ (1803), 78½ (1817) and 90¼ (1853).

From the year 1800 to 1844, the consols 3 per cent. were always below par (50¼ to 97⅛).

The most considerable stock besides the Consols is the 3 *per cent. reduced*, or *reduced* annuities; thus called from the rate of the annuities on the original stock having been reduced in 1746 from 4 to 3½ per cent., and in 1757 to 3 per cent. annuities.

The 3 per cent. Consols at the period of their consolidation were = £9,137,812; on the 5th of January, 1844, £367,813,965; on the 5th of January, 1853, £370,655,463.

The Reduced 3 per cent. amounted at the time of its reduction to £17,701,323; on the 5th of January, 1844, £125,359,557; on the 5th of January, 1853, £116,589,420.

The whole national funded debt amounted:

	£
at the Revolution in 1688, to.....	664,263
at the accession of George I., 1714	54,145,343
at the commencement of the French War, 1793...	233,733,609
at the Peace of Paris, in Sept. 1815	864,822,438
the 16th of March, 1856	775,312,694

of which £769,000,280 were applicable to the 3 per cent. stocks, (Consols, reduced annuities, Bank annuities of 1726, debt to the Bank of England and South Sea Stocks), and £6,312,414 to the 2½, 3½, and 5 per cent. stocks. The interest of this debt was for the year 1856, £23,267,361.

As Government pays the dividend half-yearly, we must compute all the annuities of these Stocks as half-yearly annuities.

Ex. 76. If Government should be disposed to exchange the 5 per cent. stock for terminable annuities of 7 per cent. per annum, for what number of years ought such annuities to be when the rate of interest is at 5½ per cent.?

It is useless in the present case to know the price of the 5 per cent.

We have found, page 10, the formula: $r^n = \frac{a}{s}$,

here, $i = 2\frac{3}{4}$ per cent., $s = 2$, $\frac{a}{s} = 3.5$, $a = 7$,

$\log 3.5 = 0.54407$
$\log^2 \quad = 9.73505$
$\log^2 . r = 8.07121$
$\log n \quad = 1.66444$
$n \quad = 46.179$ half-years, or 23.089 years.

Ex. 77. Government wishes to exchange a $3\frac{1}{2}$ per cent stock against a 3 per cent. one, in order to make use of the surplus for the extinction of said stock. In how many years could this stock be redeemed at par?

Here $a=3\frac{1}{2}$, $s=\frac{1}{2}$, $\frac{a}{s}=7$, and $t=1\frac{1}{2}$ per cent.

therefore, $\log 7 = 0,84510$
$\log^2 = 9,92691$
$C^t \log^2 r = 2,18936$
$C^t \log 2 = 9,69897$
$\log n = 1,81524$
$n = 65,349$ years.

Ex. 78. How much is to be paid annually to the sinking fund, in order to redeem at par a loan of a million pounds, $3\frac{1}{2}$ per cent. stock in 45 years?

$\log a = 4,34532\ 67$
$\log r^n = 0,67809\ 76$
$\log s = 3,66722\ 91$
$s = £4647,604$ per half-year,

or, for the complete extinction of the stock, £9.295,208 will have to be paid for amortizement, and £35.000 for annual interest, or, for the whole, £44.295,208 per annum.

Sect. 46. Whenever the stock is not at par, the half-year rate is to be calculated.

Let k represent the annual dividend corresponding to a £100 stock;

s the annual sinking fund for said stock;

c the price of this stock;

the half-yearly interest will be $t=\frac{k}{2c}$,

and $r=1+\frac{k}{2c}$.

Ex. 79. How much is to be paid per annum in order to reimburse in 4 years a loan of £45.000 consols, 3 per cent., if the mean price of these consols is £90?

£3 interest for £90 principal corresponds to the rate $3\frac{1}{3}$ per cent. per annum, or to the rate $1\frac{2}{3}$ per half year. As £90 are to be reimbursed for a £3 stock, the whole principal to be paid for a £45.000 annuity will be £1.350.000, and as the calculation is intended for 8 half-years, we have

$\log V = 6,13033\ 38$
$\log a^n = 9,12890\ 22$
$\log a = 5,25923\ 60$
$a = £181.650.$

Consequently, a mean sum = £181.650 will have to be paid at the end of every half-year.

Supposing, indeed, that these stocks are to be reimbursed without any fractions of the pound, it will be necessary to pay a little more or a little less so as to make the difference between the true annuity, and the annuity to be paid, less than £15, corresponding to £¼ rent.

Reimbursement of £45.000 Consols, 3 per cent., in 8 years, at the mean price of £90.

Half years.	Dividend payable every 6 months.	Amount paid for Amortisement.	Whole half-yearly payment.	Stocks redeemed.	Annual Dividend.
1	£22.500	£159.150	£181.650	£5.305	£39.695
2	19.847½	161.790	181.637½	5.393	34.302
3	17.151	164.490	181.641	5.483	28.819
4	14.409½	167.250	181.659½	5.575	23.244
5	11.622	170.040	181.662	5.668	17.576
6	8.788	172.860	181.648	5.762	11.814
7	5.907	175.740	181.647	5.858	5.956
8	2.978	178.680	181.658	5.956	

Ex. 80. £140,000, 5 per cent. stock annuities at the mean price of £129 are to be amortized in 24 years. What sum is to be paid per annum?

Here $t = \frac{5}{129}$,

$$\begin{aligned} &\log 140{,}000 = 5{,}14612\ 80 \\ C^t\ &\log t = 11{,}41161\ 97 \\ &\log a^n = 8{,}50774\ 53 \\ &\log a = 5{,}06549\ 30 \\ &a = £116276{,}770. \end{aligned}$$

Sect. 47. If in formula (9^s) we substitute for r its value $\left(1 + \frac{k}{2c}\right)$, in which c denotes the price of the stock and k the annual dividend, corresponding to £100 principal, and if we change n which denotes the number of half years into $\frac{n}{2}$, so as to have directly the number of years, we have:

$$\left(\frac{2c+k}{2c}\right)^{2n} = \frac{a}{s}, \text{ or } n = \frac{\log \frac{a}{s}}{2 \log \frac{2c+k}{2c}}.$$

Ex. 81. How many years are required in order to reimburse a loan borrowed for the service of Government at 3 per cent., by a

king fund equal to 1 per cent., of the capital stock created for e extinction of this loan, the mean price of the 3 per cent. being 5?

$\log 2 = 0{,}30103$ $\quad \log^2 = 9{,}47861$

$\log r = 0{,}00759\ 72$ $\quad \log^2 = 7{,}88065$

$\log n = 1{,}59796$

$n = 39{,}624$ years.

By virtue of the act 32 Geo. III. c. 55. § 3. (1792) it was ordained at for all future loans a sinking fund of 1 per cent. of the stock ould be established for the gradual redemption of the principal.

The following table shows in how many years at 3 per cent. ock may be redeemed by a sinking fund of 1 per cent. of the ock and at different prices of purchase.

Price of the 3 per cent. stock.	Number of years.
£ 75	35,003
77½	36,158
80	37,313
82½	38,469
85	39,624
87½	40,779
90	41,935
92½	43,090
95	44,245
97½	45,400
100	46,556
102½	47,711
105	48,866.

As the differences between these numbers are very nearly the ne, it is very easy to determine, by means of interpolation, the ie for all the intermediate rates; thus, for instance, if the price the 3 per cent. is £98, the number of years would be:

$$= 45{,}400 + \tfrac{1}{5} \times 1{,}156 = 45{,}631 \text{ years.}$$

Ex. 82. In how many years would a loan at 5 per cent. be ortized by a sinking fund of £1 per cent. of the stock; the mean ce of re-purchase being £115?

$$k = 5,\ s = 1,\ a = 6,\ c = 115,\ r = \frac{47}{46},$$

sequently,

$\log 6 = 0{,}77812$ $\quad \log^2 = 9{,}89106$

$\log \frac{47}{46} = 0{,}00934$ $\quad \log^2 = 7{,}97035$

$1{,}92071$

$\log 2 = 0{,}30103$

$\log n = 1{,}61968$

$n = 41{,}656$ years.

Sect. 48. The celebrated Dr Price proposed to redeem the public debt by means of a sinking fund, equal to 1 per cent. of the sum borrowed.

Ex. 83. A loan of 3½ stock has been issued at £91, with a sinking fund of 1 per cent. of the sum borrowed; how long will it take to amortize this loan if the mean redeeming price is £102?

$$k=3,5,\ s=0,91,\ a=4,41,\ c=102,\ r=\frac{415}{408},$$

$$\log\frac{a}{s}=0,68540 \qquad \log^2=9,83594$$

$$\log r=0,0073879 \qquad \log^3=\underline{7,86852}$$
$$=1,96742$$
$$\log 2=\underline{0,30103}$$
$$\log n=1,66639$$
$$n=46,386 \text{ years.}$$

Ex. 84. A loan of 3 per cent. has been issued at £87 with a sinking fund of 1 per cent. of the sum borrowed; how many years will it take to amortize it, if the price is £90?

$$k=3,\ s=0,87,\ a=3,87,\ c=90,\ r=\frac{61}{60},$$

$$\log\frac{a}{s}=0,64819 \qquad \log^2=9,81170$$

$$\log r=0,00717\ 86 \qquad C^t\log^2=2,14396$$
$$C^t\log 2=\underline{9,69897}$$
$$\log n=1,65463$$
$$n=45,147 \text{ years.}$$

Sect. 49. By virtue of the act 47 Geo. III. c. 55, (1807) it was ordained to pay annually 10 per cent. on the *sum* borrowed for the interest and redemption of the principal of future loans.

Ex. 85. If a loan is effected for the service of Government, on the 3 per cent. valued at £95, and a fund equal to 10 per cent. on the sum borrowed is established for the interest and redemption of the principal, in how many years would this loan be amortized at the mean price of 95½?

$$a=9,5,\ k=3,\ s=6,5,\ c=95\tfrac{1}{2},\ r=\frac{194}{191},$$

$$\log\frac{a}{s}=0,16481 \qquad \log^2=9,21699$$

$$\log r=0,00676\ 84 \qquad C^t\log^2=1,16952$$
$$C^t\log 2=\underline{9,69897}$$
$$\log n=1,08548$$
$$n=12,175 \text{ years.}$$

Ex. 86. In how many years will a 5 per cent. loan, contracted issued at the price of £120, be redeemed by a fund of 10 per of the sum borrowed?

$$a=12,\ k=5,\ s=7,\ c=120,\ r=\frac{49}{48},$$

$$\log\frac{a}{s}=0,23408 \qquad \log^2=9,36937$$

$$\log r=0,00895\ 48 \qquad C^t\log^2=2,04794$$
$$C^t\log 2=\underline{9,69897}$$
$$\log n=1,11628$$
$$n=13,07 \text{ years.}$$

Ex. 87. It is required to convert £84.000 of 3½ per cent. against 3 per cent. stock, with this condition, that the differ- of interest will be left for the amortizement, and £20.000 will lded per annum to it; besides, the purchase price is presumed to 96; how long will it take to amortize the debt?

Reduced interest = 72.000, $a=104,000$, $s=32.000$,

$$\frac{a}{s}=\frac{13}{14}, \qquad r=\frac{65}{64}$$

$$\log\frac{a}{s}=0,51188$$
$$\log r=0,00673\ 34$$
$$\frac{a}{s}\ldots\log^2=9,70917$$
$$C^t\log^2 r=2,17177$$
$$C^t\log 2=\underline{9,69897}$$
$$\log n=1,57991$$
$$n=38,011 \text{ years.}$$

Ex. 88. If Government contracts a loan of £112,000, 3½ per rent, and establishes a sinking fund of £88,000 per annum, hat price would it be necessary to re-purchase this loan in order nortize it in 25 years?

$$a=200,000,\ s=88,000,\ \frac{a}{s}=\frac{25}{11},\ k=3\tfrac{1}{2},\ r=1+\frac{3\frac{1}{2}}{2c},$$

$$\log\frac{a}{s}=0,35654\ 73 \qquad \log 3\tfrac{1}{2}=0,54407$$

$$\log r=0,00713095 \qquad C^t\log 2=9,69897$$
$$r=1,01655\ 515 \qquad C^t\log t=0,78107$$
$$\log C=2,02411$$
$$C=£105,71.$$

Therefore the rate is = 1,655515 per cent. per half year, that is, 31103 per cent. per annum, and the required price is = £105,71.

Ex. 89. £8,000,000 are borrowed in 4½ per cent. rents at par, h will constitute £360,000 per annum. At what mean price

could they be re-purchased in 21 years, by adding £240,000 for its amortizement, i. e. by paying yearly £600,000?

$$\frac{a}{s} = \frac{60}{24} = 2{,}5, \quad r^{42} = 2{,}5$$

$$\log 2{,}5 = 0{,}39794$$
$$\log r = 0{,}00947\ 476$$
$$r = 1{,}02705\ 62.$$

Therefore the half-yearly rate is 2,20562 for a half-year, or 4,41124 per cent. per annum, and

$$\log 2{,}5 = 0{,}35218\ 25$$
$$\log V = 8{,}34353\ 07$$
$$\log C = 2{,}00865\ 18$$
$$C = £102{,}012,$$

the mean price at which this loan is to be re-purchased, in order to amortize it in 21 years.

† If the interest and annuity were payable *yearly*, we should have

$$r^{21} = 2{,}5$$
$$\log r = 0{,}01894\ 95$$
$$r = 1{,}04459\ 88,$$

the annual rate would be = 4,45988 per cent., and the price = £100,900.

Sect. 50. *Obligations or Bonds to be reimbursed by means of drawing by lot.*

The method of redeeming loans by annual payments plays a very important part in our great industrial transactions where private means are applied to getting up the large stocks required by great companies.

Thus in most Continental States*, and in America, townships, and Canal, Mining and Railway Companies, whose grants are limited, contract heavy loans reimbursable by constant annuities.

In order to gather up the smallest savings of private individuals, companies have issued special *Bonds* called *Obligations*, representing each of them a very small fraction of the loan contracted, £20 or £50 in most cases. Such bonds, most of them issued much below par (the £20 bonds at £11 at the average), companies are obliged to reimburse by a certain number of annuities; the latest reimbursement which might be fixed being the year of the expiration of the Company's grant.

Like Public Funds, obligations bear an half-yearly interest, and are transferable at the Public Exchange.

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* In France only the obligations issued arise to more than 15 millions of pounds.

† See explanatory note, p. xiv.

Reimbursement by constant annuity differs essentially from the reimbursement by obligations, where the *sinking fund* plays an entirely different part.

Companies pay every year a constant, or nearly a constant sum, made up by two half-yearly payments, which are themselves *unequal* and *variable*. The rate of interest is always *half-yearly*, and the interest paid every six months.

The gradual redemption of nearly all loans borrowed by obligations takes place but once in the year. The *amortizement*, i. e. the surplus of the constant annuity above the total interest for the year, is not distributed over the whole number of bonds, but is made to pay up a certain number of bonds, designated by lot.

It follows that the constant annuity includes two half-yearly payments, the first of which comprehends only the interest for the first half-year, whilst the second comprehends the interest of the second half-year, and the surplus of the annuity by which the borrowed sum is gradually extinguished. In this way, principal decreasing year by year, it is evident that the first half-yearly payments are always less than the following ones, and that the number of obligations to be reimbursed at the end of the year increases.

Now, two questions remain to be solved:

(1) What is the sum payable every year?

(2) What is the present value of an obligation at a rate different from its nominal rate?

We have first to determine the annual sum to be paid for a loan of £ V in n years.

Let t = the rate of interest per annum, payable half-yearly,

a = the annuity, i. e. the sum of the two half-yearly payments for the same year which £1 will purchase in n years.

The principal remaining at the end

of the first year is $= r - x$,

... second ..... $= r^2 - \frac{x}{t}(r^2 - 1)$,

... third ..... $= r^3 - \frac{x}{t}(r^2 - 1)$,

... fourth ..... $= r^4 - \frac{x}{t}(r^4 - 1)$.

At last, at the end of the n^{th} year, the remainder of the principal will be

$$r^n - \frac{x}{t}(r^n - 1),$$

and as, at that time, the loan must be entirely extinguished, we shall have

$$r^n = \frac{x}{t}(r^n - 1),$$

whence

$$a = \frac{t}{1 - r^{-n}}$$

annuity to be paid for £1 principal; and consequently the annual sum to be paid for a principal of £V is

$$a = \frac{Vt}{1 - r^{-n}}.$$

But that is the very same annuity payable for £V, when the rate of interest and all payments are annual, consequently:

The annuity for n years, which £1 will purchase at the annual rate of interest t, is equal to the annuity paid at the half-yearly rate $\frac{t}{2}$, when interest is paid every half-year, and amortizement only once a year.

For example: For a principal of £10,000 reimbursable in 16 years, one pays every year:

(1) £8582 at the annual rate of 4 per cent., the annuity being payable at the end of every year.

(2) £8522,122, payable half-yearly, or £4261,061 every half year, at the rate of 2 per cent. half-yearly.

(3) £8582, at the rate of 2 per cent. half-yearly, when interest is payable half-yearly, and amortizement payable yearly.

Ex. 90. How much is to be paid for a £6000 loan for 6 years, at the rate of $1\frac{1}{2}$ per cent. half-yearly, if interest is payable half-yearly and amortizement only once a year.

$$\begin{aligned} \log 6000 &= 3,77815.13 \\ \log a^n &= 9,26622.58 \\ \log a &= 3,04437.71 \\ a &= £1107,585, \text{ or } £1107.\ 11s.\ 8\tfrac{1}{2}d. \end{aligned}$$

That result is shown to be exact, by the following table:

Year.	Principal at the beginning of the year.	First payment of the year.	Second payment.	
			Total.	Amortizement.
1	£6000	£90	£1017,585	£ 927,585
2	5072,415	76,086	1031,499	955,413
3	4117,002	61,755	1045,830	984,075
4	3132,927	46,994	1060,591	1013,597
5	2119,330	31,790	1075,795	1044,005
6	1075,325	16,130	1091,455	1075,325

The second question is: A given sum being converted into a constant annuity, with a half-yearly interest and an annual amortizement; required the present value of this annuity at any rate of interest, payable half-yearly.

Let V = the principal, converted into annuities,

$\frac{i}{2}$ = the half-yearly interest of £1,

a = the annuity,

$s = a - Vi = \frac{a}{r^n}$, the amortizement, or the surplus of the annuity above the annual interest; and

C = the present value of that annuity at the half-yearly rate of interest r.

The successive payments are:

1st year: first payment	$= \frac{1}{2}(a-s)$;	second payment $= \frac{1}{2}(a+s)$;
2nd	$= \frac{1}{2}(a-rs)$;	..... $= \frac{1}{2}(a+rs)$;
3rd	$= \frac{1}{2}(a-r^2s)$;	..... $= \frac{1}{2}(a+r^2s)$;
n^{th}	$= \frac{1}{2}(a-r^{n-1}s)$;	..... $= \frac{1}{2}(a+r^{n-1}s)$.

Now the present value of the annuity being equal to the sum of the present values, or its several payments, we shall have:

$$C = \frac{a}{2}(\rho^{-1}+\rho^{-2}+\rho^{-3}+\dots\rho^{-2n}) - \frac{s}{2}(\rho^{-1}+r\rho^{-3}+\dots+r^{n-1}\rho^{-2n+1})$$

$$+\frac{s}{2}(\rho^{-2}+r\rho^{-4}+\dots+r^{n-1}\rho^{-2n}),$$

$$\text{or } C = \frac{a}{2r}(1-\rho^{-2n}) - \frac{sr}{2}\cdot\frac{1-r^n\rho^{-2n}}{\rho^2-r},$$

and by substituting for s its value $\frac{a}{r^n}$,

$$C = \frac{a}{2r}(1-\rho^{-2n}) - \frac{ar}{2}\cdot\frac{r^{-n}-\rho^{-2n}}{\rho^2-r}, \text{ required value of the annuity.}$$

The first fractional term of that equation represents the *present value* of the annuity at the given rate, the annuity being supposed payable half-yearly.

(ρ^2-r) represents the *difference* between the amount of £1 after one year at the half-yearly rate r and the amount of £1 at the end of the first year and at the rate of £1 per annum.

$(r^{-n}-\rho^{-2n})$ represents the *difference** of the present value of £1 payable at the end of n years at the annual rate i, and the present value of £1 payable at the end of n years at the half-yearly rate r.

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* Difference $(r^{-n}-\rho^{-2n})$ varies with the number of years. Being first nothing, when $n=0$, it increases to a *maximum* given by the equation

$$n = \frac{\log 2 + \log^2\rho - \log^2 r}{2\log\rho - \log r},$$

Ex. 91. £6000 principal being laid out at 1½ per cent. h yearly, and reimbursable by 6 annuities, interest payable half-ye and amortisement being payable yearly; for how much are th six annuities to be bought, in order to obtain 2½ per cent. half-ye to the purchaser?

$\log 6000 = 3,77815.13$
(Tab. 15) $a^n = 9,26622.58$
$C^t \log 2 = 9,69897.00$
(Tab. 11) $a^n = 1,01105.27$
$\log = 3,75439\,98$
$= £5680,672$
$-63,051$

Value £5617,621

$\log r^{-6} = 9,87131\,36$
$\therefore r^{-6} =$,8374
$\log \rho^{-12} = 9,92297\,67$
$\therefore \rho^{-12} =$,7435
$r^{-6} - \rho^{-12} =$,0939

$\log r^{-6} - \rho^{-12} = 8,97280$
$\log \frac{a}{2} = 1,74335$

$\log r = 8,39794$
$C^t \log (\rho^2 - r) = 1,68561$
$\log = 0,79970$
£63,051.

Thus the present value at 2½ per cent. half-yearly of the p ments necessary for paying up £6000 in six years, at 1½ per c half-yearly, is £5617,621.

Reciprocally, by exchanging £5617,621 for those 12 payme the purchaser makes 2½ per cent. interest half-yearly of his money

The exactness of that result is demonstrated by the follow table:

Half-year.	Principal at the beginning of the half-year.	Half-yearly interest.	Due at the end of the half-year.	Paid up (See page 80).	Due at the end of the half-year.
1	£5617,621	£140,441	£5758,062	£ 90,	£5668,06
2	5668,062	141,702	5809,764	1017,585	4792,17
3	4792,179	119,804	4911,983	76,086	4835,89
4	4835,897	120,897	4956,794	1031,499	3925,29
5	3925,295	98,133	4023,428	61,755	3961,67
6	3961,673	99,042	4060,715	1045,830	3014,88
7	3014,885	75,372	3090,257	46,994	3043,26
8	3043,263	76,082	3119,345	1060,591	2058,75
9	2058,754	51,469	2110,223	31,790	2078,43
10	2078,433	51,961	2130,394	1075,795	1054,59
11	1054,599	26,365	1080,964	16,130	1064,83
12	1064,834	26,621	1091,455	1091,455	

---

then decreases continually until $n = \infty$, when it becomes again =

For $t = 3$ per cent. and $2r = 5\frac{1}{2}$ per cent., $n = \frac{0,26377}{0,011254} = 23,428$ yea

The present value of obligations is ordinarily calculated, admitting both rates of interest and all the payments to be either yearly or half-yearly. The result which is thus obtained is always wrong.

In the foregoing example, for instance, we find as value of the annuity:

£5621,761 when all the payments are supposed to be *yearly*, and

£5617,621 when they are supposed to be *half-yearly*.

It is very useful to know which of these two approximate values is the nearest. As the true value of the annuity is always less than any one of those two approximate values, the question is to know which is *the least*.

The present value at the rate τ of the principal of £ V invested at 1 per cent. and to be reimbursed by *yearly* payments, is

$$V.\frac{\phi(\tau)}{\phi(t)}.$$

Likewise, the present value of the same principal, when rates and all payments are *half-yearly*, is

$$V.\frac{\phi\left(\frac{\tau}{2}\right)}{\phi\left(\frac{t}{2}\right)}.$$

But we have already demonstrated (page 56) that, for a certain value of n, there exists always the following relation for the two rates t and τ (the last being the greatest):

$$\frac{\phi\left(\frac{t}{2}\right)}{\phi(t)}=\frac{\phi\left(\frac{\tau}{2}\right)}{\phi(\tau)},$$

that by *less years*, the first term is always *less* than the second; and that by *more years* it is always *greater*.

Consequently, the last equation being multiplied by $V.\frac{\phi(\tau)}{\phi\left(\frac{t}{2}\right)}$, we obtain for the same value of n the equality,

$$V.\frac{\phi(\tau)}{\phi(t)}=V.\frac{\phi\left(\frac{\tau}{2}\right)}{\phi\left(\frac{t}{2}\right)}.$$

Thus, for that number n of years found by the equation, page 56, the value by whole years is equal to the value by half-years; likewise, for a *less* number of years, the value by whole years is *less* than that by half-years; and for *more* years it is *greater* than that by half-years.

For example, if we require the present value at 5½ per cent. half-yearly of the annuity paid for a £1000 principal laid out at 3 per cent. half-yearly.

By the table given, page 55, it is seen that for these two rates of interest, the relations between annual and half-yearly payments are equal for about 39 years; consequently it does not matter whether we calculate by years or half-years.

But for less than 39 years, yearly payments are less, and more approximated than half-yearly payments, and so are, in their turn, half-yearly payments for more than 39 years.

In order to appreciate the degree of approximation between the *true value* of an annuity, and the two *approximated values* which are obtained by supposing all payments to be yearly or half-yearly, I have subjoined the following table, which shows for several years the three values at 5½ per cent., payable half-yearly, of a nominal principal of £1000 improved at 3 per cent., payable half-yearly.

Years.	True value.	Yearly.	Half-yearly.
1	£975,995	976,303	981,672
3	953,221	953,891	958,783
10	882,358	883,640	886,923
15	839,271	840,814	843,164
20	801,469	803,254	804,861
30	739,587	741,502	742,064
35	714,373	716,265	716,475
38	700,808	702,668	702,707
39	696,526	698,374	698,363
40	692,360	694,193	694,134
50	656,406	658,052	657,651
60	629,098	630,515	629,951
80	592,575	593,725	593,122
99	572,836	573,468	572,971
∞	545,454	545,454	545,454

Ex. 92. A Company issues 14271 obligations of 500 francs, bearing 5 per cent. interest, and to be reimbursed by means of 15 annuities. What sum is the Company to pay per annum, and what is the number of obligations to be reimbursed every year? (*Cordova to Seville Railway.*)

$$\log 14271 = 4,1544544$$
$$\log 500 = 2,6989700$$
$$\log a^n = 8,9338170$$
$$\log a = 5,8372414$$
$$= 687450 \text{ francs, annuity to be paid.}$$

In order to settle the accounts for the payment of the interest and successive reimbursement of the loan, we have

Mean annuity = 687450
Interest of 14271 obligations = 356775
Left for amortizement = 330675 fr.

This sum corresponds to 661,350 obligations of 500 francs, and as the number of bonds to be reimbursed must be a whole number, 561 will have to be reimbursed, which reduce the number of obligations still due to 13610.

Therefore the Company at the end of the first year will pay

for the first half-year of interest = 178387,50
... second ..... = 178387,50
for redemption of 661 obligations = 330500
Total = 687275 fr.

At the end of the *second year:*

Mean annuity = 687450
Interest of 13610 bonds = 340250
left for amortizement = 347200 fr.

This sum corresponds to 694,4 bonds; therefore 694 bonds will have to be reimbursed, which brings their number down to 12916, and the Company will pay for the second year,

for interest of 13610 obligations = 340250
for redemption of 694 ..... = 347000
Total = 687250 fr.

At the end of the *third year:*

Mean annuity = 687450
Interest of 12916 bonds = 322900
left for amortizement = 364550 fr.

which sum corresponds to 729,1 bonds; the Company will therefore re-purchase 729 obligations, which will bring their number down to 12187, and pay for the third year,

for the interest of 12196 bonds = 322900
for the redemption of 729 ..... = 364500
Total = 687400 fr.

At the end of the *fourth year:*

Mean annuity = 687450
Interest of 12187 bonds = 304675
left for amortizement = 382775 fr.

which corresponds to 765,55 bonds; the Company will therefore reimburse 766 obligations, and pay

for the interest of 12187 bonds = 304675
for the redemption of 766 ..... = 383000
Total = 687675 fr.

and so on. The whole of this calculation is shown in the following table:

Years.	Number of obligations.		Annuities.		
	Due.	Reimbursed at the end of the year.	Interest.	Amortizement.	Total.
1	14.271 fr.	661	356.775 fr.	330.500 fr.	687.275 fr.
2	13.610	694	340.250	347.000	687.250
3	12.916	729	322.900	364.500	687.400
4	12.187	766	304.675	383.000	687.675
5	11.421	804	285.525	402.000	687.525
6	10.617	844	265.425	422.000	687.425
7	9.773	886	244.325	443.000	687.325
8	8.887	931	222.175	465.500	687.675
9	7.956	977	198.900	488.500	687.400
10	6.979	1.026	174.475	513.000	687.475
11	5.953	1.077	148.825	538.500	687.325
12	4.876	1.131	121.900	565.500	687.400
13	3.745	1.188	93.625	594.000	687.625
14	2.557	1.247	63.925	623.500	687.425
15	1.310	1.310	32.750	655.000	687.750

It may be observed from the foregoing table that the annuity will ever be somewhat greater or smaller than the mean annuity.

In Sect. 22 we have mentioned this remarkable fact, that the sums paid for amortizement are in a *geometrical progression;* this is likewise applicable to the amortizement of obligations; consequently in the above example, the number of bonds reimbursed every year is *very nearly** in a geometrical progression, similar to the progression of a £1 principal, invested at the same rate of interest.

Ex. 93. A Company issues 100,000 obligations of 500 francs bearing an interest of 15 francs per annum, payable half-yearly, and to be reimbursed in 90 years. How much is this Company to pay at the end of every year for interest and amortizement, and what is the number of bonds to be reimbursed at the end of the first and of the 44th year? (*Austrian State Railways.*)

$\log V = 7{,}6989700$ $\quad \log 100{,}000 + \log a^n = 3{,}50860$

$\log a^n = 8{,}5086046$ $\quad C^t \log r^n = 8{,}84465$

$\log r^{44} = 0{,}56484$

$\log a = 6{,}2075746$ $\quad \log x = 2{,}91809$

$a = 1612778$ fr. $\quad x = 828{,}12$ oblig.

Consequently the Company must pay every year 1612778 francs viz. 1.500.000 for the interest and 112.778 for amortizement.

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* I say *very nearly*, because by redeeming the bonds by whole numbers, the fractions of bonds suppressed accumulating year by year may produce at last a difference of one bond.

The amortizement on the first year is of 226 bonds, and at the end of the 44th year it is of 828 bonds.

Ex. 94. What is the mean rate of interest of an obligation issued at 400 francs, bearing an interest of 20 fr. per annum, and to be reimbursed in 99 years, if it is re-purchased by the Company at 500 fr.? (*Swiss Western Railways.*)

log 500 = 2,69897		this number is found in the table
log a^n = 8,61110		between 5 per cent. and $5\frac{1}{8}$ per cent.
C^t log 400 = 7,39794		5 per cent. = 8,70245
log a = 8.70801		$5\frac{1}{8}$ ..... = 8,71279
		difference = 1034.

The difference between the number just found and the number corresponding to 5 per cent. is 556; the rate is consequently

$$= 5 + \frac{556}{1034} \cdot \tfrac{1}{8} = 5{,}0672 \text{ per cent. nearly,}$$

$$\text{or } r = 2{,}5336 \text{ per cent. half-yearly.}$$

In order to obtain a more exact value, the equation 15 may be divided by $\frac{C}{r}$; we have

$$r = \frac{a}{2C}(1-\rho^{-2n}) - \frac{ar^2}{2C} \cdot \frac{r^{-n}-\rho^{-2n}}{\rho^2-r},$$

and replacing, in this equation, r by the approximated value just found, we obtain

for $r = 0{,}0253$, and $\rho = 1{,}0253$,

log a	= 1,3100664	log $\frac{a}{2}$	= 1,00904
C^t log 2	= 9,6989700	log r^2	= 6.80624
log $(1-\rho^{-2n})$	= 9,9969037	log $(r^{-n}-\rho^{-2n})$	= 8,12995
C^t log C	= 7,3979400	C^t log C	= 7,39794
		C^t log $(r^2-\rho)$	= 1,94923
log	= 8,4038801	log	= 5,29240
	= 0,0253443		0,00001960 7
	− 0,0000196		
	0,0253247.		

Consequently the true rate of the loan is 2,5325 per cent. half-yearly, or 5,0650 per cent. per annum, payable half-yearly.

Ex. 95. A Company issues 100.000 obligations of 500 francs bearing 3 per cent. interest, and to be reimbursed in 99 years; at what price must they be issued in order to return $5\frac{1}{4}$ per cent., and how many bonds will there be left at the beginning of the 75th year? (*Southern of France.*)

$\log . \rho^{-2m} = 7,667719.75$
$\rho^{-2m} = 0,0046472.65$
$1-\rho^{-2m} = 0,9953527335$
$\log = 9,9979770$
$\log 500 = 2,6989700$

$\log a^m = 8,50104371$

$C^t \log 2 = 9,6989700$
$C^t \log C = 1,5606673$
$\log = 2,4576180$
$= 286,83225$ fr.
$- 0,41414$
$286,41811$, or 286 francs, 41¾ centimes.

$\log 100,000 + \log a^{35} = 3,50104$
$\log a^{35} = 8,75912$
$\log = 4,74192$
55198 obligations at the beginning of the year.

$\log . r^{-m} = 8,79311$
$r^{-m} = 0,0$
$\rho^{-2m} = 0,0$
$0,0$
$\log r^{-m} - \rho^{-2m} = 8,68971$

$\log \frac{a}{2} = 0,89856$

$\log r = 8,43933$
$C^t \log (\rho^2 - r) = 1,58912$
$\log = 9,61715$
$0,41414$

Ex. 96. A Company is desirous to contract a loan of 100 millions of francs effective at the rate of 5 per cent. payable half-yearly and issues to that effect some 1000 fr. obligations, bearing an interest of 40 francs, and to be reimbursed in 35 years by annual drawings by lot, so as to give out annually prizes to the first twelve drawn, the amount of which is 180000 francs.

How many bonds are to be issued, and at what price?

Let x = the number of bonds,

K = the present value of 35 annuities of 180,000 fr.,

S = the present value of the 35 annuities for interest and amortizement of a *single* obligation. We have:

$$100.000.000 = xS + K,$$

$$\text{therefore } x = \frac{100\,000.000 - K}{S},$$

$$K = \frac{1-\rho^{-2m}}{\rho^2-1} \times 180.000$$

$\log 90.000 = 4,9542425$
(Tab. II) $\log . \phi^{70} = 1,5171676$
$-Q_2 = -\ 53950$
$\log K = 6,4660151$
$K = 2.924.254$ fr.

$$xS = \frac{100.000.000}{97.075.746}$$

$\log = 7,987.1108$

$\log a = 1,7289810$
$C^t \log 2 = 9,6989700$
$C^t \log a = 1,5171676$
$\log = 2,9451186$
$881,2896$

$\log \frac{a}{2} = 1,42795$
$\log r = 8,39794$
$\log (r^{-m} - \rho^{-2m}) = 8,88002$
$C^t \log (\rho^2 - r) = 1,97367$
$\log = 0,67958$
$4,7817$

$$\begin{array}{rl} & 881{,}2896 \text{ fr.} \\ & \underline{4{,}7817} \\ S & = 876{,}5079 \end{array}$$

$$\begin{array}{rl} \log S & = 2{,}94275.59 \\ \log xS & = 7{,}98711.08 \\ \log x & = 5{,}04435.49 \\ x & = 110.752{,}8 \text{ number of bonds,} \end{array}$$

$$\begin{array}{rl} 8 - \log x & = 2{,}9556451 \\ & 902{,}91125 \text{ fr. price of each bond;} \end{array}$$

therefore the number of obligations is 110.753, and their price 902 fr. 91⅛ cent.

SECT. 51. By reimbursing obligations, the annuity, and consequently the rate of interest for any prize whatever, are constant with regard to the Company, which received a fixed principal for a certain number of invariable annuities.

This would likewise be the case for the *bondholder*, if the price of the bond was *at par*. However as bonds are ever below par, the bondholder received when reimbursed a larger sum than the one he paid; and this difference between the price and the reimbursed sum is to be taken into account in order to value exactly the rate of interest.

It is evident that the interest of a bond is so much greater as the number of years is smaller.

Let V denote the principal of every bond to be reimbursed,

C = the price of the bond,

T = its half-yearly interest,

τ = the *true half-yearly rate* of interest,

and n = the year at the end of which the bond is reimbursed;

the price of the bond being equal to the present value of $2n$ half-yearly payments T, including the sum V payable at the end of n years, we have

$$C = \frac{T}{\tau}(1 - \rho^{-2n}) + V\rho^{-2n},$$

$$\text{whence } \tau = \frac{T\rho^{2n} - T}{C\rho^{2n} - V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

$$\text{and } \rho^{2n} = \frac{V\tau - T}{C\tau - T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17).$$

By means of the formula 16 and of successive corrections we may find the *rate of interest;* the formula 17 may be applied to *the time* at which the bond is to be reimbursed in order to bear a given interest.

Ex. 97. 500 fr. bonds bearing an interest of 15 francs, and to be reimbursed in 90 years, are issued at 275 francs. What is the rate

of interest to be paid by the Company, and what is the r
interest of the bond reimbursed at the end of any year whatev

$$\log 500 = 2,69897 \qquad \frac{15}{275} = 0,054545\ldots$$
$$\log a^n = 8,50860$$
$$C^t \log 275 = 7,56067$$
$$\log = 8,76894$$
$$0,058846;$$

therefore the rate of interest is included between 5,86 and 5,
cent. per annum.

If $\tau = 2,91$ per cent. half-yearly, $C = 275,005$
$\tau = 2,92$ ..... $C = 274,090$
difference = 915,

consequently, the mean rate is $2,91 + \frac{5}{915} \times 0,001 = 2,9100$
cent. half-yearly, or 5,82011 per cent. per annum.

The calculation of the rate of a bond to be reimbursed
time whatever, differs from the calculations in Sect. 30, becau
annuities and a sum payable at a given time are both to be
into account.

If, for instance, it were required to know the rate of int
an obligation to be reimbursed at the end of the 40th year,
by formula 16,

$$V = 500$$
$$C = 275$$
$$T = 7,5 \quad \text{and} \quad \frac{T}{C} = \frac{7,5}{275} = \frac{3}{110} = 0,02727\ldots$$

therefore let $\tau = \frac{3}{110}$ $\qquad V\tau = 13,636\ldots$

$$\rho = \frac{113}{110}$$

$$\log \rho = 0,0116857,$$

the formula 16 becomes $\tau = 0,0272727 + \frac{6,13636}{275\ \rho^{40}}$,

$$\log 6,13636 = 0,78791$$
$$C^t \log 275 = 7,56067$$
$$C^t \log \rho^{40} = 9,06514$$
$$\log = 7,41372$$
$$= 0,0025925,$$

consequently $\tau = 0,0272727 + 0,0025925 = 0,0298652$ near
therefore, to repeat the operation, let

$$\tau = 0,0298652$$
$$\rho = 1,0298652$$
$$\log \rho = 0,0127804$$
and $V\tau = 14,9326,$

the value of r corrected *once* is

$$r = 0{,}0272727 + \frac{7.4326}{275\,\rho^{80}}$$

$$\begin{aligned} \log\ 7{,}4326 &= 0{,}87114 \\ C^{l} \log .\ 275 &= 7{,}56067 \\ C^{l} \log\ \rho^{80} &= 8{,}97757 \\ \log &= 7{,}40938 \\ & \quad 0{,}002567 \end{aligned}$$

or $r = 0{,}027273 + 0{,}002567 = 2{,}9840$ per cent. half-yearly, or 5,968 per annum.

If, at last, it was required to know at what time a bond ought to be reimbursed in order to yield $5\frac{3}{4}$ per cent. per annum, for instance, to the holder, we have:

$$\rho^{2n} = \frac{V_r - T}{C_r - T},$$

$r = 2\frac{7}{8}$ per cent. half-yearly, or $\rho^{2n} = \frac{6{,}875}{0{,}40625} = \frac{220}{13}$

$$\begin{aligned} \log\ 220 &= 2{,}34242 \\ \log\ 13 &= 1{,}11394 \\ \log \frac{220}{13} &= 1{,}22848 \\ \tfrac{1}{2} \log &= 0{,}61424 \\ \log &= 9{,}78834 \\ \text{(Tab. 57)}\ \log^{2}\rho &= 8.09025 \\ \log n &= 1{,}69809 \\ n &= 49{,}90 \text{ years.} \end{aligned}$$

In order to shew better the variety of the rate of interest of the different bonds in the preceding example, I have subjoined the following table:

Years of reimbursement.	Interest per cent. yielded.
1	74,436
2	36,677
7	13,979
10	10,288
15	8,702
20	7,324
25	6,752
30	6,399
40	5,968
46	5,821 (mean rate)
50	5,761
70	5,555
80	5,514
90	5,490

It must be observed that the rate of interest is payable half-yearly, for example, the rate of the obligation reimbursed at the end of the first year is 37,21799 per cent. *half-yearly*, which corresponds to £0,88288 interest for £1 principal at the end of a whole year.

SECT. 52. *Parities, or relative values of Annuities.* The name of *parity*, or relative value of an annuity with equal rate, has been given to the valuation of an annuity at the same rate as another annuity, the value of which is known.

When two annuities are intended for *the same number of years*, their relative values are always proportional to their annual rent. Thus, for instance, if the price of the 3 per cent. consols is £80, the price of the 3½ per cent. will be £105, at the same rate and abstraction made of all external circumstances; for the rate of interest yielded is in both cases 3¾ per cent.

If, on the contrary, both annuities are of a different duration, the rate of interest yielded by the first is determined, and then the *present value* of the other annuity at the same rate.

EX. 98. A perpetuity of £340 costs £10880; what will be the value of a £225 rent for 19 years?

The rate of the perpetuity is $\frac{34000}{10880} = 3\frac{1}{8}$ per cent.; we have now but to calculate the present value of £225 per annum at this same rate:

$$\begin{aligned} \log 225 &= 2,3521825 \\ \log a^{n} &= 8,8487351 \\ \log V &= 3,5034474 \end{aligned}$$

$V = £3187,479$, the required value.

EX. 99. £3249 are paid for a £180 rent to be continued during 38 years; what sum ought to be paid for the same rent if the number of years was doubled?

$$\begin{aligned} \log 180 &= 2,25527 \\ \log 3249 &= 3,51175 \\ \log a^{n} &= 8,74352 \text{ for 38 years;} \end{aligned}$$

this number corresponds to 4½ per cent.; therefore in order to value 76 annuities of £180 at 4½ per cent., we have

$$\begin{aligned} \log 180 &= 2,25527 \\ \log a^{n} &= 8,66880 \\ \log V &= 3,58647 \end{aligned}$$

$V = £3859$, required parity.

SECT. 53. *On Annuities increasing or decreasing in a constant ratio.*

Instead of paying invariable annuities, it may happen that the annuities increase or decrease in a constant ratio, so as to make the difference between each annuity and the preceding one constant, either positive or negative.

Let us then calculate the present value of an annuity for n years, yielding $£a$ for the first year, $£(a+d)$ for the second, $£(a+2d)$ for the third, and so on, the annuity increasing every year of $£d$.

In order to simplify the question, let us denote by

V the present value of the annuity in question,

ϕ the present value of £1 per annum, for n years,

S the present value of an annuity for n years, increasing according to the order of natural numbers, viz. £1 for the first year, £2 for the second year, £3 for the third year, etc.

Thus,

$$S=\frac{1}{r}+\frac{2}{r^2}+\frac{3}{r^3}+\ldots+\frac{n}{r^n};$$

therefore
$$\frac{\phi=\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\ldots+\frac{1}{r^n}}{S-\phi=\frac{1}{r^2}+\frac{2}{r^3}+\ldots+\frac{n-1}{r^n}};$$

$$\text{or } r(S-\phi)=\frac{1}{r}+\frac{2}{r^2}+\ldots+\frac{n-1}{r^{n-1}}=S-\frac{n}{r^n};$$

besides, we have $\phi=\frac{1}{i}(1-r^{-n})$,

$$\text{or } r^{-n}=1-i\phi;$$

$$\text{consequently, } r(S-\phi)=S-n+ni\phi,$$

$$\text{and } S=\left(n+\frac{r}{i}\right).\phi-\frac{n}{i} \quad \ldots\ldots\ldots\ldots (18),$$

present value of an annual rent corresponding to the first n natural numbers.

But
$$V=\frac{a}{r}+\frac{a+d}{r^2}+\frac{a+2d}{r^3}+\ldots+\frac{a+(n-1)d}{r^n}$$

$$=a\left(\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\ldots+\frac{1}{r^n}\right)+d\left(\frac{1}{r^2}+\frac{2}{r^3}+\ldots+\frac{n-1}{r^n}\right)$$

$$=a\phi+d(S-\phi), \text{ and substituting for } S \text{ its value,}$$

$$V=\left(a+nd+\frac{d}{i}\right).\phi-\frac{dn}{i} \quad \ldots\ldots\ldots\ldots\ldots (19).$$

If the annuity is *decreasing*, that is, if *£d less* are paid every year, the sign of d requires only to be changed.

Ex. 100. What is the present value at 4½ per cent. of an increasing annuity during 9 years, £1 the first year, £2 the second year, £3 the third, &c.

Here we have given $r=1{,}045$ and $n=9$; therefore, by formula 18,

$$S=\left(9+\frac{1,045}{0,045}\right)\phi-\frac{9}{0,045}=\frac{290}{9}\cdot\phi-200,$$

$$\begin{aligned}
&\log 290 = 2,4623980\\
C'\ &\log 9 = 9,0457575\\
&\log \phi^9 = 0,8614621\\
&\log = 2,3696176\\
&S+200 = 234,21658\\
&S = £34,217.
\end{aligned}$$

Ex. 101. Required the present value at 4½ per cent. of an increasing annuity for 9 years, £1800 for the first year, and £100 more every consecutive year.

$$V=\frac{44300}{9}\phi-20.000,$$

$$\begin{aligned}
&\log 44300 = 4,6460437\\
C'\ &\log 9 = 9,0457575\\
&\log \phi^9 = 0,8614621\\
&\log = 4,5532633\\
&V+20000 = 35778,595\\
&V = £15778,595.
\end{aligned}$$

Ex. 102. Required the present value at 3½ per cent. of a decreasing rent for 19 years, the first of £3330, the last of £2070, and generally decreasing by £70 per annum.

In this case we have given $a=3330$, $d=-70$, $n=19$, and $r=1,035$; therefore, by formula 19,

$$V=\frac{19\times 70}{0,035}=£38.000, \text{ required value.}$$

We may here notice that the present value of the 19 annuities is a whole number.

Ex. 103. Required the present value of a temporary *Life Annuity* for 19 years, held on a single life aged 20 years, at 5 per cent according to the *Northampton Table*.

According to this table (see Inwood, page 145), on 11650 new born infants there are 5132 survivors at the beginning of the 20th year, 5060 at 21 years, 4985 at 22, and every year 75 deaths until the end of the 38th year, when there are still 3710 individuals aged 39 years.

Consequently if a Life Insurance Company insures to 5132 individuals, aged 20 years, a life annuity of £1 during 19 years, it must pay according to the observations made at Northampton, £5060 at the end of the *first* year, to as many survivors, £4985 at the end of the second year, £4910 at the end of the third, &c., and £3710 at the end of the last year to the 3710 survivors, aged 39 years.

The present value of all these sums is equal to the value of 5132 annuities of £1 during 19 years, held on single lives aged 20 years

The question is thereby reduced to this: how to calculate the 132nd of the present value of a decreasing annuity for 19 years, 5060 for the first year, £75 less every year, and £3710 for the last payment.

Consequently

$$a=5060,\ d=-75,\ n=19,\ \text{and}\ t=1{,}05,$$

$$a+nd+\frac{d}{t}=5060-1425-1500=2135,$$

$$\text{and}\quad V=\frac{2135}{5132}\cdot\phi+\frac{28500}{5132},$$

$$\begin{aligned}
&\log 2135 = 3{,}3293979\\
C^t\log 5132 &= 6{,}2897134\\
\log \phi^{19} &= 1{,}0822582\\
\log\ \ &= \overline{0{,}7013695}\\
&\qquad 5{,}027701
\end{aligned}$$

$$\frac{28500}{5132}=5{,}553391$$

$$V=£10{,}581092 \text{ value of the life annuity.}$$

Ex. 104. A debt of £1.700.000 rents 3½ per cent. is to be redeemed in 15 years at the mean price of $£82\frac{6}{17}$, and by annuities which decrease by £10.000 every year. What is to be the first payment?

The formula 19 gives at once,

$$a=\frac{Vt+nd}{t\phi}-\left(nd+\frac{d}{t}\right),$$

£3½ rent at $82\frac{6}{17}$ corresponding to 4¼ per cent.;

consequently

$$Vt=1.700.000,\ d=-10.000,\ n=15,\ \text{and}\ t=0{,}0425,$$

$$\begin{aligned}
Vt &= 1.700.000\\
nd &= \underline{\ \ 150.000}\\
Vt+nd &= 1.850.000\\
\log &= 6{,}2671717\\
C^t\log t &= 11{,}3716111\\
\log a^{15} &= 8{,}9615180\\
\log &= \overline{6{,}6003008}\\
&= £3983830\\
&\quad -£385294\\
a &= £3598536 \text{ first payment.}
\end{aligned}$$

SECT. 54. *Annuities increasing or decreasing in a geometrical progression.*

These annuities are not to be found in the usual practice of business; and if such was the case, it could be but very seldom. In

order however to leave no side of the question untouched, I will shew the method of computing them.

Let a = the first annuity,

e = the relation between two consecutive annuities, or the constant ratio of the geometrical progression,

i = the rate of interest,

and V = the present value of this annuity.

Then

$$V = \frac{a}{r} + \frac{ae}{r^2} + \frac{ae^2}{r^3} + \ldots + \frac{ae^{n-1}}{r^n},$$

and

$$\frac{Ve}{r} = \frac{ae}{r^2} + \frac{ae^2}{r^3} + \ldots + \frac{ae^n}{r^{n+1}},$$

the difference between these two equations is

$$V\left(1 - \frac{e}{r}\right) = \frac{a}{r} - \frac{ae^n}{r^{n+1}},$$

whence

$$V = \frac{a}{r-e}\left(1 - \frac{e^n}{r^n}\right).$$

Ex. 105. What is the present value of 18 annuities, the first of which is £1000, the second £1010, the third £1020,100, the fourth £1030,301, &c., so as to make every annuity equal to the preceding one increased by 1 per cent. of its value, the rate of interest being 4 per cent.?

The constant ratio is $= 1{,}01$;

$$\text{therefore } V = \frac{1000}{0{,}03}\left\{1 - \left(\frac{1{,}01}{1{,}04}\right)^{18}\right\}$$

$$\log 1{,}01^{18} = 0{,}0777847$$
$$\log 1{,}04^{18} = 0{,}3066001$$
$$\overline{9{,}7711846}$$
$$0{,}5904520$$
$$0{,}4095480$$
$$V = £13651.\ 12s.$$

Ex. 106. What is the present value at 5 per cent. of an increasing *perpetuity*, the first annuity of which is £1000, the second £1010, and likewise in the foregoing example, every annuity of which is equal to 1,01 of the preceding one?

In this case n is infinite, consequently the power of the fraction $\frac{e}{r}$ becomes infinitely small, and

$$V = \frac{a}{r-e} = \frac{1000}{0{,}04} = £25.000.$$

It ought to be noticed that the payments are at last infinitely great. Already at the end of the

695th year more than 1 million should be paid.
1389th ..... 1 billion .....
2083rd ..... 1 trillion .....

and all these values at the end of a *limited* number of years become greater than any number however great it may be; nevertheless the present value of these infinitely great payments in infinite number is exactly = £25.000.

FOURTH PART.

DEFERRED ANNUITIES.

OUR investigations with regard to immediate annuities are directly applicable to any other annuities, whether deferred or with entrance.

If V denote the present value of an immediate annuity, the value of the same annuity, whether *deferred* or with *entrance*, will be

$$= Vr^m,$$

in which the exponent is *positive* when the annuity is with entrance, and *negative* when it is deferred.

Consequently the value of any annuity whatever is found by multiplying or dividing V by the amount of £1 at the time of entrance or reversion.

SECT. 55. *Present Value of a Deferred Annuity.* Its present value is equal to the sum of the present values of every single payment, that is, the sum which put out at interest will provide all these several payments of the annuity as they become due.

Let a = the annuity,

r = the amount of £1 at the end of a year,

n = the number of annual payments,

d = the number of years during which the annuity is deferred,

and D = the present value of the deferred annuity.

Then the first annuity of £a is payable at the end of $(d+1)$ years, and its present value is $ar^{-(d+1)}$; the *second* annuity is payable at the end of $(d+2)$ years, and its present value is $ar^{-(d+2)}$; the *third* annuity is payable at the end of $(d+3)$ years, and its present value is $ar^{-(d+3)}$, &c.

Whence it follows that the sum of all the present values will be

$$D = ar^{-(d+1)} + ar^{-(d+2)} + \ldots + ar^{-(d+n)},$$

Multiplying by r we have

$$rD = ar^{-d} + ar^{-(d+1)} + \ldots + ar^{-(d+n-1)}.$$

Subtracting the first equation from the second, we have

$$iD = ar^{-d} - ar^{-(d+n)};$$

whence

$$D = \frac{a}{i}\{r^{-d} - r^{-(d+n)}\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

SECT. 56. The present value of the deferred annuity which we have just obtained may be written out as follows:

$$D = \frac{a}{i}\{1 - r^{-(d+n)}\} - \frac{a}{i}(1 - r^{-d});$$

therefore *the present value of a deferred annuity is equal to the difference between two immediate annuities of the same yearly income, one for the whole term, the other to continue until the time of entering on the deferred annuity.*

Thus, the present value at 3½ per cent. of an annuity of 15 years, after the expiration of 12 years, is equal to an immediate annuity of £1 for 27 years, minus an immediate annuity of £1 for 12 years:

present value for 27 years = £17,285
..... 12 ... = 9,663
Deferred annuity = £7,622.

Formula 20 may be again written out as follows:

$$D = \frac{a}{ir^d}(1 - r^{-n});$$

and if we denote by ϕ the present value of £1 per annum for n years, or

$$\phi = \frac{1}{i}(1 - r^{-n}),$$

we have

$$D = \frac{a\phi}{r^n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21).$$

If we denote by D the deferred annuity, and by V the same annuity but immediate, the value of the deferred annuity will be

$$D = \frac{V}{r^d},$$

that is, *the present value of a deferred annuity is equal to the present value of the same annuity immediate, discounted for the number of years during which the annuity is deferred.*

Thus the present value of £1 per annum for 15 years, after the expiration of 12 years, is equal to the present value of the *immediate annuity* of £1 for 15 years discounted for 12 years, or

(Tab. 19) $\log \phi = ,06135$
$\log r^d = ,17928$
$\log D = ,88207$
$D = £7,622$ as above.

SECT. 57. **Reversions when interest and annuity are payable yearly.*

Ex. 107. What annuity to continue 19 years after the expiration of 7 years will £68.000 purchase, computing at the rate of $3\frac{5}{8}$ per cent. per annum?

$\log 68.000 = 4,8325089$
(Tab. 20) $\log a^n = 8,8676632$
$\log r^7 = 0,1082518$
$\log a = 3,8084239$
$a = £6433,153.$

Ex. 108. If the reversion of an estate for 21 years after the next 3 years 7 months costs £2450, what rent should it produce in order that the purchaser may make $4\frac{1}{4}$ per cent. out of his money?

$\log 2450 = 3,38917$
$\log a^{21} = 8,86291$
$\log r^3 = 0,05423$
$\log r^{\frac{7}{12}} = 0,01054$
$\log a = 2,31685$
$a = £207,420.$

Ex. 109. What sum ought to be given for a deferred annuity of £3600 per annum for 27 years after the next 5 years, in order that the purchaser may make $5\frac{1}{4}$ per cent. of his money?

$\log a = 3,5563025$
(Tab. 33) $\log \phi = 1,1542120$
$C^t \log r^5 = 9,8888895$
$\log D = 4,5994040$
$D = £39756,120.$

Ex. 110. Required the present value at $3\frac{5}{8}$ per cent. of an annuity of £718 for 62 years, the first annuity being payable at the end of 5 years and 9 months.

If the annuity were immediate, the first payment should be made at the end of *one* year, therefore the annuity is deferred during 4 years and 9 months.

$\log 718 = 2,8561244$
(Tab. 22) $\log \phi^{62} = 1,3685244$
$C^t \log r^4 = 9,9339559$
(Tab. 53) $C^t \log r^{\frac{9}{12}} = 9,9876167$
$\log D = 4,1462214$
$D = £14.003,010.$

Ex. 111. If an annuity of £100 per annum for 15 years be purchased for £1000, interest being reckoned at $5\frac{1}{8}$ per cent., how long will it be before the annuity commences?

* See explanatory note, p. xiv.

We shall make use in this case of the formula 21, from which may be derived

$$r^d = \frac{a\phi}{V}$$

$$\log \frac{a\phi}{V} = 0{,}01252$$

(Tab. 57)
$$\log^2 = 8{,}09760$$
$$\log^2 r = 8{,}33658$$
$$\log d = 9{,}76102$$
$$d = 0{,}5768 \text{ years or } 211 \text{ days};$$

consequently the first payment will take place at the end of 211 days.

Ex. 112. An annuity of £140 for 22 years, after the expiration of 7 years 5 months, is to be exchanged against an immediate annuity for 30 years. How much is to be paid per annum at 3½ per cent.?

$$\log 140 = 2{,}14613$$
(Tab. 17 and 52)
$$\log \phi^{22} = 1{,}19159$$
$$C^t \log r^7 = 9{,}90277$$
$$C^t \log r^{\frac{5}{12}} = 9{,}99421$$
$$\log a^{30} = 8{,}72166$$
$$\log a = 1{,}95636$$
$$a = £90{,}440.$$

Ex. 113. A has a term of 9 years in an estate worth £316 per annum; B has a term of 18 years in the same estate in reversion after the expiration of the 9 years, and C has a further term of 27 years in reversion after the 27 years: it is required to know the present value of each separate term, computing at the rate of 4 per cent.

A	B	C
$\log 316 = 2{,}4996871$	$\log 316 = 2{,}4996871$	2,4996871
$\log a^9 = 9{,}1286997$	$\log \phi^{18} = 1{,}1024096$	1,2129752
3,3709874	$C^t \log r^9 = 9{,}8466999$	9,5400998
£2349,565	3,4487966	3,2527621
	£2810,585	£1789,625.

The present value of the different terms is

$$A = £2349{,}565$$
$$B = 2810{,}585$$
$$C = 1789{,}625$$
$$\text{Total} = 6949{,}775.$$

In order to verify this result, we have

$$\log 316 = 2{,}4996871$$
$$\log a^{34} = 8{,}6577164$$
$$\log V = 3{,}8419707$$
$$V = £6949{,}775, \text{ as above.}$$

SECT. 58. *We have hitherto supposed the interest and annuity to be payable once a year; in any other case it will be easy to apply the formula

$$D=\frac{V}{r^{\lambda}},$$

that is, to find the present value of the same immediate annuity according to the rules shewn in Sect. 36, and then divide it by the amount of £1 for the time of reversion.

If the interest and annuity are payable at the *same periods of time*, ν times per annum, the annuity will be equal to the *yearly rent*, the rate of interest and annuity of which are ν times smaller, and the intervals of payments of which are ν times greater, that is, λ whole years.

At last, when the periods for paying the annuity and for paying the interest are not equal to each other, the value of the annuity, the annual payments of which correspond with the *rate of interest*, are multiplied by a constant factor independent of the number of years, and the logarithms of which have been given in Table 58 for the usual cases.

According therefore to Sect. 37, when the annuity is paid *more frequently* than the interest, the *annual rent decreases*, or the purchase-money increases; consequently the logarithm (P) of the constant ratio will be *subtracted* from the logarithm of the annuity.

If, on the contrary, the annuity is paid *less often* than the interest, the annuity *increases*, or the purchase-money decreases; consequently the logarithm (Q) of the constant ratio is to be *added* to the logarithm of the annual rent.

Ex. 114. *What annuity to continue 30 years is to be given for £880, when the interest at 3½ per cent. and annuity are payable *half-yearly*, and the first half-yearly payment is to be made in 7 years?

If the rent was immediate, the first half-yearly payment should be made at the end of half a year, consequently the rent is deferred for 6½ years, and is equal to an annual rent for 60 years, deferred for 13 years, and at the annual rate 1¾ per cent.

Consequently

$$\log 880 = 2{,}94448$$
$$\log a^{60} = 8{,}43222$$
$$\log r^{13} = \underline{0{,}09795}$$
$$\log a = 1{,}47465$$
$$a = £29{,}830 \text{ half-yearly rent, or}$$
$$£59{,}660 \text{ per annum.}$$

Ex. 115. *What should the annuity be in the last example if it was payable *monthly*?

Here, the annuity is paid 6 times more frequently than the interest, therefore the annuity ought to be less than in the preceding, and log P_6 should be subtracted.

* See explanatory note, p. xiv.

$$\begin{array}{rl} & ,47465 \\ -P_6= & 314 \\ \hline \log a= & ,47151 \end{array}$$

$a = £29,615$ half-yearly, or £4,936 monthly.

Ex. 116. *Present value of a reversion of £9700 per annum in 33 years, payable *quarterly*, and the first payment of which is to take place in 5 years, the rate of interest being 5½ per cent. payable yearly.†

The reversion being

$$\begin{array}{rl} \log 7900 = & 3,8976271 \\ \log \phi = & 1,1782593 \\ C^t \log r^4 = & 9,9069902 \\ C^t \log r^8 = & 9,9825607 \\ +P_4 = & \quad 87683 \\ \hline & =4,9742056 \end{array}$$

$V = £94.233,570$ required value.

Calculation of the Rate of Reversions.

Sect. 59. *Valuation of the rate by means of interpolation.* It is always very easy with our tables to find the rate with a very great approximation. The purchase-money has only to be calculated for two rates, between which is included the rate in question, and then the latter is to be found by means of interpolation; the logarithm of the annuity may be sufficient without looking for the numerical value.

If the simple interpolation of the first degree is resorted to, the rate is *too great*, but by a rather small quantity.

Ex. 117. An annuity of £250, to continue 37 years, after the expiration of 6 years is purchased by £3700. What is the rate of interest made by the purchase-money?

at 4 per cent.	at 4½ per cent.
$\log 250 = 2,39794$	2,39794
$\log \phi^{37} = 1,28200$	1,27438
$C^t \log r^6 = 9,89780$	9,89467
$\log \quad = 3,57774$	3,56699

difference for ½ per cent. = 1075.

But $\log 3700 = ,56820$; the difference with the 4 per cent. is 954, consequently the rate obtained is $= 4 + \frac{954}{1075} \times \frac{1}{2}$

$= 4,1109$ per cent.

The error is less than $\frac{1}{10000}$ per cent.

Ex. 118. *If £2024. 14*s.* 2*d.* be paid for an annuity of £187, payable *half-yearly*, for 23 years after 6 years and 5 months; what is the rate of interest allowed, the interest being convertible *once a year* only?

* See explanatory note, p. xiv.
† See page 128.

$4\frac{3}{8}$ per cent.	$4\frac{1}{2}$ per cent.
$\log 187 = 2,27184$	2,27184
$\log \phi^{23} = 1,15595$	1,15069
$C^t \log r^6 = 9,88842$	9,88530
$C^t \log r^{1\frac{5}{12}} = 9,99225$	9,99203
$+ P_2 = 0,00467$	0,00481
$\log = 3,31313$	3,30467,

difference for $\frac{1}{8}$ per cent. = 846,

$\log 2024,708 = ,30636,$

difference with the $4\frac{3}{8} = 667$,

$$\text{rate of interest} = 4\tfrac{3}{8} + \frac{667}{846} \cdot \tfrac{1}{8} = 4,47503 \text{ per cent.},$$

or £4. 9*s*. 6*d*. for £100.

The error is less than $\frac{1}{30000}$ per cent.

SECT. 60. *General formula for finding the rate of reversions.*

If in formula 20 we develope the two powers, by means of the Binomial Theorem, into a series ascending by the powers of t, we shall have

$$r^{-d} = (1+t)^{-d} = 1 - dt + d.(d+1)\frac{t^2}{2} - d(d+1)(d+2)\frac{t^3}{6} + \ldots$$

$$r^{-(d+n)} = 1 - (d+n)\,t + (d+n)(d+n+1).\frac{t^2}{2}$$

$$- (d+n)(d+n+1)(d+n+2)\frac{t^3}{6} + \ldots$$

and by subtracting

$$\frac{Dt}{a} = nt - (2d+n+1)\frac{nt^2}{2} + (3d^2+3dn+n^2+6d+3n+2).\frac{nt^3}{6},$$

or dividing by nt,

$$\frac{D}{an} = 1 - (2d+n+1)\frac{t}{2} + (3d^2+3dn+n^2+6d+3n+2)\frac{t^2}{6}.$$

Now if we raise both sides of this equation to the power

$$-\frac{2}{2d+n+1},$$

we shall obtain by means of the Polynomial theorem,

$$\left(\frac{D}{an}\right)^{-\frac{2}{2d+n+1}} = 1 + t - \frac{n^2-1}{12(2d+n+1)}.t^2,$$

and by assuming $$q = \left(\frac{an}{D}\right)^{\frac{2}{2d+n+1}} - 1,$$

we obtain $$q = t - \frac{n^2-1}{12(2d+n+1)}.t^2,$$

an equation of the second degree, of which the approximated root will be (according to note, page 46)

$$t=\frac{12(2d+n+1)-q(n^2-1)}{12(2d+n+1)-2q(n^2-1)}\cdot q \quad \ldots\ldots\ldots(22).$$

Ex. 119. It is required to determine at what rate of interest an annuity of £40, to continue 25 years after the expiration of the next 5 years, may be purchased for £441,719.

Here

$$a=40,\quad d=5,\quad n=25,\quad n^2-1=624,$$
$$an=1000,\quad 2d+n+1=36,$$

$\log an = 3,$

$\log D = 2,6451461$

$\log \frac{an}{D} = 0,3548539$

$\log \sqrt[18]{\ } = 0,0197141$

$q = 0,046439$

$\log q = 8,66689$

$\log .624 = 2,79518$

$1,46207$

$624q = 28,978.$

$$t=\frac{432-28,978}{432-57,956}\cdot q=\frac{403,022}{374,044}\cdot q$$

$\log ,046439 = 8,66689$

$\log 403,022 = 2,60533$

$C^t \log 374,044 = 8,42708$

$\log t = 9,69930$

$t = 5,0038$ per cent. (true rate 5 per cent.)

Sect. 61. *Formula for correcting.* Whenever by means of our tables, or by means of the last formula, an approximate value has been obtained, this may be made use of in order to determine the rate with the greatest accuracy.

Let us denote by r such approximate value of the *true* rate i, and put down

$$t=r+x,$$
$$\rho=1+r,$$

and

$$r=\rho+x,$$

then the equation 20 will become

$$\frac{D}{a}(r+x)=(\rho+x)^{-d}-(\rho+x)^{-(d+n)}.$$

Now, if we expand the powers of $(\rho+x)$ by means of the Binomial theorem, and neglect the terms into which enter the second and higher powers of x, as inconsiderable when compared with the rest, we have

$$\frac{D}{a}(r+x)=\rho^{-d}-dx\rho^{-(d+1)}-\rho^{-(d+n)}+(d+n)x\rho^{-(d+n+1)};$$

whence

$$x = \frac{\rho^{-s} - \rho^{-(d+n)} - \frac{Dr}{a}}{\frac{D}{a} + d\rho^{-(d+1)} - (d+n)\rho^{-(d+n+1)}} \quad \ldots\ldots\ldots(23).$$

In the Example 117 we have found the value of the rate to be nearly $4\frac{1}{2}$ per cent.; consequently

$\log \rho = 0,01749.70814$

$\rho^{-s} = 0,78526.73$	$\frac{D}{a} = 14,8$
$-\rho^{-43} = -0,17685.83$	$6\rho^{-7} = 4,52555$
$-\frac{Dr}{a} = -0,60844.44$	$43\rho^{-44} = -7,30461$
Numerator $= -0,00003.54$	Denominator $= 12,01094$;

whence

$$x = -\frac{0,0000354}{12,021} = -0,000002945,$$

and $i = 0,04110.8166$ true value.

Deferred Perpetuities.

SECT. 62. When the deferred annuity is a *perpetuity*, n is infinite and the general equation 20 becomes

$$D = \frac{a}{ir^d};$$

whence we deduce the following formulæ:

$$a = Dir^d,$$

and $$r^d = \frac{a}{Di}.$$

EX. 120. *Required the present value of the perpetuity of £418 per annum, to be received after 6 years and 5 months, at $3\frac{1}{2}$ per cent., interest and annuity being payable half-yearly.

The true rate is $1\frac{3}{4}$ per cent. half-yearly, each half-yearly payment is £209, therefore

$$\log \frac{418}{2} = 2,3201463$$

(Tab. 5) $\log a^{perp.} = 8,2430380$

$\log r^{12} = 0,0904130$

(Tab. 53) $\log r^{\frac{1}{2}} = 0,0062787$

$\log D = 3,9804166$

$D = £9,9559,091.$

* See explanatory note, p. xiv.

Ex. 121. If a person purchase the reversion of a fee si after 3 years 8 months, for £5000, what *quarterly* rent shou produce in order that he make 4½ per cent. of his money?

$$\begin{aligned}
\log 5000 &= 3,69897\\
\log \phi^{\text{years}} &= 8,65321\\
\log r^3 &= 0,05735\\
\log r^{\frac{1}{12}} &= 0,01274\\
-P_4 &= 0,00720\\
\log a &= 2,41507\\
a &= £260,060 \text{ per annum,}\\
&\text{or } £65,015 \text{ quarterly.}
\end{aligned}$$

Ex. 122. The reversion of an estate in fee simple prod £10 *monthly* is made over to the discharge of a debt of £2200; soon ought the creditor to take possession, if he be allowed 5 cent. interest per annum for his debt?

$$\begin{aligned}
\log 120 &= 2,07918\\
\mathcal{L}^t \log 2200 &= 6,65758\\
\log \phi^{\text{years}} &= 1,27984\\
+P_{12} &= 0,01023\\
&\quad 0,02683\\
\log^2 &= 8,42862\\
\log^2 r &= 8,34679\\
\log d &= 0,08183\\
d &= 1,2073 \text{ years, or 1 year } 75\tfrac{1}{2} \text{ day}
\end{aligned}$$

Ex. 123. A perpetuity on an estate of which the *clear* i is £540 per annum, is to be divided by reversion into 3 shares; how long will these three shareholders partake of the interest being 4 per cent. per annum?

Here

$$a = £540, \quad t = 0,04,$$

$$\text{value of the perpetuity} = £13500,$$

$$\text{value of each part} = £4500.$$

Now if we assume that the first shareholder partakes of thi during d years, then hands it over to the second during n and this one to the third, *ad perpetuum*, we shall have

$$4500 = \frac{540}{t}(1 - r^{-d}), \quad \text{and } 4500 = \frac{540}{tr^d}(1 - r^{-n}),$$

whence

$$\begin{aligned}
rd &= 1\tfrac{1}{3} & r^n &= 2\\
\log &= 0,17609 & \log &= 0,30103\\
\log^2 &= 9,24573 & \log^2 &= 9,47861\\
\log^2 r &= 8,23130 & \log^2 r &= 8,23130\\
\log d &= 1,01443 & \log n &= 1,24731\\
d &= 10,338 \text{ years,} & n &= 17,673 \text{ years.}
\end{aligned}$$

Consequently the first will partake of the rent

during 10 years 123 days

then after the second during 17 ... 246 ...

then after 28 ... 4 ...

he third will enjoy it for ever.

Ex. 124. If £5565. 8*s.* 9*d.* have been paid for the reversion of a perpetuity of £347 after 7 years, what rate of interest does the purchaser make of his money?

Present value at $4\frac{1}{2}$.	Present value at $4\frac{5}{8}$.
log 347 = 2,54033	2,54033
log r^7 = 0,13381	0,13745
log $a^{perp.}$ = 8,65321	8,66511
log = 3,75331	3,73777
£5666,3	£5467,2

difference for $\frac{1}{8}$ per cent. = 199, 1 ;

$$\text{therefore } t = 4\tfrac{1}{2} + \tfrac{1}{8} \cdot \frac{100,9}{199,1} = 4\tfrac{9}{16} \text{ per cent.}$$

The interpolation by means of proportional parts always gives a rate rather too high; in order to obtain it with the greatest accuracy, formula page 22 for the interpolation of the second degree is to be made use of; otherwise the formula for correcting annuities in reversion (Sect. 61) is to be applied.

As in this case the reversion is a perpetuity, n becomes *infinite*, and the general formula becomes

$$x = \frac{\rho^{-d} - \frac{D\tau}{a}}{\frac{D}{a} + d\rho^{-(d+1)}}.$$

Renewal of Leases.

Sect. 63. The *renewal of leases* is but a particular case of the deferred rents. The principal question is to determine the sum or *fine* which ought to be given for the renewal of a certain number of years elapsed in a lease.

The most usual number of years for which colleges, private companies and corporations, grant their leases is 10, 20, 21, 40 and 50 years.

The question then is to determine the present value of an annuity deferred for the unexpired term of the lease and then to continue for a period equal to the expired time.

Ex. 125. Required the *fine* which a tenant ought to give for the renewal of 9 years, lapsed in his lease of 21 years, supposing the

estate to produce a *clear* improved rent of £310, after dedu reserved or quit-rent, and all taxes and other annual charges being at 4 per cent.

$$\log 310 = 2,49136$$
$$\log \phi^{90} = 0,87130$$
$$\log r^{-12} = \underline{9,79560}$$
$$\log D = 3,15826$$
$$D = £1439\tfrac{3}{5}.$$

Ex. 126. A tenant has the lease of an estate for 40 which 17 years are expired; how much is to be paid for the of his lease, the *annual produce* of the estate, clear of all de being £320, the rate of interest being $5\frac{1}{2}$ per cent. per payable half-yearly?

$$\log 160 = 2,2041200$$
$$\log a^{64} = \underline{8,6594289}$$
$$\log r^{16} = 0,5419642$$
$$\log Q_2 = \underline{0,0029756}$$
$$\log D = 2,9997513$$
$$D = £999,4275, \text{ or } £999.\ 8s.\ 6\tfrac{1}{2}$$

Ex. 127. Supposing a lease of £118, the clear produce per annum has been granted for 33 years at £2250; wha fine to be paid upon the same terms, in order to renew expired of it?

This problem requires two calculations; the first for o the rate of interest of the lease, the second for the present the renewal.

$$\log 118 = 2,07188$$
$$\log 2250 = \underline{3,35218}$$
$$\log a^n = 8,71970.$$

This logarithm, which corresponds with 33 years, is foun tables to be exactly $3\frac{3}{8}$ per cent.

Consequently,

$$\log 118 = 2,07188$$
$$\log a^{14} = \underline{8,96539}$$
$$\log r^{19} = 0,29383$$
$$\log D = 2,81266$$
$$D = £649,620, \text{ the fine required}$$

FIFTH PART.

ANNUITIES WITH ENTRANCE.

ALL annuities with entrance may be represented by

$$Vr^m,$$

in which V denotes the present value of the same immediate annuity, and r^m the amount of £1 from the time of entrance.

These annuities may be divided into three distinct classes:

(1) *Amount of annuities in arrear*, or amount of annuities which being payable or becoming due *at the end* of every year, remain unpaid any number of years, and improved annually at compound interest.

(2) Amount of annual investments made *at the beginning* of every year.

(3) Present value of annuities *with growing dividend.*

I.

Amount of Annuities in arrear.

SECT. 64. In order to find the amount of an annuity laid up at the end of every year and improved at compound interest, the amount of each year's annuity is determined, supposing every one to have been improved at compound interest from the time of its becoming due.

Let a = the annuity, or yearly rent,
n = the number of years or the number of annuities,
S = the amount of the annuities.

Now the first payment of £a having been $(n-1)$ years ago, the second $(n-2)$ years ago, the third $(n-3)$ years ago, and so forth, the amount of these payments will be,

$$\text{for the first} = ar^{n-1},$$
$$\ldots\ldots \text{ second} = ar^{n-2},$$
$$\ldots\ldots \text{ third} = ar^{n-3},$$

and so forth, until the last or n^{th}, which being just due, its amount will be £a.

Consequently the sum of all these values for n years will be the amount of the annuity for n years,

$$\text{or } S = ar^{n-1} + ar^{n-2} + ar^{n-3} + \ldots + a,$$
$$\text{and } rS = ar^n + ar^{n-1} + ar^{n-2} + \ldots + ar.$$

Subtracting the first series from the second,

$$iS = ar^n - a,$$

whence

$$S = \frac{a}{i}(r^n - 1) \quad \ldots\ldots (24),$$

$$\text{and } r^n = 1 + \frac{iS}{a} \quad \ldots\ldots (25).$$

Now formula 24 may be written as follows:

$$S = \frac{ar^n}{i}(1 - r^{-n}),$$

and if ϕ denotes the present value of £1 for n years, (page 17),

$$\phi = \frac{1}{i}(1 - r^{-n}),$$

the above formula will become

$$S = ar^n\phi, \quad \ldots\ldots (26),$$

amount of n annuities of £a laid up at the end of every year.

SECT. 65. *It would be useless to repeat the calculations made in Sect. 37, when the annuity and interest are payable several times per annum.

If the annuity and interest are payable *half-yearly*, take half the annuity and rate of interest and then double the periods.

Generally, if both are payable at *the same periods*, ν times per annum and at ν equal intervals, *divide* the annuity and rate by the number ν, and *multiply* the number of payments by ν. The amount increases according to the increasing number of payments per annum, and it reaches its *maximum* when ν is infinite, that is, when the payments of annuity and interest are supposed to be made instantly.

Thus, for instance, the amount of £1 per annum for 25 years, and at 5 per cent. when the annuity and interest are payable, is,

yearly	= £47,72719
half-yearly	= 48,74217
quarterly	= 49,26809
monthly	= 49,62580
instantly	= 49,80686.

Finally, when the periods for paying the annuity and interest are not equal to each other, the indications given with regard to immediate annuities should be made use of.

If the annuity is paid *more frequently* than the interest, the amount is first computed supposing the annuity to be paid at the same periods as the interest, then multiply the result by the constant ratio P (page 63),

$$S = ar^n\phi P \quad \ldots\ldots (26^a)$$

* See explanatory note, p. xiv.

*When, on the contrary, the annuity is paid *less frequently* than the interest, *divide* the amount which corresponds to the case of the annuity according to the interest by a constant ratio Q (page 63)†,

$$S = \frac{ar^n\phi}{Q}.$$

Ex. 128. If £1000 yearly rent be forborn 82 years, what will it amount to, at 5½ per cent. per annum?

Here: $a = 1000$ $t = 5\frac{1}{2}$ $n = 82$

$r = 1{,}055$

$\log 1000 = 3,$
$\log r^{82} = 1{,}9067017$
$\log \phi^{82} = 1{,}2542200$
$\log S = \overline{6{,}1609217}$
$S = £1{,}448{,}570.67.$

Ex. 129. To what sum will a half-yearly rent of £259 amount in 37 years, when laid up and improved at compound interest at the end of every six months; the rate being 4½ per cent. payable *half-yearly*?

$\log 259 = 2{,}41329.98$
(Tab. 9) $\log \phi = 1{,}55484.46$
$\log r^{74} = 0{,}71508.54$
$\log S = 4{,}68322.98$
$S = £48220{,}290.$

Ex. 130. *Required the amount of a rent of £18 per annum for 50 years, and payable *half-yearly*, when the interest is at 5 per cent. convertible *half-yearly*.

$\log 9 = 0{,}95424.25$
$\log \phi^{100} = 1{,}56364.84$
$\log r^{100} = 1{,}07238.65$
$\log S = 3{,}59027.74$
$S = £3892{,}937.$

What would be the amount of the same annuity payable *quarterly*?

$3{,}5902774$
P_2 (Tab. 63) $0{,}0026892$
$\log S = 3{,}5929666$
$S = £3917{,}117,$ required amount.

Ex. 131. *To what sum would an annuity of £540, forborn 23 years, amount, when received and laid up at interest in equal monthly parts of £45 each, the interest at the rate of 3¾ per cent. being converted into principal *twice* a year?

$\log 270 = 2{,}43136.38$
$\log \phi^{46} = 1{,}48629.63$
$\log r^{46} = 0{,}37111.06$
$P_6 = 0{,}00336.76$
$\log S = 4{,}29213.83$
$S = £19594{,}685.$

* See explanatory note, p. xiv.
† See p. 128.

Balance between the amounts of a principal and an annuity.

Ex. 132. £3000 are laid out at $3\frac{1}{2}$ per cent. per annum; a end of every year £150 are drawn; how much of the princi left after 21 years?

$\log 3000 = 3,4771213$
$\log r^{21} = 0,2806432$
$\log = 3,7577645$
$£5724,854$, amount of the princip

$\log 150 = 2,1760913$
$\log r^{21} = 0,2806432$
$\log \phi = 1,1827288$
$\log S = 3,6394633$
$S = £4359,767$, amount of the annuity,

remainder $= £1365,087$.

If we denote by C the principal, by a the annuity, and by x difference between both the amounts, we have (form. 24),

$$x = Cr^n - \frac{a}{i}(r^n - 1),$$

$$\text{or } x = \frac{a}{i} + r^n\left(C - \frac{a}{i}\right);$$

therefore $x = 4800 + (3000 - 4800) \cdot r^{21}$
$= 4800 - 1800 \cdot r^{21}$
$= 4800 - 3434,913$
or $x = £1365,087$, as above.

Ex. 133. *From £730, improved at $4\frac{1}{2}$ per cent. per annu £20 are drawn every year. How much will be owing after 35 yea if annuity and interest are payable *half-yearly*?

$\log 730 = 2,8633229$	,6392449
$\log r^{70} = 0,6392449$	,4405781
$\log = 3,5025678$	,1986668
$= £3180,030$	£1580,035
Amount of the principal.	Amount of the annuity.

Rest of the principal $= £1600,995$.

Ex. 134. If £1500 be put out to compound interest at $3\frac{1}{2}$ p cent. per annum, and at the end of every year £40 be added there which is also improved at the same rate of interest; what will be t total amount at the end of 22 years?

$\log 1500 = 3,1760913$	$\log 40 = 1,6020600$
$\log r^{22} = 0,3517383$	$\log r^{22} = 0,3517383$
$\log = 3,5278296$	$\log \phi^{22} = 1,1703405$
$= £3371,550$	$\log = 3,1241388$
	$= £1330,880$

therefore, £4702,430 required amount.

* See explanatory note, p. xiv.

Annual rent and time.

Ex. 135. What annual rent, improved at the rate of 5½ per cent. will at the end of 28 years amount to £3600?

$$\begin{aligned}\log 3600 &= 3,55630\\ \log a^{28} &= 8,83858\\ C^t \log r^{28} &= 9,37778\\ \log a &= 1,77266\end{aligned}$$

$a = £59,246$ or £59. 4*s*. 11*d*. annual rent.

Ex. 136. How much is to be laid out at the end of every quarter of a year in order to obtain £7500 after 24 years, the rate of interest being 4¾ per cent. convertible half-yearly?

$$\begin{aligned}\log 7500 &= 3,87506\\ C^t \log r^{48} &= 9,51069\\ \log a^{48} &= 8,54579\\ -P_2 &= 0,00256\\ \log a &= 1,92898\end{aligned}$$

$a = £84,915$ half-yearly, or £169,830 per annum.

Ex. 137. In what number of years will an annuity of £312 per annum amount to £20.000 at the rate of 3¾ per cent.?

Here we make use of the formula 25.

$$r^n = 1 + \frac{3\frac{3}{4}\cdot 20000}{312} = \frac{531}{156}.$$

$$\begin{aligned}\log 531 &= 2,72509\\ \log 156 &= 2,19312\\ \log r^n &= 0,53197\\ \log^s r^n &= 9,72589\\ \text{(Tab. 57)}\ \log^s r &= 8,20380\\ \log n &= 1,52209\\ n &= 33,273 \text{ or } 33 \text{ years } 100 \text{ days.}\end{aligned}$$

Ex. 138. An annuity of £225 per annum, payable *quarterly*, and improved at 4½ per cent. *per annum*, amounts to £10,000. What is the number of years?

Here we make use of the formula 26ª.

$$S = ar^n \phi P$$

$$= \frac{aP}{i}(r^n - 1),$$

whence $r^n = 1 + \frac{Si}{aP}$,

$$\frac{St}{a}=\frac{17}{9}=1,889.$$

$\log 1,8889=0,27621$

$-P_4=\quad 681$

$\log\frac{St}{aP}=0,26940$

$\frac{St}{aP}=1,8595$

$\log r^m=0,45629$

$\log^2 =0,65924$

(Tab. 57) $\log^2 r=8,25710$

$\log n=1,40214$

$n=25,243$, or 25 years, 89 days

SECT. 66. *Amount of Annuities increasing or decreasing in constant ratio.*

Let $£a$ be the annuity invested at the expiration of the first year, and d the annual increase of this annuity, the amount will be

$$S=ar^{n-1}+(a+d).r^{n-2}+\ldots+a+(n-1)d$$

$$=r^n.\left\{\frac{a}{r}+\frac{a+d}{r^2}+\frac{a+2d}{r^3}+\ldots+\frac{a+(n-1)d}{r^n}\right\}.$$

But we have already found (form. 19) that this last series is equal to

$$\left(a+nd+\frac{d}{i}\right)\phi-\frac{nd}{i}.$$

Consequently,

$$S=\left(a+nd+\frac{d}{i}\right)r^n\phi-\frac{ndr^n}{i};$$

and as

$$r^n\phi=\frac{1}{i}(r^n-1),$$

$$S=\frac{r^n}{i}\left(a+\frac{d}{i}\right)-\frac{1}{i}\left(a+nd+\frac{d}{i}\right).$$

If the annuity *decreases*, the sign of d will require to be changed in this last equation.

Ex. 139. Let us determine the amount of 21 annuities invested at 4 per cent. per annum at the end of every year, and the first of which is £36, increasing by £1 every year.

Here,

$$a=36,\quad d=1,\quad n=21,\quad i=0,04,$$

therefore

$$S=1525.r^{21}-2050$$

$\log 1525=3,18321968$

$\log.r^{21}=0,3577001$

$\log\quad =3,5409969$

$=£3475,121$

$-£2050$

$S=£1425,121$, amount of the annuities.

:CT. 67. *Rate of Interest.* The rate of interest is found by olation of our logarithmic tables, which gives for general ce a more than sufficient accuracy, or by means of the fol- g formula.

:. 140. £7104,436 have been received for 27 annuities of payable at the end of every year; what is the rate of interest?

ere we make use of the formula,

$$\frac{S}{a} = r^n \phi,$$

$$\begin{aligned} \log \quad S &= 3,85153 \\ \log\, a &= 2,17609 \\ \log .\, r^n \phi &= 1,67544 \end{aligned}$$

it $\log . r^n \phi$ becomes at

4 per cent.	$4\frac{1}{2}$ per cent.
$\log r^{27} = 0,45990$	$= 0,47399$
$\log a^{27} = 8,78702$	$= 8,79309$
$\log = 9,67288$	$= 9,68090$
difference $= 802$,	difference between the 4 per cent. and the given log $= 256$;

:refore the rate of interest is $= 4 + \frac{1}{2} \cdot \frac{256}{802} = 4,0399$ per cent. per

).

e rate thus obtained by simple interpolation is always a little nall; thus in the foregoing example its true value is $= 4,04$ nt.

t 4,04 per cent. rate corresponds exactly to 2 per cent. half-, or to 4 per cent. per annum, convertible *half-yearly.*

68. *General formula for finding the rate of annuity in arrear.*

the value of *deferred* annuities $D = a\phi r^{-d}$ becomes equal to the it of *annuity in arrear,* when n is put instead of $(-d)$, we have n all formulæ concerning deferred annuities, to convert d into for obtaining directly analogous formulæ applicable to an- i in arrear.

us, in formula 22, $(2n + d + 1)$ becomes $= 1 - n$, and the ap- iated formula which determines the rate, becomes directly,

$$q = \left(\frac{S}{an}\right)^{\frac{2}{n-1}} - 1,$$

$$i = \frac{12 + q(n+1)}{12 + 2q(n+1)} \cdot q.$$

Ex. 141. At what rate per cent. will an annuity of £150, forborn 27 years, amount to £7104,436?

Here,

$$S=7104{,}436,\quad a=150,\quad n=27,\quad \frac{2}{n-1}=\frac{1}{13},\quad \text{and}\ \frac{n+1}{12}=$$

$$\begin{aligned}
\log S &= 3{,}8515296\\
C^{o}\log a &= 7{,}8239087\\
C^{o}\log n &= 8{,}5686362\\
\log\frac{S}{an} &= 0{,}2440745\\
\frac{1}{13}\log &= 0{,}01877496\\
1+q &= 1{,}0441790\\
q &= 0{,}044179
\end{aligned}$$

$$t=q.\frac{1{,}103084}{1{,}206169}$$

$$\begin{aligned}
\log q &= \bar{2}{,}645\\
\log 1{,}103084 &= 0{,}042\\
C^{o}\log 1{,}206169 &= \bar{9}{,}918\\
\log t &= \bar{2}{,}606\\
t &= 0{,}0404
\end{aligned}$$

true rate = 4,04 per cent. per annum, or 2 per cent. half-yearly.

Formula of correction. The formula of correction for an in arrear is obtained in the same way, by converting, in formu d into $(-n)$.

Then,

$$d+n=0,$$

and

$$\varepsilon=\frac{\rho^{n}-1-\frac{S\tau}{a}}{\frac{S}{a}-n\rho^{n-1}}.$$

Thus in Ex. 140, we have found

$$\tau=0{,}040399$$
$$\rho=1{,}040399,$$

thence

$$\begin{aligned}
\log\ \rho &= 0{,}01719.992609\\
\log\rho^{27} &= 0{,}46439.80044\\
\rho^{27} &= 2{,}91338.585
\end{aligned}$$

$$\varepsilon=\frac{2{,}91338585-2{,}91341407}{47{,}363-75{,}607}$$

$$=\frac{0{,}00002824\ldots}{28{,}244}=0{,}000001;$$

and the corrected value of rate is $t=4{,}04$ per cent.

II.

Amount of annual investments.

SECT. 69. The *annual investments* are essentially distinct from ıe *annuities in arrear*, these being invested at the end of every year, hereas the former are invested at the beginning of every year as single principal.

The amount of an annuity in arrear proceeds by an annual rent aving always been laid up as it became due, and improved at com-ound interest; *the amount of an annual investment* is the product of ıccessive annual loans effected by the creditor at the beginning of very year.

The amount of an annuity being equal to the sum of the amounts f all its partial investments, the amount of each annuity at the end f the given time is only to be determined. If therefore we assume

K = the amount of the annuity,
a = the annuity or annual rent,
n = the number of years,

ıe first payment having been made n years ago, the second $(n-1)$ ears ago, &c., the amount of these investments will be, for the first r^n, for the second ar^{n-1}, and so forth, until the n^{th} and last is at-ained, which having been made one year ago will be $=ar$.

Consequently the sum of all these values for n years will be

$$K = ar^n + ar^{n-1} + ar^{n-2} + \ldots + ar,$$

ıultiplying by r, $\quad Kr = ar^{n+1} + ar^n + ar^{n-1} + \ldots + ar^2,$

nd subtracting $\quad Kt = ar^{n+1} - ar,$

herefore
$$K = \frac{a}{t}(r^{n+1} - r) \quad \ldots\ldots\ldots\ldots (27).$$

If, for instance, it is required to find the amount of £1 per an-ıum, invested at 5 per cent. at the beginning of every year during 4 years, we have:

$r^{35} = 3,386355$
$r = 1,05$
$r^{35} - r = 2,336355$
$K = £46,72710$

Formula 27 may be written as follows:

$$K = \frac{ar^{n+1}}{t}(1 - r^{-n}),$$

ınd as

$$\phi = \frac{1}{t}(1 - r^{-n}),$$

we have

$$K = a\phi r^{n+1} \quad \ldots\ldots\ldots\ldots (28),$$

a formula very convenient for logarithmic calculations.

Besides, we have found (form. 26) that the amount of an annuity of £1 in arrear is

$$S = a\phi r^n,$$

therefore $$K = rS.$$

Or, the amount of an annual investment is equal to the amount of an annuity in arrear for the *same number of years*, increased by the interest of a year.

Thus, in the above example:

Amount of an annuity of £1 in arrear for 24 years...=£44

Annual interest of this sum, at 5 per cent.....= 2

Amount of £1 per annum, improved during 24 years=£46

Lastly, if we denote by S' the amount of an annuity of a for $(n+1)$ years, we have by means of form. 24,

$$S' = \frac{a}{t}(r^{n+1} - 1),$$

and as $$K = \frac{a}{t}(r^{n+1} - r),$$

therefore $$K = S' - a;$$

or, the amount of an annual investment for n years is equal to the same annuity in arrear for $(n+1)$ years, minus an annuity.

Thus, for instance, in the preceding example:

Amount of an annuity of £1 in arrear for 25 years = £47,7

Amount of an annual investment of £1 for 24 years = 46,7

From formulæ 27 and 28 others may be deduced for determining any one of the quantities n and a when the rest are given.

$$r^n = 1 + \frac{Kt}{ar} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(29}$$

$$a = \frac{Kr^{-(n+1)}}{\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(30}$$

SECT. 70. *Whatever may generally be the kind of payment adopted for the annuity and interest, whether yearly, half-yearly, quarterly, &c., *the amount of the annuity is always equal to the present value of the same annuity with immediate entrance, multiplied by the amount of* £1 *at the end of* $(n+1)$ *years.*

If the annuity and interest are payable at *the same periods* of time, ν times per annum, *divide* the annual rent and the rate of interest by the number ν, and *multiply* the number of years by ν.

This case is rather worthy of notice when the payments, instead of being annual, are half-yearly, quarterly, monthly, &c.; the annuities in such cases may either decrease or increase.

* See explanatory note, p. xiv.

Thus, for instance, the amount of £1 per annum payable at the beginning of every year, is at 5 per cent.,

	Yearly.	Half-yearly.
for 2 years	£ 2,153	£ 2,128
5 ...	5,802	5,742
10 ...	13,207	13,092
20 ...	34,719	34,544
32 ...	79,064	79,059
33 ...	84,067	84,099
40 ...	126,840	127,296
50 ...	219,815	221,681.

We may see from this that when the annuity is for 32 years or less, the amount of the annuity payable yearly is *greater* than the amount of the annuity payable half-yearly. If, on the contrary, the annuity is for 33 years or more, the amount of the annuity payable yearly is *less* than the amount of the annuity payable half-yearly.

And universally: If the number of payments increases, the amount of the yearly rent is *greater* than the amount of the half-yearly, quarterly, &c. investments, up to a certain number* of annuities; if the number of annuities exceeds it, the amount of the yearly rent will be *less* than the half-yearly, quarterly, &c. one.

Finally, when the payments of interest and annuity are not uniform, the amount is first computed supposing the annuity to be paid at the same periods as the interest; then multiply or divide by the constant ratio P or Q. (See page 61.)

Ex. 142. £5 are invested at the beginning of every month, during 13 years. What will be their amount at the rate of $4\frac{1}{2}$ per cent. payable half-yearly?

$$\log \phi^{30} = 30 = 1,4771213$$
$$\log r^{27} = 0,2609096$$
$$\log \phi^{26} = 1,2905491$$
$$+P_6 = 0,0040351$$
$$\log K = 3,0326151$$
$$K = £1077,991$$

Ex. 143. How much is to be put out at $4\frac{3}{8}$ per cent. at the beginning of every year, to obtain £9037 after 61 years?

---

* This number, which is the limit, is determined by the general equation

$$(1+t)^{n+1} = \left(1+\frac{t}{\nu}\right)^{n\nu+1} + t \cdot \frac{\nu-1}{\nu};$$

it diminishes according as the rate increases.

When $\nu = 2$, we have

$$(1+t)^{n+1} = \left(1+\frac{t}{2}\right)^{2n+1} + \frac{t}{2}.$$ (See page 56).

Here we make use of the formula 30:

$$\log 9037 = 3,95602$$
$$\log r^{-42} = 8,84702$$
$$\log a^{42} = 8,67408$$
$$\log a = 1,47712$$
$$a = £30.$$

Ex. 144. *How many years are required for obtaining £ from an annual investment of £180 at 4½ per cent., annuity i terest being half-yearly?

Here we shall make use of the formula 29.

$$\log K = 4,00557$$
$$\log t = 2,35218$$
$$C^t \log a = 8,04576$$
$$C^t \log r = \underline{9,99934}$$
$$\log \frac{Kt}{ar} = 0,39385,$$

$$\frac{Kt}{ar} = 2,4765$$
$$r^n = 3,4765$$
$$\log . r^n = 0,54114$$

and (Tab. 9) $n = 56$ half-years, or 28 years.

SECT. 71. *Amount of annual investments increasing or decre by a constant ratio.*

As the amount of an annual investment is always r times g than the amount of an annuity in arrear, we shall have, acco to page 114,

$$K = \left(a + \frac{d}{i}\right)\frac{r^{n+1}}{i} - \left(a + dn + \frac{d}{i}\right)\frac{r}{i}.$$

Ex. 145. What is the amount of 9 annuities laid out beginning of every year, the first instalment of which being £ the second £2100, the third £2200, and every next year's insta being greater by £100 than that of the foregoing year, interest 5 per cent.?

Here: $a = 2000$ $d = 100$ $n = 9$ and $i = 0,05$,

and $K = 20 \times 4000\, r^{10} - 4900 \times 21.$

$$\log 80,000 = 4,9030900$$
$$\log r^{10} = 0,2118930$$
$$\log = 5,1149830$$
$$£130311,570$$
$$- 102900$$
$$£27.411,570$$ required amount

Ex. 146. What will be the amount of an annuity for 20 laid out at the beginning of every year, and bearing 5 per cen terest; the first annuity being £800, the second £760, the third and so on, every instalment being less by £40 than the preceding

* See explanatory note, p. xiv.

Here,

$$a=800, \quad d=40, \quad n=20, \quad t=0{,}05;$$

therefore,

$$K=800\times\frac{1{,}05}{0{,}05}=£16{,}800, \text{ required amount.}$$

Rate of interest. The rate of interest of annual investments is determined by means of our logarithmic tables, or by a general formula.

As the present value of a deferred annuity is (form. 21)

$$D=a\phi r^{-d},$$

and the amount of an annual investment, at the same rate of interest,

$$K=a\phi r^{n+1},$$

it suffices, in all formulæ relating to deferred rents, to replace $(-d)$ by $(n+1)$ in order to obtain directly the respective formulæ relating to the amount of annual investments.

Wherefore, the general formula for finding the rate (page 103) becomes:

$$q=\left(\frac{K}{an}\right)^{\frac{2}{n+1}}-1,$$

$$t=\frac{12+(n-1)\,q}{12+(n-1)\,2q}\cdot q.$$

Ex. 147. £39.586 have been received for 67 annuities of £50 laid out at the beginning of every year. What is the rate of interest?

The amount of £50 per annum is at

(Tab. 37) $5\frac{3}{4}$	(Tab. 38) $5\frac{5}{8}$
$\log a = 1{,}69897$	$= 1{,}69897$
$\log r^{68} = 1{,}65107$	$= 1{,}68595$
$\log \phi^{67} = 1{,}22995$	$= 1{,}22141$
$\log K = 4{,}57999$	$= 4{,}60633$

difference $= 2634$,

but $\log 39586 = 4{,}59754$;

difference with the $5\frac{3}{4}$ per cent. $= 1755$.

Consequently the rate of interest will be

$$=5\tfrac{3}{4}+\frac{1755}{2634}\cdot\tfrac{1}{8}=5\tfrac{5}{8} \text{ per cent.}$$

Ex. 148. The amount of £200 per annum, invested at the b ginning of every year, for 51 years, is £46.371. Required the ra of interest.

Here we make use of the general formula,

$$n=51,\quad \frac{2}{n+1}=\frac{1}{26},\quad \frac{n-1}{12}=\frac{25}{6},$$

therefore

$$\begin{aligned}
\log K &= 4,66625 \\
C^l \log a &= 7,69897 \\
C^l \log n &= 8,29243 \\
&\quad 0,65765 \\
\log (1+q) &= 0,0252942 \\
1+q &= 1,059971 \\
q &= 0,059971
\end{aligned}$$

$$\begin{aligned}
\frac{25}{6}q &= 0,24988 \\
\frac{25}{3}q &= 0,49976 \\
t &= \frac{1,24988}{1,49976} \\
\log q &= 8,77794 \\
&\quad 0,09687 \\
&\quad 9,82398 \\
\log t &= 8,69879 \\
t &= 4,9979 \text{ per cent.}
\end{aligned}$$

True rate = 5 per cent.

III.

Annuities with growing dividend.

SECT. 72. It happens most frequently that perpetual or termi- nal annuities are bought between the falling due of two consecutive annuities. In that case, not only the first payment, but all others, are put forward by a certain part of the year.

For example, if 3 per cent. Consols are bought the 5th of May, the first half-yearly dividend being payable 2 months afterwards, the 5th of July, and all subsequent ones from 6 to 6 months, all pay- ments have been advanced by 4 months, consequently the annuity is said to have 4 months entrance.

Generally, as every payment is brought nearer by the same part of the year, it is needed only to determine the value of that same annuity with immediate entrance, and to multiply it by the amount of £1 put out for the time included between the entrance and the moment of the purchase of the annuity.

Thus if this time is a $\left(\frac{1}{k}\right)$th of the interval between two succes- sive payments, if t be the true rate of interest of each interval, and V the present value of the same annuity with immediate entrance (page 17), the value of the annuity with past entrance will be

$$S = V v^{\frac{1}{k}}.$$

Purchase of Annuities already entered upon.

Ex. 149. Required the present value of an annual rent of £358 for 27 years at $5\frac{1}{8}$ per cent. per annum; the first payment being due after a 7 months' lapse.

$$\begin{array}{rl} & \log a = 2,55388\ 30 \\ \text{(Tab. 32)} & \log \phi = 1,15990\ 12 \\ \text{(Tab. 51)} & \log r^{\frac{1}{2}} = \quad 904\ 42 \\ & \log S = 3,72282\ 84 \\ & S = £5282,365. \end{array}$$

Ex. 150. *Required the present value of 15 annuities of £160, payable *quarterly*, at $4\frac{3}{4}$ per cent. per annum, payable *half-yearly*, when the first payment of £40 is due after a 2 months' lapse.

The interest being half-yearly, take half the annual rent and the rate of interest and double the periods; consequently we have 30 annuities of £80, payable half-yearly, at $2\frac{3}{8}$ per cent., with 4 months' entrance; therefore

$$\begin{array}{rl} & \log 80 = 1,90309\ 00 \\ \text{(Tab. 10)} & \log \phi = 1,32804\ 16 \\ \text{(Tab. 51)} & \log r^{1\frac{1}{2}} = \quad 33980 \\ & P_4 = \quad 25560 \\ & \log S = 3,23708\ 56 \\ & S = £1726,178. \end{array}$$

Sect. 73. *Rate of interest* of annuities with growing dividend is obtained by means of interpolation, like rate of immediate annuities.

Ex. 151. What is the rate of interest of a £680 annuity for 21 years, of which the first payment is allowed in 10 months, and which has been bought for £8742. 5*s*. 8*d*.?

at 5 per cent.	at $5\frac{1}{8}$ per cent.
$\log 680 = 2,83251$	$= 2,83251$
$\log \phi^{21} = 1,10793$	$= 1,10316$
$\log r^{1\frac{2}{3}} = 0,00353$	$= 0,00362$
$\log \quad = 3,94397$	$= 3,93929$, difference $= 468$,
$\log 8742,283 = \ ,94162$,	difference with the 5 per cent. $= 235$;

therefore $t = 5 + \frac{1}{8} \cdot \frac{235}{468} = 5\frac{1}{16}$ per cent., or $2\frac{1}{32}$ per cent. half-yearly.

Ex. 152. Required the *annual* rate of interest for a £280 annuity for 31 years, payable *quarterly*, with 2 months entrance, which has been sold for £4452. 5*s*. 11*d*.

* *See* explanatory note, p. xiv.

at 5 per cent.	at $5\frac{1}{8}$ per cent.
log 280 = 2,447 16	= 2,447 16
log ϕ^{m} = 1,192 92	= 1,186 62
log P_4 = 0,007 99	= 0,008 18
log $r^{\frac{1}{2}}$ = 0,003 53	= 0,003 62
log = 3,651 60	= 3,645 58,

difference = 602,

log 4452,3 = ,648 58, difference with the 5 per cent. = 302;

therefore $t = 5 + \frac{1}{8} \cdot \frac{302}{602} = 5\frac{1}{16}$ per cent. per annum, or $2\frac{1}{32}$ per cent. half-yearly.

SECT. 74. *Rate of Government Stocks.*

The half-yearly dividends of Stocks are paid at fixed periods:

The 5th of Jan. and the 5th of July:

3 per cent. Consols, 3 per cent. 1726, New 5 per cent. annuities, and Life Annuities transferred between Jan. 5 and April 4, or between July 5 and Oct. 9.

The 5th of April and the 10th of October:

Bank Stock, 3 per cent. Reduced, Long Annuities, Life Annuities transferred between April 5 and July 4, or between Oct. 10 and Jan. 4.

Consequently, when Stocks are bought, *the time of entrance*, i. e. the time past since the last dividend was paid, is to be taken into consideration.

For example: If 3 per cent. Consols are bought the 5th of March, we notice on one side the interest of 2 months, nearly £$\frac{1}{2}$, which is payable but 4 months afterwards, and on the other, all payments are advanced by 2 months.

Let C = the whole *cost* of the 3 per cent. Stock (i. e. the *price* of this Stock and its *Brokerage**), with entrance of $\frac{1}{k}$th half year,

and t = the true half-yearly rate of interest,

we shall have

$$C = \frac{1\frac{1}{2}}{t} \cdot r^{\frac{1}{k}}.$$

---

* The Brokerage upon either sale or purchase of Stocks is $\frac{1}{8}$ per cent. on the *quantity* of Stock, and is never less than 1*s.*, however small may be the sum transferred.

The charge for brokerage is not always the same; some brokers charging sixpence for every £10 Stocks or under; others charge 1*s.* for £25, 1*s.* 6*d.* for £50, 2*s.* for £75, or under these sums; and 2*s.* 6*d.* for above £75.

Ex. 153. Required the price of the 3 per cent. Consols on the 17th of March, for laying out one's money at $3\frac{1}{4}$ per cent. payable half yearly?

From Jan. 5 to July 5, is 181 days,
..... March 17, is 76 ...

therefore $\frac{1}{k} = \frac{76}{181}$, and $t = 1\frac{5}{8}$ per cent.

$$\begin{aligned} \log . \ 1\tfrac{1}{2} &= 0,17609 \\ Ct \log t &= 9,78915 \\ \tfrac{1}{k} \log r &= 0,00294 \\ \log C &= \overline{1,96818} \\ C &= £92,935, \text{ whole cost,} \\ \text{or Price} &= 92,810 \\ \text{Brokerage} &= 0,125. \end{aligned}$$

For finding the *rate per cent.* made of money invested in a Stock 3 per cent. we have

$$C = \frac{1\frac{1}{2}}{t} \cdot (1+t)^{\frac{1}{k}};$$

then developing by the Binomial Theorem, and neglecting the third and higher powers of t,

$$\begin{aligned} Ct &= \tfrac{3}{2}\left\{1 + \frac{t}{k} + \frac{t^2}{2k}\left(\frac{1}{k} - 1\right)\right\} \\ &= \tfrac{3}{2} + \frac{3t}{2k} - \frac{3t^2}{4k^2}\left(1 - \frac{1}{k}\right), \end{aligned}$$

thence

$$\left(C - \frac{3}{2k}\right)t = \tfrac{3}{2} - \frac{3t^2}{4k^2}\left(1 - \frac{1}{k}\right),$$

and

$$t = \frac{1,5}{C - \frac{3}{2k}} - \frac{3t^2}{4k} \cdot \frac{1 - \frac{1}{k}}{C - \frac{3}{2k}} \quad \ldots\ldots\ldots\ldots\ldots (31).$$

Neglecting the last term of that equation as very little:

$$t = \frac{1,5}{C - \frac{3}{2k}}.$$

But $\frac{3}{2k}$ is the *Simple interest* of £100 stock corresponding to $\left(\frac{1}{k}\right)^{\text{th}}$ half year; therefore, for finding, by approximation of the

from before the rate of interest of a 3 per cent. Stock, calculate the value of the *proportionate or growing dividend* from the day when the last half yearly dividend was due, to the given day, and subtract it from the given price of the Stock, in order to find the *net price;* the half yearly rate of interest will be £1½ divided by this net price.

Ex. 134. To find the rate per cent. which is made of money invested in the 3 per cent. Consols, at 94¾ per cent., on the 19th of January, 1838.

From Jan. 5 to July 5 is 181 days,
from Jan. 5 to Jan. 19 is 14 days,

the Dividend for 181 days being £1½, the proportional dividend will be 0,116;

therefore £94,750, Price,
0,125, Brokerage,
C=94,875, Whole Cost,

$\frac{3}{26}$= 0,116, Proportional Dividend,

$C-\frac{3}{26}$=94,759,

net price of the Stocks, excluding the dividend, and £1½ divided by this last number will give

1,58301 per cent. half-yearly, or 3,1659 per cent. per annum.

Ex. 135. What is the rate of interest of the Consols at 98¼ per cent. on the 10th Nov. 1838?

From July 5 to Jan. 5 is 184 days,
from July 5 to November 10 is 128 days,

the proportional dividend will be $=\frac{128}{184}\cdot 1\frac{1}{2}$=£1,043478,

and

£98,250, Price,
0,125, Brokerage,
98,375, Whole Cost,
1,043478, Growing Dividend,
£97,331522, Net Price,

and 1½ divided by this number will give 1,541125 per cent.

The rate obtained by the simple application of the first term of equation (3) is always too great by a quantity smaller than the value of the second fractionary term; so if we take the result just obtained as the value of i, we have

$$\log \tfrac{3}{4} = 9{,}875$$
$$\log . t^2 = 6{,}376$$
$$\log \tfrac{1}{k} = 9{,}842$$
$$\log\left(1-\tfrac{1}{k}\right) = 9{,}483$$
$$Ct.\log\left(C-\tfrac{3}{2k}\right) = 8{,}012,$$
$$\overline{3{,}588},$$

the number corresponding to this logarithm is 0,00000039,
and subtracting it from 0,01541125,

we have the corrected value of t=0,01541086, true to the last figure; consequently the rate of interest is=3,082172 per cent. per annum, payable half-yearly.

IV.

Comparison of the Logarithmic Tables No. 1 *with Smart's* 5 *Tables for the computation of money interests.*

Sect. 75. It may tend to facilitate a due understanding of the following Logarithmic Tables, and their application to the process of computation, if we present Examples for comparison between these Tables and such as Smart's 5 Tables upon money computation, since the Table I. in this book will give, in logarithms, all the values contained in the 5 Tables by Smart.

Thus Example 1.

Amount of £1 in 25 years at 6 per cent. per annum.
By Smart's Tab. 1 this is=£4.2914; Log of this is =0,6326466.
By Thoman's Tab. 1 (at p. 39 of Tab.) column r^n, and 25 years } =0,6326466.

Ex. 2. Present value of £1 due in 25 years, int. at 6 per cent.
By Smart's Tab. 2 this is=£0.233; and log=$\bar{1}$,3673554.
By Thoman's Tab. 1=the *recip.* of r^n, or r^{-n}, =$\bar{1}$,3673534.

Ex. 3. Amount of £1 per annum in 25 years, int. at 6 per cent.
By Smart's Tab. 3 this is=£54·8645; and the Log =1,7392915.
By Thoman's Tab. 1 take log r^n+recip. log a, or ϕ,...=1,7392915.

Ex. 4. Present value of £1 per annum for 25 years, int. at 6 per cent.
By Smart's Tab. 4 this is=£12.7834, and the Log =1,1066449.
By Thoman's Tab. 1 take recip. of log a, or log ϕ... =1,1066449.

Ex. 5. Annuity which £1 will purchase for 25 years, int. at 6 per cent.
By Smart's Tab. 5 this is=£·0782; and the Log =$\bar{2}$,8933551.
By Thoman's Tab. 1 take log a, here increased by 10 to avoid negative indices } =8,8933551.

Note to p. 102.

By Example 116, based upon the formula $V=\frac{a.\phi^n.}{r^n}$ see when P_n of Tab. IV. is a *multiplier*, and when a *divis*

Thus

$$V=\frac{a.\phi^n.P_n}{r^n};$$

$$a=\frac{V.r^n}{\phi^n.P_n}$$

$$=\frac{V.r^n.a^n}{P_n}.$$

Likewise

$$r^n=\frac{a.\phi^n.P_n}{V};$$

$$r=\left(\frac{a.\phi^n.P_n}{V}\right)^{\frac{1}{n}};$$

$$n=\frac{\log\frac{a.\phi^n.P_n}{V}}{\log r}.$$

Note to p. 111.

From formula $S=\frac{a.r^n.\phi}{Q_n}$, we may find when Q is to *multiplier*, and when a *divisor*; thus from the above we have,

$$S.Q_n=a.r^n.\phi^n;$$

$$a=\frac{S.Q_n}{r^n.\phi^n}$$

$$=\frac{S.a^n.Q_n}{r^n};$$

$$r^n=\frac{S.a^n.Q_n}{a}$$

$$=\frac{S.Q_n}{a.\phi^n};$$

$$\phi^n=\frac{S.Q_n}{a.r^n}.$$

LOGARITHMIC TABLES

OF

Compound Interest and Annuities.

BY

FEDOR THOMAN.

TABLE I.

SHOWING

A. The Logarithms of the Amount of £1 at the end of any number of years from 1 to 100 years.

$$\text{Log } r^n$$

B. The Logarithms of the Annuity £a per annum, WHICH £1 WILL PURCHASE, for any number of years from 1 to 100 years.

$$\text{Log } a^n.$$

le 1. Shewing: 1st Logarithm re amount of £1. at the end of n number of years. 1/2 2nd the Logarithm of the annuity £a. per annum which £1. will purchase for any number of years.

s	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
	0,00216·61	0,00216·61	8,34757·77	0,11046·91	51
	0,00433·21	9,70221·77	8,34018·12	0,11263·52	52
	0,00649·82	9,52720·73	8,33294·45	0,11480·13	53
	0,00866·42	9,40334·84	8,32586·16	0,11696·73	54
	0,01083·03	9,30751·74	8,31892·67	0,11913·34	55
	0,01299·64	9,22941·43	8,31213·45	0,12129·95	56
	0,01516·24	9,16354·47	8,30547·99	0,12346·55	57
	0,01732·85	9,10662·90	8,29895·81	0,12563·16	58
	0,01949·46	9,05655·18	8,29256·45	0,12779·76	59
	0,02166·06	9,01186·88	8,28629·48	0,12996·37	60
	0,02382·67	8,97154·97	8,28014·49	0,13212·98	61
	0,02599·27	8,93483·38	8,27411·08	0,13429·58	62
	0,02815·88	8,90114·35	8,26818·87	0,13646·19	63
	0,03032·49	8,87002·97	8,26237·52	0,13862·80	64
	0,03249·09	8,84113·64	8,25666·70	0,14079·40	65
	0,03465·70	8,81417·67	8,25106·05	0,14296·01	66
	0,03682·30	8,78891·61	8,24555·30	0,14512·61	67
	0,03898·91	8,76515·97	8,24014·12	0,14729·22	68
	0,04115·52	8,74274·50	8,23482·25	0,14945·83	69
	0,04332·12	8,72153·41	8,22959·42	0,15162·43	70
	0,04548·73	8,70140·94	8,22445·35	0,15379·04	71
	0,04765·34	8,68226·97	8,21939·82	0,15595·64	72
	0,04981·94	8,66402·73	8,21442·58	0,15812·25	73
	0,05198·55	8,64660·57	8,20953·39	0,16028·86	74
	0,05415·15	8,62993·79	8,20472·05	0,16245·46	75
	0,05631·76	8,61396·47	8,19998·33	0,16462·07	76
	0,05848·37	8,59863·34	8,19532·05	0,16678·68	77
	0,06064·97	8,58389·75	8,19073·01	0,16895·28	78
	0,06281·58	8,56971·49	8,18621·02	0,17111·89	79
	0,06498·19	8,55604·81	8,18175·89	0,17328·49	80
	0,06714·79	8,54286·33	8,17737·46	0,17545·10	81
	0,06931·40	8,53012·96	8,17305·56	0,17761·71	82
	0,07148·00	8,51781·95	8,16880·04	0,17978·31	83
	0,07364·61	8,50590·74	8,16460·73	0,18194·92	84
	0,07581·22	8,49437·03	8,16047·48	0,18411·52	85
	0,07797·82	8,48318·69	8,15640·16	0,18628·13	86
	0,08014·43	8,47233·78	8,15238·61	0,18844·74	87
3	0,08231·03	8,46180·52	8,14842·72	0,19061·34	88
9	0,08447·64	8,45157·26	8,14452·35	0,19277·95	89
0	0,08664·25	8,44162·47	8,14067·37	0,19494·56	90
1	0,08880·85	8,43194·75	8,13687·67	0,19711·16	91
2	0,09097·46	8,42252·77	8,13313·12	0,19927·77	92
3	0,09314·07	8,41335·33	8,12943·61	0,20144·37	93
4	0,09530·67	8,40441·31	8,12579·04	0,20360·98	94
5	0,09747·28	8,39569·62	8,12219·29	0,20577·59	95
6	0,09963·88	8,38719·30	8,11864·27	0,20794·19	96
7	0,10180·49	8,37889·42	8,11513·86	0,21010·80	97
8	0,10397·10	8,37079·11	8,11167·99	0,21227·41	98
9	0,10613·70	8,36287·57	8,10826·55	0,21444·01	99
0	0,10830·31	8,35514·03	8,10489·45	0,21660·62	100
			7,69897·00		Perp

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log.
1	0.004332·14	0.004332·14	8.40013·73	0.235
2	0.008664·27	9.70544·67	8.39368·10	0.242
3	0.012996·41	9.53150·71	8.38738·20	0.229
4	0.017328·55	9.40871·66	8.38123·39	0.233
5	0.021660·69	9.31395·11	8.37523·13	0.237
6	0.025992·82	9.23691·09	8.36936·88	0.241
7	0.030324·96	9.17210·14	8.36364·13	0.246
8	0.034657·10	9.11624·34	8.35804·40	0.250
9	0.038989·24	9.06722·10	8.35257·23	0.254
10	0.043321·37	9.02359·02	8.34722·17	0.259
11	0.047653·51	8.98432·06	8.34198·82	0.263
12	0.051985·65	8.94805·15	8.33686·80	0.267
13	0.056317·79	8.91600·53	8.33185·73	0.272
14	0.060649·92	8.88503·30	8.32695·27	0.276
15	0.064982·06	8.85807·85	8.32215·04	0.280
16	0.069314·20	8.83215·49	8.31744·74	0.285
17	0.073646·34	8.80792·76	8.31284·06	0.289
18	0.077978·47	8.78520·21	8.30832·70	0.293
19	0.082310·61	8.76381·54	8.30390·41	0.298
20	0.086642·75	8.74362·98	8.29956·87	0.302
21	0.090974·88	8.72452·77	8.29531·85	0.306
22	0.095307·02	8.70640·81	8.29115·10	0.311
23	0.099639·16	8.68918·31	8.28706·38	0.315
24	0.103971·30	8.67277·62	8.28305·45	0.319
25	0.108303·43	8.65712·04	8.27912·12	0.324
26	0.112635·57	8.64215·65	8.27526·16	0.328
27	0.116967·71	8.62783·20	8.27147·36	0.332
28	0.121299·85	8.61410·00	8.26795·54	0.337
29	0.125631·98	8.60091·87	8.26410·51	0.341
30	0.129964·12	8.58825·06	8.26052·10	0.345
31	0.133296·26	8.57606·17	8.25700·14	0.350
32	0.138628·40	8.56432·14	8.25354·42	0.354
33	0.142960·53	8.55300·19	8.25014·80	0.358
34	0.147292·67	8.54207·78	8.24681·18	0.362
35	0.151624·81	8.53152·59	8.24353·36	0.367
36	0.155956·95	8.52132·52	8.24031·21	0.371
37	0.160289·08	8.51145·62	8.23714·56	0.375
38	0.164621·22	8.50190·10	8.23403·34	0.380
39	0.168953·36	8.49264·30	8.23097·39	0.384
40	0.173285·50	8.48366·72	8.22796·52	0.388
41	0.177617·63	8.47495·92	8.22500·72	0.393
42	0.181949·77	8.46650·62	8.22209·81	0.397
43	0.186281·91	8.45829·59	8.21923·68	0.401
44	0.190614·04	8.45031·70	8.21642·22	0.406
45	0.194946·18	8.44255·89	8.21365·35	0.410
46	0.199278·32	8.43501·17	8.21092·93	0.414
47	0.203610·46	8.42766·63	8.20824·89	0.419
48	0.207942·59	8.42051·42	8.20561·12	0.423
49	0.212274·73	8.41354·69	8.20301·53	0.427
50	0.216606·87	8.40675·69	8.20046·04	0.432
			8.00000·00	

$1\frac{1}{2}$ Per Cent.

Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
0,00646·60	0,00646·60	8,45016·73	0,32976·82	51
0,01293·21	9,70865·71	8,44455·80	0,33623·42	52
0,01939·81	9,53577·87	8,43910·15	0,34270·02	53
0,02586·42	9,41404·49	8,43379·19	0,34916·63	54
0,03233·02	9,32033·19	8,42862·34	0,35563·23	55
0,03879·63	9,24433·95	8,42359·09	0,36209·84	56
0,04526·23	9,18057·36	8,41868·90	0,36856·44	57
0,05172·83	9,12575·45	8,41391·32	0,37503·04	58
0,05819·44	9,07776·69	8,40925·87	0,38149·65	59
0,06466·04	9,03516·62	8,40472·13	0,38796·25	60
0,07112·65	8,99692·23	8,40029·69	0,39442·86	61
0,07759·25	8,96227·45	8,39598·15	0,40089·46	62
0,08405·85	8,93064·52	8,39177·13	0,40736·07	63
0,09052·46	8,90158·54	8,38766·27	0,41382·67	64
0,09699·06	8,87473·89	8,38365·27	0,42029·27	65
0,10345·67	8,84981·90	8,37973·78	0,42675·88	66
0,10992·27	8,82659·08	8,37591·50	0,43322·48	67
0,11638·88	8,80486·00	8,37218·13	0,43969·09	68
0,12285·48	8,78446·37	8,36853·40	0,44615·69	69
0,12932·08	8,76526·42	8,36497·01	0,45262·30	70
0,13578·69	8,74714·37	8,36148·74	0,45908·90	71
0,14225·29	8,73000·10	8,35808·32	0,46555·50	72
0,14871·90	8,71374·88	8,35475·53	0,47202·11	73
0,15518·50	8,69831·03	8,35150·12	0,47848·71	74
0,16165·11	8,68361·84	8,34831·89	0,48495·32	75
0,16811·71	8,66961·41	8,34520·62	0,49141·92	76
0,17458·31	8,65624·46	8,34216·13	0,49788·52	77
0,18104·92	8,64346·33	8,33918·21	0,50435·13	78
0,18751·52	8,63122·84	8,33626·68	0,51081·73	79
0,19398·13	8,61950·22	8,33341·34	0,51728·34	80
0,20044·73	8,60825·10	8,33062·05	0,52374·94	81
0,20691·34	8,59744·40	8,32788·65	0,53021·55	82
0,21337·94	8,58705·33	8,32520·95	0,53668·15	83
0,21984·54	8,57705·37	8,32258·81	0,54314·75	84
0,22631·15	8,56742·20	8,32002·07	0,54961·36	85
0,23277·75	8,55813·71	8,31750·61	0,55607·96	86
0,23924·36	8,54917·95	8,31504·28	0,56254·57	87
0,24570·96	8,54053·14	8,31262·96	0,56901·17	88
0,25217·56	8,53217·62	8,31026·49	0,57547·78	89
0,25864·17	8,52409·87	8,30794·79	0,58194·38	90
0,26510·77	8,51628·49	8,30567·71	0,58840·98	91
0,27157·38	8,50872·18	8,30345·14	0,59487·59	92
0,27803·98	8,50139·68	8,30126·97	0,60134·19	93
0,28450·59	8,49429·90	8,29913·08	0,60780·80	94
0,29097·19	8,48741·78	8,29703·39	0,61427·40	95
0,29743·79	8,48074·32	8,29497·78	0,62074·01	96
0,30390·40	8,47426·61	8,29296·14	0,62720·61	97
0,31037·00	8,46797·79	8,29098·41	0,63367·21	98
0,31683·61	8,46187·03	8,28904·46	0,64013·82	99
0,32330·21	8,45593·57	8,28714·24	0,64660·42	100
		8,17600·13		Perp.

1 5/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log.
1	0,00700·06	0,00700·06	8,46228·76	0,3570
2	0,01400·11	9,70945·67	8,45687·59	0,3040
3	0,02100·17	9,53684·22	8,45161·58	0,3710
4	0,02800·22	9,41537·09	8,44650·12	0,3780
5	0,03500·28	9,32191·89	8,44152·64	0,3850
6	0,04200·34	9,24618·62	8,43666·64	0,3920
7	0,04900·39	9,18267·86	8,43197·57	0,3990
8	0,05600·45	9,12811·63	8,42738·97	0,4060
9	0,06300·50	9,08038·42	8,42292·39	0,4130
10	0,07000·56	9,03803·76	8,41857·38	0,4200
11	0,07700·61	9,00004·66	8,41433·55	0,4270
12	0,08400·67	8,96565·02	8,41020·47	0,4340
13	0,09100·73	8,93427·09	8,40617·81	0,4410
14	0,09800·78	8,90545·97	8,40225·20	0,4480
15	0,10500·84	8,87886·05	8,39842·30	0,4550
16	0,11200·89	8,85418·64	8,39468·79	0,4620
17	0,11900·95	8,83120·29	8,39104·37	0,4690
18	0,12601·01	8,80971·52	8,38748·73	0,4760
19	0,13301·06	8,78956·07	8,38401·59	0,4830
20	0,14001·12	8,77060·16	8,38062·69	0,4900
21	0,14701·17	8,75272·00	8,37731·79	0,4970
22	0,15401·23	8,73581·51	8,37408·61	0,5040
23	0,16101·28	8,71979·91	8,37092·93	0,5110
24	0,16801·34	8,70459·55	8,36784·51	0,5180
25	0,17501·40	8,69013·73	8,36483·15	0,5250
26	0,18201·45	8,67636·51	8,36188·64	0,5320
27	0,18901·51	8,66322·65	8,35900·78	0,5390
28	0,19601·56	8,65067·47	8,35619·36	0,5460
29	0,20301·62	8,63866·80	8,35344·23	0,5530
30	0,21001·68	8,62716·86	8,35075·18	0,5600
31	0,21701·73	8,61614·29	8,34812·05	0,5670
32	0,22401·79	8,60555·99	8,34554·67	0,5740
33	0,23101·84	8,59539·20	8,34302·89	0,5810
34	0,23801·90	8,58561·38	8,34056·55	0,5880
35	0,24501·96	8,57620·21	8,33815·50	0,5950
36	0,25202·01	8,56713·59	8,33579·60	0,6020
37	0,25902·07	8,55839·57	8,33348·74	0,6090
38	0,26602·12	8,54996·37	8,33122·74	0,6160
39	0,27302·18	8,54182·32	8,32901·50	0,6230
40	0,28002·23	8,53395·91	8,32648·89	0,6300
41	0,28702·29	8,52635·73	8,32472·80	0,6370
42	0,29402·35	8,51900·48	8,32265·12	0,6440
43	0,30102·40	8,51188·94	8,32061·72	0,6510
44	0,30802·46	8,50499·97	8,31862·48	0,6580
45	0,31502·51	8,49832·51	8,31667·34	0,6650
46	0,32202·57	8,49185·59	8,31476·15	0,6720
47	0,32902·63	8,48558·30	8,31288·83	0,6790
48	0,33602·68	8,47949·75	8,31105·32	0,6860
49	0,34302·74	8,47359·15	8,30925·47	0,6930
50	0,35002·79	8,46785·72	8,30749·23	0,7000
			8,21085·34	

$1\frac{3}{4}$ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
	0,00753·44	0,00753·44	8,47425·55	0,38425·53	51
	0,01506·88	9,71025·53	8,46903·59	0,39178·97	52
	0,02260·33	9,53790·40	8,46396·64	0,39932·41	53
	0,03013·77	9,41669·43	8,45904·12	0,40685·86	54
	0,03767·21	9,32350·26	8,45425·46	0,41439·30	55
	0,04520·65	9,24802·86	8,44960·11	0,42192·74	56
	0,05274·09	9,18477·83	8,44507·55	0,42946·18	57
	0,06027·53	9,13047·18	8,44067·34	0,43699·62	58
	0,06780·98	9,08299·40	8,43639·01	0,44453·07	59
	0,07534·42	9,04090·03	8,43222·12	0,45206·51	60
	0,08287·86	9,00316·04	8,42816·25	0,45959·95	61
	0,09041·30	8,96901·39	8,42421·04	0,46713·39	62
	0,09794·74	8,93788·29	8,42036·10	0,47466·83	63
	0,10548·19	8,90931·86	8,41661·07	0,48220·27	64
	0,11301·63	8,88296·48	8,41295·62	0,48973·72	65
	0,12055·07	8,85853·47	8,40939·42	0,49727·16	66
	0,12808·51	8,83579·35	8,40592·18	0,50480·60	67
	0,13561·95	8,81454·67	8,40253·58	0,51234·04	68
	0,14315·39	8,79463·17	8,39923·37	0,51987·48	69
	0,15068·84	8,77591·04	8,39601·25	0,52740·93	70
	0,15822·28	8,75826·56	8,39287·02	0,53494·37	71
	0,16575·72	8,74159·57	8,38980·37	0,54247·81	72
	0,17329·16	8,72581·33	8,38681·08	0,55001·25	73
	0,18082·60	8,71084·18	8,38388·95	0,55754·69	74
	0,18836·04	8,69661·43	8,38103·74	0,56508·13	75
	0,19589·49	8,68307·13	8,37825·25	0,57261·58	76
	0,20342·93	8,67016·03	8,37553·26	0,58015·02	77
	0,21096·37	8,65783·49	8,37287·63	0,58768·46	78
	0,21849·81	8,64605·30	8,37028·14	0,59521·90	79
	0,22603·25	8,63477·70	8,36774·59	0,60275·34	80
	0,23356·70	8,62397·31	8,36526·83	0,61028·78	81
	0,24110·14	8,61361·06	8,36284·73	0,61782·23	82
	0,24863·58	8,60366·17	8,36048·09	0,62535·67	83
	0,25617·02	8,59410·10	8,35816·78	0,63289·11	84
	0,26370·46	8,58490·54	8,35590·63	0,64042·55	85
	0,27123·90	8,57605·41	8,35369·49	0,64795·99	86
	0,27877·35	8,56752·71	8,35153·28	0,65549·44	87
	0,28630·79	8,55930·68	8,34941·82	0,66302·88	88
	0,29384·23	8,55137·67	8,34734·99	0,67056·32	89
	0,30137·67	8,54372·16	8,34532·67	0,67809·76	90
	0,30891·11	8,53632·73	8,34334·75	0,68563·20	91
	0,31644·56	8,52918·09	8,34141·12	0,69316·64	92
	0,32398·00	8,52227·03	8,33951·65	0,70070·09	93
	0,33151·44	8,51558·38	8,33766·23	0,70823·53	94
	0,33904·88	8,50911·12	8,33584·77	0,71576·97	95
	0,34658·32	8,50284·25	8,33407·18	0,72330·41	96
	0,35411·76	8,49676·86	8,33233·34	0,73083·85	97
	0,36165·21	8,49088·08	8,33063·19	0,73837·30	98
	0,36918·65	8,48517·10	8,32896·56	0,74590·74	99
	0,37672·09	8,47963·16	8,32733·42	0,75344·18	100
			8,24303·80		Perp.

1 7/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^x.	Log. r^x.
1	0.008961·76	0.008961·76	8.486071·21	0,411448
2	0.018131·52	9.711051·27	8.481031·92	0,419163
3	0,024201·29	9.538961·40	8.476151·50	0.427839
4	0.032271·05	9.418511·54	8.471411·35	0,435616
5	0.040331·81	9.325081·31	8.466801·90	0.443719
6	0.048401·57	9.240861·69	8.462331·64	0,451860
7	0.056471·34	9.180871·27	8.457991·04	0.459854
8	0.064541·10	9.132821·10	8.453761·62	0.467922
9	0.072601·86	9.085591·62	8.449651·94	0.475899
10	0.080671·62	9.043751·38	8,445661·57	0,484057
11	0.088741·38	9.006261·40	8.441781·08	0,492124
12	0.096811·15	8.972361·58	8.438001·08	0.500191
13	0.104871·91	8.941481·14	8.434321·22	0.508265
14	0.113041·67	8.913161·21	8.430741·13	0.516327
15	0.121011·43	8.887051·19	8.427251·47	0.524393
16	0.129081·19	8.862861·36	8.423851·95	0.532463
17	0.137141·96	8.840361·28	8.420551·23	0.540533
18	0.145211·72	8.819351·49	8.417331·03	0.548598
19	0.153281·48	8.799671·69	8.414191·08	0,556665
20	0,161351·24	8,781191·14	8,411131·08	0,564731
21	0.169421·01	8,763781·05	8.408141·80	0,572801
22	0.177481·77	8,747341·31	8.405241·00	0,580868
23	0.185551·53	8,731791·16	8.402401·42	0,588935
24	0,193621·29	8,717041·94	8,399631·85	0,597004
25	0.201681·05	8,703041·05	8,396941·07	0,605071
26	0.209751·82	8,689731·27	8,394301·88	0 613139
27	0.217821·58	8.677041·65	8,391741·08	0,621201
28	0.225881·34	8.664041·41	8,389231·47	0 629274
29	0.233961·10	8,653381·36	8,386781·87	0.637342
30	0.242021·87	8,642321·76	8,384401·11	0,645409
31	0.250091·63	8,631741·21	8,382071·02	0.653477
32	0.258161·39	8,621501·65	8,379791·41	0.661545
33	0.266231·15	8,611801·28	8,377571·16	0.669612
34	0.274291·01	8,602511·60	8,375401·09	0.677680
35	0.282361·68	8,593531·27	8,373281·06	0.685747
36	0.290431·44	8,584801·10	8,371201·95	0.693815
37	0.298501·20	8,576571·41	8,369181·60	0.701883
38	0.306561·96	8,568361·14	8,367201·89	0.709951
39	0.314631·72	8,560831·74	8,365271·70	0.718018
40	0.322701·49	8,553381·71	8,363381·88	0.726086
41	0.330771·25	8,546101·50	8,361541·34	0.734154
42	0.338841·01	8,539251·09	8,359731·95	0.742222
43	0.346901·77	8,532541·01	8,357971·61	0.750289
44	0.354971·54	8,526051·23	8,356251·21	0.758357
45	0.363041·30	8,519771·66	8,354561·66	0.766425
46	0.371111·06	8,513701·37	8,352901·82	0.774493
47	0.379171·82	8,507821·30	8,351301·63	0.782561
48	0.387241·58	8,502121·85	8,349721·98	0.790629
49	0.395311·35	8,496601·98	8,348181·80	0.798697
50	0.403381·11	8,491261·61	8,346671·98	0.806765
			8,273001·13	

2 Per Cent.

Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
0,00860·02	0,00860·02	8,49773·88	0,43860·88	51
0,01720·03	9,71184·90	8,49288·73	0,44720·89	52
0,02580·05	9,54002·23	8,48818·28	0,45580·91	53
0,03440·07	9,41933·40	8,48361·96	0,46440·93	54
0,04300·09	9,32666·02	8,47919·18	0,47300·94	55
0,05160·10	9,25170·10	8,47489·44	0,48160·96	56
0,06020·12	9,18896·21	8,47072·20	0,49020·98	57
0,06880·14	9,13516·38	8,46667·00	0,49881·00	58
0,07740·15	9,08819·08	8,46273·38	0,50741·01	59
0,08600·17	9,04659·87	8,45890·93	0,51601·03	60
0,09460·19	9,00935·71	8,45519·19	0,52461·05	61
0,10320·21	8,97570·56	8,45157·82	0,53321·07	62
0,11180·22	8,94506·63	8,44806·42	0,54181·08	63
0,12040·24	8,91699·05	8,44464·66	0,55041·10	64
0,12900·26	8,89112·17	8,44132·18	0,55901·12	65
0,13760·27	8,86717·35	8,43808·68	0,56761·13	66
0,14620·29	8,84491·09	8,43493·85	0,57621·15	67
0,15480·31	8,82413·95	8,43187·38	0,58481·17	68
0,16340·33	8,80469·66	8,42889·01	0,59341·19	69
0,17200·34	8,78644·42	8,42598·47	0,60201·20	70
0,18060·36	8,76926·48	8,42315·50	0,61061·22	71
0,18920·38	8,75305·73	8,42039·86	0,61921·24	72
0,19780·40	8,73773·40	8,41771·31	0,62781·25	73
0,20640·41	8,72321·83	8,41509·63	0,63641·27	74
0,21500·43	8,70944·33	8,41254·59	0,64501·29	75
0,22360·45	8,69634·97	8,41006·01	0,65361·31	76
0,23220·46	8,68388·50	8,40763·67	0,96221·32	77
0,24080·48	8,67200·24	8,40527·39	0,67081·34	78
0,24940·50	8,66066·02	8,40296·99	0,67941·36	79
0,25800·52	8,64982·07	8,40072·29	0,68801·37	80
0,26660·53	8,63945·01	8,39853·11	0,69661·39	81
0,27520·55	8,62951·77	8,39639·29	0,70521·41	82
0,28380·57	8,61999·58	8,39430·69	0,71381·43	83
0,29240·58	8,61085·89	8,39227·14	0,72241·44	84
0,30100·60	8,60208·40	8,39028·51	0,73101·46	85
0,30960·62	8,59364·99	8,38834·65	0,73961·48	86
0,31820·64	8,58553·72	8,38645·44	0,74821·49	87
0,32680·65	8,57772·80	8,38460·72	0,75681·51	88
0,33540·67	8,57020·59	8,38280·38	0,76541·53	89
0,34400·69	8,56295·56	8,38104·31	0,77401 55	90
0,35260·70	8,55596·32	8,37932·39	0,78261·56	91
0,36120·72	8,54921·53	8,37764·48	0,79121·58	92
0,36980·74	8,54270·02	8,37600·50	0,79981·60	93
0,37840·76	8,53640·61	8,37440·34	0,80841·61	94
0,38700·77	8,53032·30	8,37283·88	0,81701·63	95
0,39560·79	8,52444·06	8,37131·04	0,82561·65	96
0,40420·81	8,51874·97	8,36981·72	0,83421·67	97
0,41280·82	8,51324·20	8,36835·82	0,84281·68	98
0,42140·84	8,50790·93	8,36693·25	0,85141·70	99
0,43000·86	8,50274·40	8,36553·93	0,86001·72	100
		8,30103·00		Perp.

$2\frac{1}{8}$ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log
1	0,00913·21	0,00913·21	8,50925·70	0,465
2	0,01826·41	9,71264·41	8,50458·14	0,474
3	0,02739·62	9,54107·89	8,50005·14	0,483
4	0,03652·83	9,42065·01	8,49566·10	0,493
5	0,04566·03	9,32823·42	8,49140·45	0,502
6	0,05479·24	9,25353·10	8,48727·66	0,511
7	0,06392·45	9,19104·63	8,48327·23	0,520
8	0,07305·66	9,13750·04	8,47938·67	0,529
9	0,08218·86	9,09077·79	8,47561·55	0,538
10	0,09132·07	9,04943·46	8,47195·41	0,547
11	0,10045·28	9,01244·01	8,46839·85	0,557
12	0,10958·48	8,97903·37	8,46494·50	0,566
13	0,11871·69	8,94863·78	8,46158·98	0,575
14	0,12784·90	8,92080·35	8,45832·92	0,584
15	0,13698·10	8,89517·46	8,45516·00	0,593
16	0,14611·31	8,87146·43	8,45207·90	0,602
17	0,15524·52	8,84943·79	8,44908·33	0,611
18	0,16437·73	8,82890·09	8,44616·96	0,620
19	0,17350·93	8,80969·06	8,44333·55	0,630
20	0,18264·14	8,79166·90	8,44057·81	0,639
21	0,19177·35	8,77471·88	8,43789·50	0,648
22	0,20090·55	8,75873·85	8,43528·36	0,657
23	0,21003·76	8,74364·08	8,43274·17	0,666
24	0,21916·97	8,72934·89	8,43026·70	0,675
25	0,22830·17	8,71579·57	8,42785·75	0,684
26	0,23743·38	8,70292·24	8,42551·09	0,694
27	0,24656·59	8,69067·61	8,42322·54	0,703
28	0,25569·79	8,67901·02	8,42099·91	0,712
29	0,26483·00	8,66788·30	8,41883·00	0,721
30	0,27396·21	8,65725·67	8,41671·66	0,730
31	0,28309·42	8,64709·75	8,41465·70	0,739
32	0,29222·62	8,63737·48	8,41264·97	0,748
33	0,30135·83	8,62806·09	8,41069·32	0,757
34	0,31049·04	8,61913·02	8,40878·59	0,767
35	0,31962·24	8,61056·00	8,40692·63	0,776
36	0,32875·45	8,60232·86	8,40511·31	0,785
37	0,33718·66	8,59441·70	8,40334·50	0,794
38	0,34701·86	8,58680·73	8,40162·05	0,803
39	0,35615·07	8,57948·28	8,39993·86	0,812
40	0,36528·28	8,57242·85	8,39829·79	0,821
41	0,37441·48	8,56563·03	8,39669·74	0,831
42	0,38354·69	8,55907·50	8,39513·59	0,840
43	0,39267·90	8,55275·08	8,39361·23	0,849
44	0,40181·11	8,54664·60	8,39212·54	0,858
45	0,41094·31	8,54075·03	8,39067·46	0,867
46	0,42007·52	8,53505·38	8,38925·85	0,877
47	0,42920·73	8,52954·74	8,38787·63	0,886
48	0,43833·93	8,52422·22	8,38652·73	0,895
49	0,44747·14	8,51907·05	8,38521·04	0,904
50	0,45660·35	8,51408·44	8,38392·46	0,913
			8,32735·89	

$2\frac{1}{4}$ Per Cent.

Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
0,00966·33	0,00960·33	8,52062·79	0,49282·92	51
0,01932·66	9,71343·81	8,51612·32	0,50249·25	52
0,02899·00	9,54213·37	8,51176·23	0,51215·58	53
0,03865·33	9,42196·39	8,50753·94	0,52181·91	54
0,04831·66	9,32980·50	8,50344·87	0,53148·24	55
0,05797·99	9,25535·68	8,49948·50	0,54114·57	56
0,06764·32	9,19312·53	8,49564·32	0,55080·91	57
0,07730·65	9,13983·07	8,49191·85	0,56047·24	58
0,08696·99	9,09335·76	8,48830·64	0,57013·57	59
0,09663·32	9,05226·17	8,48480·26	0,57979·90	60
0,10629·65	9,01551·27	8,48140·31	0,58946·23	61
0,11595·98	8,98235·00	8,47810·38	0,59912·56	62
0,12562·31	8,95219·58	8,47490·13	0,60878·90	63
0,13528·64	8,92460·13	8,47179·18	0,61845·23	64
0,14494·98	8,89921·03	8,46877·22	0,62811·56	65
0,15461·31	8,87573·61	8,46583·92	0,63777·89	66
0,16427·64	8,85394·38	8,46298·99	0,64744·22	67
0,17393·97	8,83363·92	8,46022·11	0,65710·55	68
0,18360·30	8,81465·92	8,45753·02	0,66676·89	69
0,19326·63	8,79686·61	8,45491·46	0,67643·22	70
0,20292·97	8,78014·23	8,45237·16	0,68609·55	71
0,21259·30	8,76438·69	8,44989·91	0,69575·88	72
0,22225·63	8,74951·19	8,44749·44	0,70542·21	73
0,23191·96	8,73544·10	8,44515·54	0,71508·54	74
0,24158·29	8,72210·71	8,44288·00	0,72474·88	75
0,25124·62	8,70945·09	8,44066·62	0,73441·21	76
0,26090·96	8,69742·01	8,43851·20	0,74407·54	77
0,27057·29	8,68596·79	8,43641·53	0,75373·87	78
0,28023·62	8,67505·22	8,43437·47	0,76340·20	79
0,28989·95	8,66463·57	8,43238·82	0,77306·53	80
0,29956·28	8,65468·47	8,43045·41	0,78272·87	81
0,30922·61	8,64516·82	8,42857·08	0,79239·20	82
0,31888·95	8,63605·86	8,42673·69	0,80205·53	83
0,32855·28	8,62733·05	8,42495·08	0,81171·86	84
0,33821·61	8,61896·08	8,42321·10	0,82138·19	85
0,34787·94	8,61092·85	8,42151·63	0,83104·52	86
0,35754·27	8,60321·40	8,41986·53	0,84070·86	87
0,36720·60	8,59579·96	8,41825·66	0,85037·19	88
0,37686·94	8,58866·87	8,41668·90	0,86003·52	89
0,38653·27	8,58180·61	8,41516·13	0,86969·85	90
0,39619·60	8,57519·79	8,41367·26	0,87936·18	91
0,40585·93	8,56883·09	8,41222·14	0,88902·51	92
0,41552·26	8,56269·29	8,41080·69	0,89868·85	93
0,42518·59	8,55677·28	8,40942·79	0,90835·18	94
0,43484·93	8,55106·01	8,40808·36	0,91801·51	95
0,44451·26	8,54554·47	8,40677·28	0,92767·84	96
0,45417·59	8,54021·77	8,40549·46	0,93734·17	97
0,46383·92	8,53517·03	8,40424·83	0,94700·50	98
0,47350·25	8,53009·45	8,40303·28	0,95666·84	99
0,48316·58	8,52528·28	8,40184·74	0,96633·17	100
		8,35218·25		Perp.

$2\frac{3}{8}$ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.
1	0,01019·39	0,01019·39	8,53185·29	0,51988·97
2	0,02038·78	9,71423·09	8,52751·40	0,53008·36
3	0,03058·17	9,54318·69	8,52331·72	0,54027·75
4	0,04077·57	9,42327·52	8,51925·65	0,55047·14
5	0,05096·96	9,33137·25	8,51532·64	0,56066·53
6	0,06116·35	9,25717·86	8,51152·15	0,57085·92
7	0,07135·74	9,19519·91	8,50783·67	0,58105·31
8	0,08155·13	9,14215·48	8,50426·73	0,59124·71
9	0,09174·52	9,09592·97	8,50080·89	0,60144·10
10	0,10193·91	9,05508·01	8,49745·71	0,61163·49
11	0,11213·31	9,01857·51	8,49420·78	0,62182·88
12	0,12232·70	8,98565·46	8,49105·72	0,63202·27
13	0,13252·09	8,95574·04	8,48800·16	0,64221·66
14	0,14271·48	8,92838·40	8,48503·75	0,65241·05
15	0,15290·87	8,90322·91	8,48216·16	0,66260·45
16	0,16310·26	8,87998·90	8,47937·06	0,67279·84
17	0,17329·66	8,85842·88	8,47666·16	0,68299·23
18	0,18349·05	8,83835·42	8,47403·17	0,69318·62
19	0,19368·44	8,81960·24	8,47147·80	0,70338·01
20	0,20387·83	8,80203·55	8,46899·81	0,71357·40
21	0,21407·22	8,78553·60	8,46658·92	0,72376·80
22	0,22426·61	8,77000·26	8,46424·91	0,73396·19
23	0,23446·00	8,75534·79	8,46197·53	0,74415·58
24	0,24465·40	8,74149·52	8,45976·59	0,75434·97
25	0,25484·79	8,72837·75	8,45761·83	0,76454·36
26	0,26504·18	8,71593·57	8,45553·08	0,77473·75
27	0,27523·57	8,70411·73	8,45350·14	0,78493·14
28	0,28542·96	8,69287·54	8,45152·83	0,79512·54
29	0,29562·35	8,68216·82	8,44960·94	0,80531·93
30	0,30581·74	8,67195·84	8,44774·23	0,81551·32
31	0,31601·14	8,66221·19	8,44592·81	0,82570·71
32	0,32620·53	8,65289·81	8,44416·24	0,83590·10
33	0,33639·92	8,64398·93	8,44244·45	0,84609·49
34	0,34659·31	8,63546·00	8,44077·30	0,85628·88
35	0,35678·70	8,62728·74	8,43914·65	0,86648·28
36	0,36698·09	8,61945·00	8,43756·35	0,87667·67
37	0,37717·48	8,61192·87	8,43602·29	0,88687·06
38	0,38736·88	8,60470·55	8,43452·32	0,89706·45
39	0,39756·27	8,59776·41	8,43306·33	0,90725·84
40	0,40775·66	8,59108·91	8,43164·21	0,91745·23
41	0,41795·05	8,58466·66	8,43025·82	0,92764·62
42	0,42814·44	8,57848·34	8,42891·07	0,93784·02
43	0,43833·83	8,57252·74	8,42759·85	0,94803·41
44	0,44853·23	8,56678·76	8,42632·05	0,95822·80
45	0,45872·62	8,56125·31	8,42507·58	0,96842·19
46	0,46892·01	8,55591·44	8,42386·35	0,97861·58
47	0,47911·40	8,55076·19	8,42268·24	0,98880·97
48	0,48930·79	8,54578·74	8,42153·20	0,99900·37
49	0,49950·18	8,54098·27	8,42041·11	1,00919·76
50	0,50969·57	8,53634·03	8,41931·90	1,01939·15
			8,37566·36	

$2\frac{1}{2}$ Per Cent.

rs	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
.	0,01072·39	0,01072·39	8,54293·36	0,54691·71	51
)	0,02144·77	9,71502·27	8,53875·55	0,55764·10	52
)	0,03217·16	9,54423·83	8,53471·76	0,56836·49	53
)	0,04289·55	9,42458·42	8,53081·40	0,57908·87	54
)	0,05361·93	9,33293·68	8,52703·92	0,58981·26	55
)	0,06434·32	9,25899·62	8,52338·78	0,60053·65	56
)	0,07506·71	9,19726·80	8,51985·48	0,61126·03	57
)	0,08579·09	9,14447·26	8,51643·54	0,62198·42	58
)	0,09651·48	9,09849·45	8,51312·52	0,63270·81	59
)	0,10723·87	9,05788·95	8,50991·99	0,64343·19	60
)	0,11796·25	9,02162·73	8,50681·53	0,65415·58	61
)	0,12868·64	8,98894·73	8,50380·78	0,66487·97	62
)	0,13941·03	8,95927·17	8,50089·34	0,67560·35	63
)	0,15013·41	8,93215·16	8,49806·90	0,68632·74	64
)	0,16085·80	8,90723·11	8,49533·10	0,69705·13	65
)	0,17158·18	8,88422·30	8,49267·63	0,70777·51	66
)	0,18230·57	8,86289·29	8,49010·20	0,71849·90	67
)	0,19302·96	8,84304·63	8,48760·50	0,72922·28	68
)	0,20375·34	8,82452·04	8,48518·27	0,73994·67	69
)	0,21447·73	8,80717·72	8,48283·24	0,75067·06	70
)	0,22520·12	8,79089·94	8,48055·17	0,76139·44	71
)	0,23592·50	8,77558·57	8,47833·80	0,77211·83	72
)	0,24664·89	8,76114·85	8,47618·91	0,78284·22	73
)	0,25737·28	8,74751·14	8,47410·29	0,79356·60	74
)	0,26809·66	8,73460·72	8,47207·72	0,80428·99	75
)	0,27882·05	8,72237·68	8,47010·99	0,81501·38	76
)	0,28954·44	8,71076·77	8,46819·91	0,82573·76	77
)	0,30026·82	8,69973·31	8,46634·31	0,83646·15	78
)	0,31099·21	8,68923·13	8,46453·99	0,84718·54	79
)	0,32171·60	8,67922·47	8,46278·79	0,85790·92	80
)	0,33243·98	8,66967·94	8,46108·53	0,86863·31	81
)	0,34316·37	8,66056·49	8,45943·08	0,87935·70	82
)	0,35388·76	8,65185·33	8,45782·26	0,89008·08	83
)	0,36461·14	8,64351·93	8,45625·94	0,90080·47	84
)	0,37533·53	8,63553·99	8,45473·97	0,91152·86	85
)	0,38605·92	8,62789·39	8,45326·23	0,92225·24	86
)	0,39678·30	8,62056·18	8,45182·56	0,93297·63	87
)	0,40750·69	8,61352·59	8,45042·85	0,94370·02	88
)	0,41823·08	8,60676·98	8,44906·99	0,95442·40	89
)	0,42895·46	8,60027·83	8,44774·84	0,96514·79	90
)	0,43967·85	8,59403·72	8,44646·31	0,97587·18	91
)	0,45040·23	8,58803·36	8,44521·27	0,98659·56	92
)	0,46112·62	8,58225·53	8,44399·64	0,99731·95	93
)	0,47185·01	8,57669·11	8,44281·29	1,00804·33	94
)	0,48257·39	8,57133·06	8,44166·15	1,01876·72	95
)	0,49329·78	8,56616·35	8,44054·10	1,02949·11	96
)	0,50402·17	8,56118·12	8,43945·07	1,04021·49	97
)	0,51474·55	8,55637·48	8,43838·95	1,05093·88	98
)	0,52546·94	8,55173·64	8,43735·68	1,06166·27	99
)	0,53619·33	8,54725·84	8,43635·16	1,07238·65	100
			8,39704·00		Perp.

2 5/8 Per Cent.

Years	Log. r^n.	Log. d^n.	Log. e^n.	Log. r^n.
1	0,01125·32	0,01125·32	8,55387·14	0,57391·17
2	0,02250·63	9,71581·33	8,54984·90	0,58516·48
3	0,03375·95	9,54528·79	8,54596·51	0,59641·80
4	0,04501·27	9,42589·07	8,54221·36	0,60767·12
5	0,05626·59	9,33449·80	8,53858·90	0,61892·44
6	0,06751·90	9,26080·97	8,53508·61	0,63017·75
7	0,07877·22	9,19933·16	8,53169·96	0,64143·07
8	0,09002·54	9,14678·41	8,52842·49	0,65268·39
9	0,10127·85	9,10105·19	8,52525·77	0,66393·70
10	0,11253·17	9,06069·03	8,52219·35	0,67519·02
11	0,12378·49	9,02466·04	8,51922·83	0,68644·34
12	0,13503·80	8,99222·85	8,51635·83	0,69769·65
13	0,14629·12	8,96278·97	8,51357·98	0,70894·97
14	0,15754·44	8,93590·42	8,51088·94	0,72020·29
15	0,16879·76	8,91121·61	8,50828·38	0,73145·61
16	0,18005·07	8,88843·83	8,50575·97	0,74270·92
17	0,19130·39	8,86733·61	8,50331·43	0,75396·24
18	0,20255·71	8,84771·55	8,50094·45	0,76521·56
19	0,21381·02	8,82941·31	8,49864·78	0,77646·87
20	0,22506·34	8,81229·15	8,49642·15	0,78772·19
21	0,23631·66	8,79623·30	8,49426·29	0,79897·51
22	0,24756·97	8,78113·65	8,49217·00	0,81022·82
23	0,25882·29	8,76691·42	8,49014·01	0,82148·14
24	0,27007·61	8,75348·99	8,48817·13	0,83273·46
25	0,28132·93	8,74079·64	8,48626·14	0,84398·78
26	0,29258·24	8,72877·45	8,48440·48	0,85524·09
27	0,30383·56	8,71737·17	8,48261·04	0,86649·41
28	0,31508 88	8,70654·14	8,48086·55	0,87774·73
29	0,32634·19	8,69624·17	8,47917·20	0,88900·04
30	0,33759·51	8,68643·52	8,47752·80	0,90025·36
31	0,34884·83	8,67708·78	8,47593·21	0,91150·68
32	0,36010·14	8,66816·90	8,47438·26	0,92275·99
33	0,37135·46	8,65965·11	8,47287·81	0,93401·31
34	0,38260·78	8,65150·88	8,47141·70	0,94526·63
35	0,39386·10	8,64371·89	8,46999·80	0,95651·95
36	0,40511·41	8,63626·03	8,46861·98	0,96777·26
37	0,41636·73	8,62911·38	8,46728·10	0,97902·58
38	0,42762·05	8,62226·13	8,46598·05	0,99027·90
39	0,43887·36	8,61568·65	8,46471·69	1,00153·21
40	0,45012·68	8,60937·42	8,46348·91	1,01278·53
41	0,46138·00	8,60331·05	8,46229·62	1,02403·85
42	0,47263·31	8,59748·22	8,46113·68	1,03529·16
43	0,48388·63	8,59187·72	8,46001·01	1,04654·48
44	0,49513·95	8,58648·42	8,45891·50	1,05779·80
45	0,50639·27	8,58129·29	8,45785·06	1,06905·12
46	0,51764·58	8,57629·35	8,45681·59	1,08030·43
47	0,52889·90	8,57147·65	8,45581·00	1,09155·75
48	0,54015·22	8,56683·36	8,45483·22	1,10281·07
49	0,55140·53	8,56235·68	8,45388·13	1,11406·38
50	0,56265·85	8,55803·84	8,45295·69	1,12531·70
			8,41912·93	

2 3/4 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01178·18	0,01178·18	8,56466·78	0,60087·34	51
2	0,02356·37	9,71660·28	8,56079·64	0,61265·52	52
3	0,03534·55	9,54633·59	8,55706·15	0,62443·70	53
4	0,04712·73	9,42719·49	8,55345·71	0,63621·88	54
5	0,05890·92	9,33605·59	8,54997·78	0,64800·07	55
6	0,07069·10	9,26261·92	8,54661·81	0,65978·25	56
7	0,08247·28	9,20139·02	8,54337·31	0,67156·43	57
8	0,09425·46	9,14908·96	8,54023·81	0,68334·62	58
9	0,10603·65	9,10360·17	8,53720·85	0,69512·80	59
10	0,11781·83	9,06348·24	8,53428·02	0,70690·98	60
11	0,12960·01	9,02770·14	8,53144·91	0,71869·17	61
12	0,14138·20	8,99549·79	8,52871·14	0,73047·35	62
13	0,15316·38	8,96629·44	8,52606·34	0,74225·53	63
14	0,16494·56	8,93964·19	8,52350·17	0,75403·72	64
15	0,17672·75	8,91518·44	8,52102·30	0,76581·90	65
16	0,18850·93	8,89263·51	8,51862·41	0,77760·08	66
17	0,20029·11	8,87175·89	8,51630·21	0,78938·26	67
18	0,21207·29	8,85236·18	8,51405·41	0,80116·45	68
19	0,22385·48	8,83428·09	8,51187·74	0,81294·63	69
20	0,23563·66	8,81737·85	8,50976·93	0,82472·81	70
21	0,24741·84	8,80153·69	8,50772·75	0,83656·00	71
22	0,25920·03	8,78665·49	8,50574·95	0,84829·18	72
23	0,27098·21	8,77264·51	8,50383·30	0,86007·36	73
24	0,28276·39	8,75943·09	8,50197·59	0,87185·55	74
25	0,29454·58	8,74694·52	8,50017·61	0,88363·73	75
26	0,30632·76	8,73512·89	8,49843·16	0,89541·91	76
27	0,31810·94	8,72392·96	8,49674·05	0,90720·09	77
28	0,32989·13	8,71330·05	8,49510·10	0,91898·28	78
29	0,34167·31	8,70319·98	8,49351·12	0,93076·46	79
30	0,35345·49	8,69359·00	8,49196·96	0,94254·64	80
31	0,36523·67	8,68443·72	8,49047·45	0,95432·83	81
32	0,37701·86	8,67571·08	8,48902·44	0,96611·01	82
33	0,38880·04	8,66738·32	8,48761·76	0,97789·19	83
34	0,40058·22	8,65942·89	8,48625·28	0,98967·38	84
35	0,41236·41	8,65182·48	8,48492·87	1,00145·56	85
36	0,42414·59	8,64455·00	8,48364·40	1,01323·74	86
37	0,43592·77	8,63758·51	8,48239·72	1,02501·93	87
38	0,44770·96	8,63091·21	8,48118·72	1,03680·11	88
39	0,45949·14	8,62451·47	8,48001·29	1,04858·29	89
40	0,47127·32	8,61837·77	8,47887·30	1,06036·47	90
41	0,48305·51	8,61248·72	8,47776·64	1,07214·66	91
42	0,49483·69	8,60682·99	8,47669·23	1,08392·84	92
43	0,50661·87	8,60139·40	8,47564·94	1,09571·02	93
44	0,51840·05	8,59616·81	8,47463·68	1,10749·21	94
45	0,53018·24	8,59114·17	8,47365·35	1,11927·39	95
46	0,54196·42	8,58630·51	8,47269·87	1,13105·57	96
47	0,55374·60	8,58164·91	8,47177·15	1,14283·76	97
48	0,56552·79	8,57716·51	8,47087·10	1,15461·94	98
49	0,57730·97	8,57284·52	8,46999·64	1,16640·12	99
50	0,58909·15	8,56868·18	8,46914·69	1,17818·31	100
			8,43033·27		Perp.

2 7/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0.01230·98	0,01230·98	8,575,32·41	0,62780·23	51
2	0.02461·97	9.717,39·12	8.57159·90	0,64011·21	52
3	0.03692·95	9.54738·21	8.56800·85	0.65242·20	53
4	0.04923·94	9.42849·66	8.56454·64	0.66473·18	54
5	0.06154·92	9.33761·07	8.56120·75	0,6770417	55
6	0.07385·91	9.26442·45	8.55798·62	0.68935·15	56
7	0.08616·89	9.20344·38	8.55487·76	0,70166·13	57
8	0.09847·88	9.151,38·89	8.55187·72	0,71397·12	58
9	0.11078·86	9.10614·43	8.54898·03	0,72628·10	59
10	0.12309·85	9,06626·59	8,54618·27	0,73859·09	60
11	0.13540·83	9.03072·33	8.54348·06	0,75090·07	61
12	0.14771·82	8.998,75·58	8.54086·99	0 76321·06	62
13	0.16002·80	8.969,78 60	8.53834·72	0,77552·04	63
14	0.17233·79	8.94336·46	8.53590·89	0,78783·03	64
15	0.18464·77	8.91913·59	8.53355·18	0.80014·01	65
16	0.19695·76	8 89681·29	8.53127·28	0.81245·00	66
17	0.20926·74	8 8·616·09	8.52906·89	0 82475·98	67
18	0.22157·73	8 85698·54	8.52693·72	0 83706·97	68
19	0.23388·71	8.83912·38	8.52487·51	0.84937·95	69
20	0.24619·70	8.82243 82	8.52288 00	0,86168·94	70
21	0,25850·68	8.80681·12	8.52094·94	0.87399·92	71
22	0.27081·67	8.79214·13	8.51908·09	0,88630·91	72
23	0 28312·65	8 77834·13	8,51727·23	0.89861·89	73
24	0.29543·64	8.76533·45	8.51552·14	0.91092·88	74
25	0.30774·62	8.75305·39	8.51382·63	0,92323·86	75
26	0.32005·61	8.74144·05	8,51218·47	0·93554·85	76
27	0.33236·59	8.73044·16	8.51059·50	0 94785 83	77
28	0.34467·57	8.72001·06	8.50905·53	0,96016·82	78
29	0 35698·56	8.71010·58	8.50756·38	0.97247·80	79
30	0,36929·54	8,70068·95	8,50611·90	0,98478·79	80
31	0,38160·53	8.69172·80	8.50471·90	0.99709·77	81
32	0,39391·51	8.68319·07	8.50336·25	1,00940·76	82
33	0.40622·50	8.67504·98	8.50204·80	1.02171·74	83
34	0.41853·48	8.66728·00	8.50077·41	1,03402·72	84
35	0.43084·47	8.65985·83	8.49953·92	1,04633·71	85
36	0·44315·45	8.65276·36	8.49834·22	1,05864·69	86
37	0 45546·44	8.64597·66	8.49718·19	1,07095·68	87
38	0 46777·42	8.63947·92	8.49605·69	1,08326·66	88
39	0 48008·41	8.63325·52	8.49496·62	1,09557·65	89
40	0 49239·39	8,62728·96	8,49390 86	1,10788·63	90
41	0 50470·38	8,62156·81	8.49288·30	1,12019·62	91
42	0.51701·36	8.61607·79	8,49188·83	1,13250·60	92
43	0,52932·35	8,61080·69	8,49092·36	1,14481·59	93
44	0.54163·33	8.60574·36	8,48998·80	1,15712·57	94
45	0,55394·32	8.60087·79	8,48908·04	1,16943·56	95
46	0.56625·30	8.59619·98	8,48820·00	1,18174·54	96
47	0.57856·29	8,59170·02	8.48734·59	1,19405·53	97
48	0.59087·27	8,58737·06	8,48651·73	1,20636·51	98
49	0 60318·26	8,58320·31	8,48571·34	1,21867·50	99
50	0,61549·24	8,57918·99	8,48493·33	1,23098·48	100
			8,45863·78		Perp.

3 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01283·72	0,01283·72	8,58584·21	0,65469·85	51
2	0,02567·44	9,71817·84	8,58225·86	0,66753·57	52
3	0,03851·17	9,54842·67	8,57880·78	0,68037·29	53
4	0,05134·89	9,42979·59	8,57548·32	0,69321·01	54
5	0,06418·61	9,33916·23	8,57227·98	0,70604·74	55
6	0,07702·33	9,26622·58	8,56919·22	0,71888·46	56
7	0,08986·06	9,20549·22	8,56621·52	0,73172·18	57
8	0,10269·78	9,15368·19	8,56334·42	0,74455·90	58
9	0,11553·50	9,10867·95	8,56057·53	0,75739·63	59
10	0,12837·22	9,06904·07	8,55790·35	0,77023·35	60
11	0,14120·95	9,03373·51	8,55532·53	0,78307·07	61
12	0,15404·67	9,00200·22	8,55283·67	0,79590·79	62
13	0,16688·39	8,97326·43	8,55043·41	0,80874·52	63
14	0,17972·11	8,94707·25	8,54811·42	0,82158·24	64
15	0,19255·84	8,92307·08	8,54587·36	0,83441·96	65
16	0,20539·56	8,90097·23	8,54370·93	0,84725·68	66
17	0,21823·28	8,88054·22	8,54161·83	0,86009·41	67
18	0,23107·00	8,86158·64	8,53959·78	0,87293·13	68
19	0,24390·73	8,84394·18	8,53764·51	0,88576·85	69
20	0,25674·45	8,82747·08	8,53575·76	0,89860·57	70
21	0,26958·17	8,81205·58	8,53393·29	0,91144·30	71
22	0,28241·89	8,79759·57	8,53216·86	0,92428·02	72
23	0,29525·62	8,78400·29	8,53046·27	0,93711·74	73
24	0,30809·34	8,77120·09	8,52881·27	0,94995·46	74
25	0,32093·06	8,75912·27	8,52721·68	0,96279·19	75
26	0,33376·78	8,74770·92	8,52567·30	0,97562·91	76
27	0,34660·51	8,73690·79	8,52417·94	0,98846·63	77
28	0,35944·23	8,72667·20	8,52273·41	1,00130·35	78
29	0,37227·95	8,71696·00	8,52133·56	1,01414·08	79
30	0,38511·67	8,70773·41	8,51998·21	1,02697·80	80
31	0,39795·40	8,69896·07	8,51867·20	1,03981·52	81
32	0,41079·12	8,69060·91	8,51740·38	1,05265·24	82
33	0,42362·84	8,68265·15	8,51617·61	1,06548·97	83
34	0,43646·56	8,67506·27	8,51498·76	1,07832·69	84
35	0,44930·29	8,66781·98	8,51383·67	1,09116·41	85
36	0,46214·01	8,66090·15	8,51272·22	1,10400·13	86
37	0,47497·73	8,65428·85	8,51164·30	1,11683·85	87
38	0,48781·45	8,64796·30	8,51059·77	1,12967·58	88
39	0,50065·18	8,64190·86	8,50958·53	1,14251·30	89
40	0,51348·90	8,63611·04	8,50860·46	1,15535·02	90
41	0,52632·62	8,63055·41	8,50765·46	1,16818·74	91
42	0,53916·34	8,62522·68	8,50673·42	1,18102·47	92
43	0,55200·07	8,62011·63	8,50584·25	1,19386·19	93
44	0,56483·79	8,61521·17	8,50497·85	1,20669·91	94
45	0,57767·51	8,61050·23	8,50414·14	1,21953·63	95
46	0,59051·23	8,60597·85	8,50333·02	1,23237·36	96
47	0,60334·96	8,60163·10	8,50254·40	1,24521·08	97
48	0,61618·68	8,59745·14	8,50178·22	1,25804·80	98
49	0,62902·40	8,59343·16	8,50104·37	1,27088·52	99
50	0,64186·12	8,58956·42	8,50032·80	1,28372·25	100
			8,47712·13		Perp.

3 1/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01336·40	0,01336·40	8.59622·33	0,68156·20	51
2	0,02672·79	9,71896·45	8.59277·69	0,69492·60	52
3	0,04009·19	9.54946·96	8.58946·09	0,70829·00	53
4	0,05345·58	9,43109·29	8.58626·95	0,72165·39	54
5	0,06681·98	9.34071·08	8.58319·69	0,73501·79	55
6	0,08018·38	9,26802·30	8.58023·81	0,74838·18	56
7	0,09354·77	9.20753·57	8.57738·81	0,76174·58	57
8	0,10691·17	9.15596·89	8.57464·21	0,77510·98	58
9	0,12027·57	9.11120·74	8.57199·59	0,78847·37	59
10	0,13363·96	9,07180·68	8.56944·52	0,80183·77	60
11	0,14700·36	9.03673·69	8.56698·59	0,81520·17	61
12	0,16036·75	9.00523·70	8.56461·45	0.82856·56	62
13	0,17373·15	8.97672·96	8.56232·71	0,84192·96	63
14	0,18709·55	8,95076·56	8.56012·06	0,85529·35	64
15	0,20045·94	8.92698·91	8.55799·15	0,86865·75	65
16	0,21382·34	8.90511·32	8.55593·69	0,88202·15	66
17	0,22718·73	8.88490·32	8.55395·39	0,89538·54	67
18	0,24055·13	8.86616·47	8.55203·95	0,90874·94	68
19	0,25391·53	8.84873·51	8.55019·12	0,92211·34	69
20	0,26727·92	8,83247·65	8.54840·62	0,93547·73	70
21	0,28064·32	8,81727·12	8.54668·24	0,94884·13	71
22	0,29400·72	8,80301·83	8.54501·74	0,96220·52	72
23	0,30737·11	8,78963·02	8.54340·89	0,97556·92	73
24	0,32073·51	8,77703·03	8.54185·47	0,98893·32	74
25	0,33409·90	8.76515·18	8.54035·30	1,00229·71	75
26	0,34746·30	8,75393·54	8.53890·17	1,01566·11	76
27	0,36082·70	8.74332·87	8.53749·91	1,02902·50	77
28	0,37419·09	8.73328·51	8.53614·32	1,04238·90	78
29	0,38755·49	8,72376·27	8.53483·25	1,05575·30	79
30	0,40091·88	8,71472·42	8.53356·52	1,06911·69	80
31	0,41428·28	8,70613·55	8.53233·99	1,08248·09	81
32	0,42764·68	8.69796·63	8.53115·50	1,09584·49	82
33	0,44101·07	8.69018 87	8.53000 90	1,10920·88	83
34	0,45437·47	8.68277·75	8.52890 07	1,12257·28	84
35	0,46773·87	8.67570·97	8.52782·86	1,13593·67	85
36	0,48110·26	8.86896·42	8.52679·16	1,14930·07	86
37	0,49446·66	8,66252·16	8.52578·84	1,16266·47	87
38	0,50783·05	8,65636·44	8.52481·77	1,17602·86	88
39	0,52119·45	8.65047·58	8.52387·86	1,18939·26	89
40	0,53455·85	8,64484·10	8.52296·98	1,20275·65	90
41	0,54792·24	8.63944·58	8.52209·04	1,21612·05	91
42	0,56128·64	8,63427·75	8.52123·93	1,22948·45	92
43	0,57465·03	8.62932·37	8.52041·56	1,24284·84	93
44	0,58801·43	8,62457·34	8.51961·84	1,25621·24	94
45	0,60137·83	8,62001·62	8.51884·67	1,26957·64	95
46	0,61474·22	8,61564·23	8.51809·97	1,28294·03	96
47	0,62810·62	8,61144·26	8.51737·66	1,29630·43	97
48	0,64147·02	8,60740·86	8.51667·65	1,30966·82	98
49	0,65483·41	8,60353·23	8.51599·87	1,32303·22	99
50	0,66819·81	8,59980·62	8.51534·25	1,33639·62	100
			8.49385·00		Perp

3 ¼ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01389·01	0,01389·01	8,60646·92	0,70839·31	51
2	0,02778·01	9,71974·96	8,60315·55	0,72228·31	52
3	0,04167·02	9,55051·08	8,59997·00	0,73617·32	53
4	0,05556·02	9,43238·76	8,59690·69	0,75006·33	54
5	0,06945·03	9,34225·60	8,59396·07	0,76395·33	55
6	0,08334·04	9,26981·63	8,59112·62	0,77784·34	56
7	0,09723·04	9,20957·41	8,58839·84	0,79173·34	57
8	0,11112 05	9,15824·08	8,58577·28	0,80562·35	58
9	0,12501·05	9,11372·80	8,58324·47	0,81951·36	59
10	0,13890·06	9,07456·43	8,58081·02	0,83340·36	60
11	0,15279·07	9,03972·87	8,57846·52	0,84729·37	61
12	0,16668·07	9,00846·04	8,57620 62	0.86118·37	62
13	0,18057·08	8,98018·18	8,57402·93	0,87507·38	63
14	0,19446·08	8,95444·39	8,57193·13	0,88896·39	64
15	0,20835·09	8,93089·08	8,56990·90	0,90285·39	65
16	0,22224·10	8,90923·57	8,56795·93	0,91674·40	66
17	0,23613·10	8,88924·38	8,56607·92	0,93063·40	67
18	0,25002·11	8,87072·07	8,56426·61	0,94452·41	68
19	0,26391·11	8,85350·37	8,56251·72	0,95841·42	69
20	0,27780·12	8,83745·51	8,56083·01	0,97230·42	70
21	0,29169·13	8,82245·73	8,55920·23	0,98619·43	71
22	0,30558·13	8,80840·92	8,55763·15	1,00008·43	72
23	0,31947·14	8,79522·32	8,55611·56	1,01397·44	73
24	0,33336·14	8,78282·30	8,55465·23	1,02786·45	74
25	0,34725·15	8,77114·14	8,55323·98	1,04175·45	75
26	0,36114·16	8,76011·94	8,55187·62	1,05564·46	76
27	0,37503·16	8,74970·45	8,55055·96	1,06953·46	77
28	0,38892·17	8,73985 00	8,54928 82	1,08342·47	78
29	0,40281·17	8,73051·44	8,54806·03	1,09731·48	79
30	0,41670·18	8,72165·99	8,54687·46	1,11120·48	80
31	0,43059·19	8,71325·20	8,54572·89	1,12509·49	81
32	0,44448·19	8,70526·28	8,54462·24	1,13898·49	82
33	0,45837·20	8,69766·18	8,54355·34	1,15287·50	83
34	0,47226·21	8 69042·48	8 54252·05	1,16676·51	84
35	0,48615·21	8,68352 86	8 54152·24	1,18065·51	85
36	0,50004·22	8 67695·24	8,54055·79	1,19454·52	86
37	0,51393·22	8,67067·67	8,53962·59	1,20843·52	87
38	0,52782·23	8,66468·37	8,53872·51	1,22232·53	88
39	0,54171·24	8,65895·73	8,53785·45	1,23621·54	89
40	0,55560·24	8,65348·21	8,53701·28	1,25010·54	90
41	0,56910·25	8,64824·42	8,53619·93	1,26399·55	91
42	0,58338·25	8,64323·07	8 53541·28	1,27788·55	92
43	0,59727·26	8,63842·97	8,53465·23	1,29177·56	93
44	0,61116·27	8,63382·97	8,53391·71	1,30566·57	94
45	0,62505·27	8,62942·06	8,53320·63	1,31955·57	95
46	0,63894·28	8,62519·25	8,53251·89	1,33344·58	96
47	0,65283·28	8,62113·63	8,53185·41	1,34733·58	97
48	0,66672·29	8,61724·36	8,53121·14	1,36122·59	98
49	0,68061·30	8,61350·65	8,53058·96	1,37511·60	99
50	0,69450·30	8,60991·73	8,52998·83	1,38900·60	100
			8,52188·34		Perp.

$3\frac{3}{8}$ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01441·55	0,01441·55	8,61658·14	0,73519·17	51
2	0,02883·10	9,72053·35	8,61339·61	0,74960·72	52
3	0,04324·66	9,55155·02	8,61033·67	0,76402·27	53
4	0,05766·21	9,43367·98	8,60739·76	0,77843·82	54
5	0,07207·76	9,34379·82	8,60457·33	0,79285·37	55
6	0,08649·31	9,27160·56	8,60185·85	0,80726·93	56
7	0,10090·87	9,21160·74	8,59924·85	0,82168·48	57
8	0,11532·42	9,16052·46	8,59673·85	0,83610·03	58
9	0,12973·97	9,11624·14	8,59432·41	0,85051·58	59
10	0,14415·52	9,07731·34	8,59200·13	0,86493·14	60
11	0,15857·07	9,04271·06	8,58976·61	0,87934·69	61
12	0,17298·63	9,01167·24	8,58761·47	0,89376·24	62
13	0,18740·18	8,98362·11	8,58554·37	0,90817·79	63
14	0,20181·73	8,95810·76	8,58354·97	0,92259·34	64
15	0,21623·28	8,93477·62	8,58162·94	0,93700·90	65
16	0,23064·84	8,91333·99	8,57977·98	0,95142·45	66
17	0,24506·39	8,89356·40	8,57799·81	0,96584·00	67
18	0,25947·94	8,87525·43	8,57628·15	0,98025·55	68
19	0,27389·49	8,85824·80	8,57462·75	0,99467·11	69
20	0,28831·05	8,84240·71	8,57303·32	1,00908·66	70
21	0,30272·60	8,82761·44	8,57149·67	1,02350·21	71
22	0,31714·15	8,81376·87	8,57001·56	1,03791·76	72
23	0,33155·70	8,80078·24	8,56858·74	1,05233·31	73
24	0,34597·25	8,78857·90	8,56721·04	1,06674·87	74
25	0,36038·81	8,77709·16	8,56588·25	1,08116·42	75
26	0,37480·36	8,76626·12	8,56460·18	1,09557·97	76
27	0,38921·91	8,75603·53	8,56336·65	1,10999·52	77
28	0,40363·46	8,74636·72	8,56217·50	1,12441·08	78
29	0,41805·02	8,73721·51	8,56102·52	1,13882·63	79
30	0,43246·57	8,72854·17	8,55991·60	1,15324·18	80
31	0,44688·12	8,72031·32	8,55884·57	1,16765·73	81
32	0,46129·67	8,71249·89	8,55781·29	1,18207·29	82
33	0,47571·22	8,70507·13	8,55681·61	1,19648·84	83
34	0,49012·78	8,69800·50	8,55585·40	1,21090·39	84
35	0,50454·33	8,69127·71	8,55492·53	1,22531·94	85
36	0,51895·88	8,68486·66	8,55402·88	1,23973·49	86
37	0,53337·43	8,67875·41	8,55316·34	1,25415·05	87
38	0,54778·99	8,67292·19	8,55232·79	1,26856·60	88
39	0,56220·54	8,66735·37	8,55152·12	1,28298·15	89
40	0,57662·09	8,66203·45	8,55074·23	1,29739·70	90
41	0,59103·64	8,65695·01	8,54999·00	1,31181·26	91
42	0,60545·19	8,65208·77	8,54926·35	1,32622·81	92
43	0,61986·75	8,64743·53	8,54856·21	1,34064·36	93
44	0,63428·30	8,64298·18	8,54788·44	1,35505·91	94
45	0,64869·85	8,63871·66	8,54723·01	1,36947·46	95
46	0,66311·40	8,63463·02	8,54659·79	1,38389·02	96
47	0,67752·96	8,63071·35	8,54598·73	1,39830·57	97
48	0,69194·51	8,62695·79	8,54539·76	1,41272·12	98
49	0,70636·06	8,62335·56	8,54482·77	1,42713·67	99
50	0,72077·61	8,61989·91	8,54427·70	1,44155·23	100
			8,52827·38		Perp.

3 ¼ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01404·03	0,01494·03	8,62656·17	0,76195·78	51
2	0,02988·07	9,72131·63	8,62350·03	0,77689·82	52
3	0,04482·10	9,55258·81	8,62056·28	0,79183·85	53
4	0,05976·14	9,43496·97	8,61774·35	0,80677·89	54
5	0,07470·17	9,34533·72	8,61503·66	0,82171·92	55
6	0,08964·21	9,27339·07	8,61243·73	0,83665·96	56
7	0,10458·24	9,21363·59	8,60994·06	0,85159·99	57
8	0,11952·28	9,16279·32	8,60754·19	0,86654·03	58
9	0,13446·31	9,11874·74	8,60523·67	0,88148·06	59
10	0,14940·35	9,08005·39	8,60302·11	0,89642·10	60
11	0,16434·38	9,04568·27	8,60089·11	0,91136·13	61
12	0,17928·42	9,01487·30	8,59884·30	0,92630·17	62
13	0,19422·45	8,98704·74	8,59687·33	0,94124·20	63
14	0,20916·49	8,96175·67	8,59497·87	0,95618·24	64
15	0,22410·52	8,93864·51	8,59345·60	0,97112·27	65
16	0,23904·56	8,91742·58	8,59140·20	0,98606·31	66
17	0,25398·59	8,89786·42	8,58971·42	1,00100·34	67
18	0,26892·63	8,88976·57	8,58808·95	1,01594·38	68
19	0,28386·66	8,86296·78	8,58652·57	1,03088·41	69
20	0,29880·70	8,84733·25	8,58502·01	1,04582·45	70
21	0,31374·73	8,83274·26	8,58357·02	1,06076·48	71
22	0,32868·77	8,81909·67	8,58217·40	1,07570·52	72
23	0,34362·80	8,80630·76	8,58082·92	1,09064·55	73
24	0,35856·84	8,79429·86	8,57953·39	1,10558·59	74
25	0,37350·87	8,78300·29	8,57828·60	1,12052·62	75
26	0,38844·91	8,77236·13	8,57708·38	1,13546·66	76
27	0,40238·94	8,76232·15	8,57592·53	1,15040·69	77
28	0,41832·98	8,75283·67	8,57480·89	1,16534·73	78
29	0,43327·01	8,74386·54	8,57373·31	1,18028·76	79
30	0,44821·05	8,73536 99	8,57269·61	1,19522·80	80
31	0,46315·08	8,72731·67	8,57169·65	1,21016·83	81
32	0,47809·12	8,71967·52	8,57073·29	1,22510·87	82
33	0,49303·15	8,71241·75	8,56980·39	1,24004·90	83
34	0,50797·19	8,70551·87	8,56890·83	1,25498·94	84
35	0,52291·22	8,69895·57	8,56804·46	1,26992·97	85
36	0,53785·26	8,69270·74	8,56721·18	1,28487·01	86
37	0,55279·29	8,68675·46	8,56640·87	1,29981·04	87
38	0,56773·33	8,68107·96	8,56563·41	1,31475·08	88
39	0,58267·36	8,67566 61	8,56488·71	1,32969·11	89
40	0,59761·40	8,67049·89	8,56416 65	1,34463·15	90
41	0,61255·43	8,66556·43	8,56347·14	1,35957·18	91
42	0,62749·47	8,66084·92	8,56280·09	1,37451·22	92
43	0,64243·50	8,65634·16	8,56215·41	1,38945·25	93
44	0,65737·54	8,65203·04	8,56153·00	1,40439·29	94
45	0,67231·57	8,64790·53	8,56092·79	1,41933·32	95
46	0,68725·61	8,64395·66	8,56034·69	1,43427·36	96
47	0,70219·64	8,64017·52	8,55978·64	1,44921·39	97
48	0,71713·68	8,63655·27	8,55924·54	1,46415·43	98
49	0,73207·71	8,63308·11	8,55872·35	1,47909·46	99
50	0,74701·75	8,62975·31	8,55821·96	1,49403·50	100
			8,54406·80		Perp.

$3\frac{5}{8}$ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0.01546·45	0.01546·45	8,63641·15	0,78869·17	51
2	0,03092 91	9·72209·80	8,63347·00	0,80415·63	52
3	0,04639·36	9·55362·44	8,63065·02	0,81962·08	53
4	0,06185·82	9·43625·72	8,62794·62	0,83508·54	54
5	0,07732·27	9·34687·31	8,62535·28	0,85054·99	55
6	0.09278·73	9·27517·19	8,62286·45	0,86601·44	56
7	0.10825·18	9.21565·94	8,62047·69	0,88147·90	57
8	0,12371·63	9,16505·610	8,61818·52	0,89694·35	58
9	0.13918·09	9,12124·62	8,61598·50	0,91240·81	59
10	0,15464·54	9,08278·59	8,61387·22	0,92787·26	60
11	0.17011·00	9,04864·47	8.61184·31	0,94333·72	61
12	0,18557·45	9 01806·22	8,60989·39	0,95880·17	62
13	0.20103·91	8.99246·07	8,60802·13	0,97426·62	63
14	0.21650·36	8.96539·11	8,60622·17	0,98973·08	64
15	0,23196·82	8·94249·77	8,60449·21	1,00519·53	65
16	0.24743·27	8.92149·36	8,60282·94	1,02065·99	66
17	0,26289·72	8.90214·42	8,60123·11	1,03612·44	67
18	0.27836·18	8,88425·49	8,59969·41	1,05158·90	68
19	0,29382·63	8.86766·32	8,59821·61	1,06705·35	69
20	0,30929·09	8,85223·13	8,59679·46	1,08251·81	70
21	0.32475·54	8 83784·18	8,59542·71	1,09798·26	71
22	0,34022·00	8 82439·36	8,59411·16	1,11344·71	72
23	0,35568·45	8.81179·91	8,59284·57	1,12891·17	73
24	0,37114·90	8,79998·20	8,59162·77	1,14437·62	74
25	0,38661·36	8,7888,·52	8,59045·57	1,15984·08	75
26	0.40207·81	8,7,841·97	8,58932·74	1,17530·53	76
27	0.41754·27	8,76856·32	8,58824·15	1,19076·99	77
28	0.43,00·72	8,75925 90	8,58719·61	1,20623·44	78
29	0.44847·18	8,75046·55	8,58618 97	1,22169·89	79
30	0,46393·63	8,74214·50	8,58522·07	1,23716·35	80
31	0.47940·09	8,73426·39	8.58428·77	1,25262·80	81
32	0.49486·54	8,72679·19	8.58338 91	1,26809·26	82
33	0,51032·99	8,71970·10	8,58252·38	1,28355·71	83
34	0.52579·45	8,71296·62	8,58169·04	1,29902·17	84
35	0,54125·90	8,70656·48	8,58088·76	1,31448·62	85
36	0,55672·36	8.70047·54	8,58011·44	1,32995·07	86
37	0,57218 81	8.69467·89	8,57936·95	1,34541·53	87
38	0,58765·27	8.68915·74	8,57865·18	1,36087·98	88
39	0,60311·72	8.68,89·49	8,57796·03	1,37634·44	89
40	0,61858·17	8,6,887·63	8,57729·41	1,39180·89	90
41	0,63404·63	8.67408·76	8,57665·23	1,40727·35	91
42	0,64951·08	8.66951·59	8,57603·37	1,42273·80	92
43	0.66497·54	8.66514·94	8,57543·75	1,43820·26	93
44	0.68043·99	8.66097·69	8,57486·30	1,45366·71	94
45	0.69590·45	8,65698·79	8,57430·94	1,46913·16	95
46	0,71136·90	8,65317·29	8,57377·57	1,48459·62	96
47	0,72683·35	8,64952·28	8,57326·15	1,50006·07	97
48	0,74229 81	8,64602·92	8,57276·57	1,51552·53	98
49	0,75776·26	8,64268·44	8,57228·78	1,53098·98	99
50	0,77322·72	8,63948·08	8,57182·71	1,54645·44	100
			8,55030·80		Perp.

3 3/4 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01598·81	0,01598·81	8,64613·25	0,81539·34	51
2	0,03197·62	9,72287·86	8,64330·69	0,83138·15	52
3	0,04796·43	9,55465·88	8,64060·06	0,84736·96	53
4	0,06395·24	9,43754·25	8,63800·81	0,86335·77	54
5	0,07994·05	9,34840·59	8,63552·38	0,87934·58	55
6	0,09592·86	9,27694·92	8,63314·26	0,89533·39	56
7	0,11191·67	9,21767·79	8,63085·99	0,91132·20	57
8	0,12790·48	9,16731·26	8,62867·09	0,92731·01	58
9	0,14389·29	9,12373·79	8,62657·14	0,94329·82	59
10	0,15988·11	9,08550·94	8,62455·74	0,95928·63	60
11	0,17586·92	9,05159·71	8,62262·50	0,97527·44	61
12	0,19185·73	9,02124·02	8,62077·05	0,99126·25	62
13	0,20784·54	8,99386·12	8,61899·66	1,00725·06	63
14	0,22383·35	8,96901·11	8,61728·18	1,02323·87	64
15	0,23982·16	8,94633·40	8,61564·11	1,03922·69	65
16	0,25580·97	8,92554·32	8,61406·56	1,05521·50	66
17	0,27179·78	8,90640·40	8,61255·26	1,07120·31	67
18	0,28778·59	8,88872·20	8,61109·89	1,08719·12	68
19	0,30377·40	8,87233·45	8,60970·26	1,10317·93	69
20	0,31976·21	8,85710·38	8,60836·09	1,11916·74	70
21	0,33575·02	8,84291·25	8,60707·15	1,13515·55	71
22	0,35173·83	8,82965·95	8,60583·25	1,15114·36	72
23	0,36772·64	8,81725·72	8,60464·16	1,16713·17	73
24	0,38371·45	8,80562·03	8,60349·67	1,18311·98	74
25	0,39970·26	8,79470·89	8,60239·62	1,19910·79	75
26	0,41569·07	8,78443·69	8,60133·80	1,21509·60	76
27	0,43167·88	8,77476·09	8,60032·05	1,23108·41	77
28	0,44766·70	8,76563·43	8,59934·21	1,24707·22	78
29	0,46365·51	8,75701·55	8,59840·11	1,26306·03	79
30	0,47964·32	8,74886·70	8,59749·60	1,27904·84	80
31	0,49563·13	8,74115·52	8,59662·54	1,29503·65	81
32	0,51161·94	8,73384·95	8,59578·79	1,31102·46	82
33	0,52760·75	8,72692·23	8,59498·22	1,32701·27	83
34	0,54359·56	8,72034·85	8,59420·71	1,34300·09	84
35	0,55958·37	8,71410·50	8,59346·13	1,35898·90	85
36	0,57557·18	8,70817·10	8,59274·37	1,37497·71	86
37	0,59155·99	8,70252·72	8,59205·31	1,39096·52	87
38	0,60754·80	8,69715·60	8,59138·85	1,40695·33	88
39	0,62353·61	8,69204·10	8,59074·89	1,42294·14	89
40	0,63952·42	8,68716·73	8,59013·33	1,43892·95	90
41	0,65551·23	8,68252·09	8,58954·08	1,45491·76	91
42	0,67150·04	8,67808·90	8,58897·05	1,47090·57	92
43	0,68748·85	8,67385·97	8,58842·14	1,48689·38	93
44	0,70347·66	8,66982·19	8,58789·29	1,50288·19	94
45	0,71946·47	8,66596·52	8,58738·41	1,51887·00	95
46	0,73545·28	8,66228·01	8,58689·42	1,53485·81	96
47	0,75144·10	8,65875·75	8,58642·26	1,55084·62	97
48	0,76742·91	8,65538·91	8,58596·86	1,56683·43	98
49	0,78341·72	8,65216·69	8,58553·13	1,58282·24	99
50	0,79940·53	8,64908·37	8,58511·03	1,59881·05	100
			8,57403·13		Perp.

3 7/8 Per Cent.

Years	Log. r^n.	Log. d^n.	Log. a^n.	Log. r^n.	Years
1	0.01051·10	0.01051·10	8.65572·64	0,84206·29	51
2	0.03302·21	9,72365·81	8.65301·26	0.85857·39	52
3	0.04953·31	9.55569·16	8.65041·60	0,87508·50	53
4	0.06604·41	9 43882·53	8.64793·08	0,89159·60	54
5	0.08255·52	9 34903·56	8.64555·17	0,90810·70	55
6	0.09906·62	9.27822·24	8.64327·35	0,92461·81	56
7	0.11557·73	9.21969·15	8.64109·16	0.94112·91	57
8	0.13208·83	9.16956·32	8.63900·14	0,95764·01	58
9	0.14859·93	9.12622·23	8,63699·86	0,97415·12	59
10	0.16511·04	9.08822·45	8,63507·92	0,99066·22	60
11	0.18162·14	9.05453·96	8.63323·94	1,00717·32	61
12	0.19813·24	9.02440·69	8 63147·56	1,02368·43	62
13	0.21464·35	8 99724·88	8.62978·43	1,04019·53	63
14	0.23115·45	8.97261·65	8.62816·23	1,05670·64	64
15	0.24766·56	8.95015·42	8.62660·65	1,07321·74	65
16	0.26417·66	8.92957·49	8.62511·40	1,08972·84	66
17	0.28068·76	8 91064·59	8.62368·20	1,10623·95	67
18	0 29719·87	8 89316·72	8,62230·79	1,12275·05	68
19	0,31370·97	8.87698·17	8.62098·91	1,13926·15	69
20	0,33022·07	8.86195·01	8,61972·33	1,15577·26	70
21	0.34673·18	8 84795·47	8.61850·82	1,17228·36	71
22	0,36324·28	8.83489·44	8.61734·17	1,18879·46	72
23	0.37975·38	8.82268·19	8.61622·15	1,20530·57	73
24	0,39626·49	8.81124·08	8.61514·59	1,22181·67	74
25	0.41277·59	8 80050·41	8.61411·30	1,23832·78	75
26	0.42928·70	8.79041·29	8,61312·08	1,25483·88	76
27	0.44579·80	8.78091·47	8.61216·79	1,27134·98	77
28	0.46230·90	8.77196·29	8.61125·24	1,28786·09	78
29	0.47882·01	8.76351·60	8.61037·30	1,30437·19	79
30	0.49533·11	8,75553·66	8,60952·79	1,32088·29	80
31	0,51184·21	8.74799·08	8.60871·60	1,33739·40	81
32	0.52835·32	8.74084·85	8.60730·58	1,35390·50	82
33	0.54486·42	8.73408·17	8.60718·60	1,37041·61	83
34	0.56137·53	8.72766·54	8.60646·54	1,38692·71	84
35	0,57788·63	8.72157·69	8.60577·28	1,40343·81	85
36	0.59439·73	8.71579·50	8.60510·72	1,41994·92	86
37	0.61090·84	8.71030·06	8.60446·73	1,43646·02	87
38	0.62741·94	8.70507·60	8.60385·22	1,45297·12	88
39	0.64393·04	8.70010·51	8.60326·08	1,46948·23	89
40	0.66044·15	8.69537·26	8,60269·23	1,48599·33	90
41	0.67695·25	8.69086·51	8.60214·55	1,50250·43	91
42	0.69346·35	8.68656·93	8.60162·00	1,51901·54	92
43	0.70997·46	8.68247·36	8.60111·46	1,53552·64	93
44	0.72648·56	8.67856·68	8.60062·86	1,55203·75	94
45	0.74299·67	8.67483·87	8.60016·13	1,56854·85	95
46	0.75950·77	8.67127·97	8.59971·18	1,58505·95	96
47	0.77601·87	8.66788·07	8.59927·96	1,60157·06	97
48	0.79252·98	8.66463·35	8.59886·40	1,61808·16	98
49	0.80904·08	8.66153·03	8,59846·42	1,63459·26	99
50	0,82555·18	8,65856·35	8,59807·96	1,65110·37	100
			8.58827·17		Perp.

4 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01703·33	0,01703·33	8.66519·48	0,86870·03	51
2	0,03406·67	9,72443·65	8,66258·90	0,88573·36	52
3	0,05110·00	9.55672·28	8.66009·81	0,90276·70	53
4	0,06813·34	9.44010·59	8.65771·64	0,91980·03	54
5	0,08516·67	9,35146·22	8.65543·85	0,93683·37	55
6	0,10220·00	9,28049·16	8,65325·96	0,95386·70	56
7	0,11923·34	9,22170·01	8,65117·45	0,97090·03	57
8	0,13626·67	9,17180·78	8,64917·91	0,98793·37	58
9	0,15330·01	9,12869·97	8.64726·91	1,00496·70	59
10	0,17033·34	9,09093·12	8,64544·04	1,02200·04	60
11	0,18736·67	9.05747·23	8,64368·93	1,03903·37	61
12	0,20440·01	9,02756·23	8,64201·22	1,05606·70	62
13	0,22143·34	9,00062·38	8,64040·56	1,07310·04	63
14	0,23846·68	8.97620·77	8 63886·64	1,09013·37	64
15	0,25550·01	8.95395·82	8 63739·16	1,10716·71	65
16	0,27253·34	8 93358 85	8.63597·82	1,12420·04	66
17	0,28956·68	8,91486·40	8.63462·35	1,14123·37	67
18	0,30660·01	8 89759·04	8,63332·48	1,15826·71	68
19	0,32363·34	8.88160·50	8.63207·97	1,17530·04	69
20	0,34066·68	8,86677·02	8,63088·59	1,19233·38	70
21	0,35770·01	8.85296·84	8.62974·11	1,20936·71	71
22	0,37473·35	8,84009·86	8.62864·32	1,22640·04	72
23	0,39176·68	8.82807·35	8.62759·01	1.24343·38	73
24	0,40880·01	8,81681·66	8.62657·99	1,26046·71	74
25	0,42583·35	8 80626·12	8,62561·08	1,27750·04	75
26	0,44286·68	8.79634·80	8.62468·09	1,29453·38	76
27	0,45990·02	8.78702·48	8.62378·89	1,31156·71	77
28	0,47693·35	8.77824·52	8,62293·26	1,32860·05	78
29	0,49396·68	8,76996·73	8,62211·11	1,34563·38	79
30	0,51100·02	8,76215·40	8,62132·25	1,36266·71	80
31	0,52803·35	8,75477·14	8,62056·56	1,37970·05	81
32	0,54506·69	8,74778·92	8,61983·91	1,39673·38	82
33	0,56210·02	8,74117·97	8,61914·16	1,41376·72	83
34	0,57913·35	8,73491·80	8,61847·21	1,43080·05	84
35	0,59616·69	8,72898·10	8.61782·92	1,44783·38	85
36	0,61320·02	8,72334·80	8,61721·20	1,46486·72	86
37	0,63023·36	8,71799·95	8,61661·94	1,48190·05	87
38	0,64726·69	8,71291·83	8,61605·03	1,49893·39	88
39	0,66430·02	8,70808·79	8,61550·38	1,51596·72	89
40	0,68133·36	8,70349·33	8,61497·89	1,53300·05	90
41	0,69836·69	8,69912·09	8,61447·48	1,55003·39	91
42	0,71540·03	8,69495·78	8,61399·08	1,56706·72	92
43	0,73243·36	8,69099·20	8,61352·58	1,58410·06	93
44	0,74946·69	8,68721·27	8,61307·91	1,60113·39	94
45	0,76650·03	8,68360·94	8,61265·01	1,61816·72	95
46	0,78353·36	8,68017·27	8.61223·80	1,63520·06	96
47	0,80056·69	8,67689·37	8,61184·21	1,65223·39	97
48	0,81760·03	8,67376·39	8,61146·18	1,66926·73	98
49	0,83463·36	8,67077·56	8,61109·64	1,68630·06	99
50	0,85166·70	8,66792·16	8,61074·53	1,70333·39	100
			8,60206·00		Perp.

4 1/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01755·50	0,01755·50	8,67453·95	0,89530·57	51
2	0,03511·00	9,72521·38	8,67203·78	0,91286·07	52
3	0,05266·50	9,55775·22	8,66964·88	0,93041·58	53
4	0,07022·01	9,44138·41	8,66736·68	0,94797·08	54
5	0,08777·51	9,35298·56	8,66518·65	0,96552·58	55
6	0,10533·01	9,28225·70	8,66310·27	0,98308·08	56
7	0,12288·51	9,22370·38	8,66111·09	1,00063·58	57
8	0,14044·01	9,17404·65	8,65920·66	1,01819·08	58
9	0,15799·51	9,13116·98	8,65738·54	1,03574·58	59
10	0,17555·01	9,09362·95	8,65564·36	1,05330·09	60
11	0,19310·52	9,06039·52	8,65397·73	1,07085·59	61
12	0,21066·02	9,03070·66	8,65238·31	1,08841·09	62
13	0,22821·52	9,00398·60	8,65085·75	1,10596·59	63
14	0,24577·02	8,97978·44	8,64939·74	1,12352·09	64
15	0,26332·52	8,95774·61	8,64799·98	1,14107·59	65
16	0,28088·02	8,93758·43	8,64666·16	1,15863·10	66
17	0,29843·52	8,91906·43	8,64538·04	1,17618·60	67
18	0,31599·03	8,90199·19	8,64415·35	1,19374·10	68
19	0,33354·53	8,88620·44	8,64297·84	1,21129·60	69
20	0,35110·03	8,87156·42	8,64185·29	1,22885·10	70
21	0,36865·53	8,85795·38	8,64077·48	1,24640·60	71
22	0,38621·03	8,84527·23	8,63974·18	1,26396·10	72
23	0,40376·53	8,83343·21	8,63875·20	1,28151·61	73
24	0,42132·03	8,82235·70	8,63780·36	1,29907·11	74
25	0,43887·54	8,81198·01	8,63689·47	1,31662·61	75
26	0,45643·04	8,80224·25	8,63602·38	1,33418·11	76
27	0,47398·54	8,79309·17	8,63518·88	1,35173·61	77
28	0,49154·04	8,78448·13	8,63438·84	1,36929·11	78
29	0,50909·54	8,77636·97	8,63362·11	1,38684·61	79
30	0,52665·04	8,76871·95	8,63288·56	1,40440·12	80
31	0,54420·54	8,76149·70	8,63218·03	1,42195·62	81
32	0,56176·05	8,75467·20	8,63150·40	1,43951·12	82
33	0,57931·55	8,74821·68	8,63085·55	1,45706·62	83
34	0,59687·05	8,74210·64	8,63023·37	1,47462·12	84
35	0,61442·55	8,73631·76	8,62963·73	1,49217·62	85
36	0,63198·05	8,73083·04	8,62906·52	1,50973·12	86
37	0,64953·55	8,72562·48	8,62851·67	1,52728·63	87
38	0,66709·05	8,72068·34	8,62799·04	1,54484·13	88
39	0,68464·56	8,71599·02	8,62748·56	1,56239·63	89
40	0,70220·06	8,71153·00	8,62700·13	1,57995·13	90
41	0,71975·56	8,70728·94	8,62653·67	1,59750·63	91
42	0,73731·06	8,70325·53	8,62609·10	1,61506·13	92
43	0,75486·56	8,69941·61	8,62566·34	1,63261·63	93
44	0,77242·06	8,69576·03	8,62525·32	1,65017·14	94
45	0,78997·56	8,69227·85	8,62485·95	1,66772·64	95
46	0,80753·07	8,68896·05	8,62448·18	1,68528·14	96
47	0,82508·57	8,68579·77	8,62411·93	1,70283·64	97
48	0,84264·07	8,68278·17	8,62377·15	1,72039·14	98
49	0,86019·57	8,67990·48	8,62343·77	1,73794·64	99
50	0,87775·07	8,67715·96	8,62311·74	1,75550·14	100
			8,61542·40		Perp.

$4\frac{1}{4}$ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01807·61	0,01807·61	8,68376·19	0,92187·92	51
2	0,03615·21	9,72599·01	8,68136·08	0,93995·53	52
3	0,05422·82	9,55878·01	8,67907·02	0,95803·14	53
4	0,07230·43	9,44266·01	8,67688·42	0,97610·74	54
5	0,09038·03	9,35450·62	8,67479·75	0,99418·35	55
6	0,10845·64	9,28401·84	8,67280·54	1,01225·96	56
7	0,12653·24	9,22570·26	8,67090·29	1,03033·56	57
8	0,14460·85	9,17627·92	8,66908·58	1,04841·17	58
9	0,16268·46	9,13363·29	8,66735·01	1,06648·78	59
10	0,18076·06	9,09631·94	8,66569·14	1,08456·38	60
11	0,19883·67	9,06330·85	8,66410·64	1,10263·99	61
12	0,21691·28	9,03333·97	8,66259·14	1,12071·59	62
13	0,23498·88	9,00733·55	8,66114·32	1,13879·20	63
14	0,25306·49	8,98334·69	8,65975·84	1,15686·81	64
15	0,27114·10	8,96151·80	8,65843·43	1,17494·41	65
16	0,28921·70	8,94156·22	8,65716·79	1,19302·02	66
17	0,30729·31	8,92324·48	8,65595·66	1,21109·63	67
18	0,32536·91	8,90637·17	8,65479·78	1,22917·23	68
19	0,34344·52	8 89078 01	8,65368·92	1,24724·84	69
20	0,36152·13	8,87633·24	8,65262·84	1,26532·45	70
21	0,37959·73	8,86291·12	8,65161·33	1,28340·05	71
22	0,39767·34	8 85041·55	8,65064·18	1,30147·66	72
23	0,41574·95	8 83875·80	8,64971·21	1,31955·26	73
24	0,43382·55	8,82786·22	8 64882·20	1,33762·87	74
25	0,45190·16	8,81766·13	8,64796·99	1,35570·48	75
26	0,46997·77	8,80809·65	8,64715·40	1,37378 08	76
27	0,48805·37	8,79911·55	8,64637·30	1,39185·69	77
28	0,50612·98	8,79067·16	8,64562·51	1,40993·30	78
29	0,52420·58	8,78272·34	8,64490·89	1,42800·90	79
30	0,54228·19	8,77523·36	8,64422·30	1,44608·51	80
31	0,56035·80	8,76816·83	8,64356·61	1,46416·12	81
32	0,57843·40	8,76149·75	8,64293·68	1,48223·72	82
33	0,59651·01	8,75519·36	8,64233·41	1,50031·33	83
34	0,61458·62	8,74923·13	8,64175·69	1,51838·93	84
35	0,63266·22	8,74358·80	8,64120·37	1,53646·54	85
36	0,65073·83	8,73824·30	8,64067·38	1,55454·15	86
37	0,66881·44	8,73317·68	8,64016·61	1,57261·75	87
38	0,68689·04	8,72837·21	8,63967·97	1,59069·36	88
39	0,70496·65	8,72381·26	8,63921·36	1,60876·97	89
40	0,72304·25	8,71948·36	8,63876·70	1,62684·57	90
41	0,74111·86	8,71537·12	8,63833·91	1,64492·18	91
42	0,75919·47	8,71146·29	8,63792·89	1,66299·79	92
43	0,77727·07	8,70774·66	8,63753·58	1,68107·39	93
44	0,79534·68	8,70421·13	8,63715·92	1,69915·00	94
45	0,81342·29	8,70084·71	8,63679·81	1,71722·60	95
46	0,83149·89	8,69764·42	8,63645·21	1,73530·21	96
47	0,84957·50	8,69459·41	8,63612·04	1,75337·82	97
48	0,86765·11	8,69168·82	8,63580·26	1,77145·42	98
49	0,88572·71	8,68891·89	8,63549·78	1,78953·03	99
50	0,90380·32	8,68627·91	8,63520·58	1,80760·64	100
			8,62838·89		Perp.

4 3/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0.018559·65	0.018559·65	8,69286·38	0,9484·09	51
2	0.037119·30	9.726776·52	8.690559·98	0,96701·74	52
3	0,055678·95	9.559806·63	8,688366·38	0,98561·39	53
4	0.074238·60	9.443937·37	8,686270·02	1,00421·04	54
5	0.092798·24	9 356021·35	8,684271·38	1,02280·69	55
6	0.111357·89	9 285775·8	8.682366·96	1,04140·34	56
7	0.129917·54	9 22·69·65	8.680553·30	1,05999·98	57
8	0.148477·19	9 17850·60	8 67881·98	1,07859·63	58
9	0.167036 84	9.13608·89	8,67716·57	1,09719·28	59
10	0.185596·49	9.09900·10	8,67558·67	1,11578·93	60
11	0.204156·14	9.06621·22	8,67407·93	1,13438·58	61
12	0.222715·79	9.03696·18	8,67263·99	1,15298·23	62
13	0.241275·44	9.01067·23	8,67126·54	1,17157·88	63
14	0,259835·08	8.98689·51	8,66995·25	1,19017·53	64
15	0.278394·73	8.96527·40	8,66869·85	1,20877·18	65
16	0.296954·38	8.94552·24	8,66750·03	1,22736·82	66
17	0,31614·03	8 92740·58	8.66635·55	1,24596·47	67
18	0,33473·68	8.91073·00	8,66526·14	1,26456·12	68
19	0,35333·33	8 80533·21	8,66421·58	1,28315·78	69
20	0,37192·98	8,88107·48	8,66321·63	1,30175·42	70
21	0,39052·63	8.86784·06	8,66226·11	1,32035·07	71
22	0,40912·27	8.85552·85	8.66134·77	1,33894·72	72
23	0,42771·92	8 84405·12	8,66047·44	1,35754·37	73
24	0,44631·57	8 83333·23	8,65963·94	1,37614·01	74
25	0.46491·22	8 82330·50	8.65884·09	1,39473·66	75
26	0.48350·87	8 81391·05	8.65807·72	1,41333·31	76
27	0,50210·52	8.80509·64	8.65734·67	1,43192·96	77
28	0.520·0·17	8.79681·64	8.65664·81	1,45052·61	78
29	0,53929·82	8,78902·88	8.65597·98	1,46912·26	79
30	0,55789·47	8,78169·64	8,65534·05	1,48771·91	80
31	0,57649·11	8.77478·56	8,65472·89	1,50631·56	81
32	0,59508·76	8.76826·60	8,65414·36	1,52491·21	82
33	0.61368·41	8.76211·02	8.65358·36	1,54350·85	83
34	0.63228·06	8.75629·31	8.65304·78	1,56210·50	84
35	0,65087·71	8.75079·21	8.65253·53	1,58070·15	85
36	0,66947·36	8.74558·62	8,65204·46	1,59929·80	86
37	0,68807·01	8.74065·64	8.65157·50	1,61789·45	87
38	0,70666·66	8.73598·50	8.65112·55	1,63649·10	88
39	0,72526·31	8.73155·62	8,65069·54	1,65508·75	89
40	0,74385·95	8,72735·50	8,65028·37	1,67368·40	90
41	0,76245·60	8.72336·76	8.64988·95	1,69228·05	91
42	0,78105·25	8,71958·14	8,64951·24	1,71087·69	92
43	0,79964·90	8.71598·45	8,64915·13	1,72947·34	93
44	0,81824·55	8,71256·62	8,64880·56	1,74806·99	94
45	0,8368·4·20	8,70931·63	8,64847·46	1,76666·64	95
46	0,85543·85	8,70622·51	8,64815·78	1,78526·29	96
47	0,8·403·50	8,70328·41	8,64785·44	1,80385·94	97
48	0,89263·14	8,70048·49	8,64756·40	1,82245·59	98
49	0,91122·79	8,69781·98	8,64728·59	1,84105·24	99
50	0,92982·44	8,69528·17	8,64701·96	1,85964·88	100
			8,64097·81		Perp.

4 ½ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01911·63	0,01911·63	8,70184·69	0,97493·08	51
2	0,03823·26	9,72753·93	8,69963·65	0,99404·71	52
3	0,05734·89	9,56083·09	8,69753·17	1,01316·34	53
4	0,07646·52	9,44520·50	8,69552·71	1,03227·97	54
5	0,09558·15	9,35753·78	8,69361·74	1,05139·60	55
6	0,11469·77	9,28752·94	8,69179·78	1,07051·23	56
7	0,13381·40	9,22968·56	8,69006·37	1,08962·86	57
8	0,15293·03	9,18072·68	8,68841·06	1,10874·48	58
9	0,17204·66	9,13853·79	8,68683·47	1,12786·11	59
10	0,19116·29	9,10167·43	8,68533·19	1,14697·74	60
11	0,21027·92	9,06910·61	8,68389·87	1,16609·37	61
12	0,22939·55	9,04007·27	8,68253·16	1,18521·00	62
13	0,24851·18	9,01399·67	8,68122·74	1,20432·63	63
14	0,26762·81	8,99042·91	8,67998·31	1,22344·26	64
15	0,28674·44	8,96901·41	8,67879·57	1,24255·89	65
16	0,30586·06	8,94946·50	8,67766·24	1,26167·52	66
17	0,32497·69	8,93154·73	8,67658·07	1,28079·15	67
18	0,34409·32	8,91506·67	8,67554·80	1,29990·78	68
19	0,36320·95	8,89986·07	8,67456·22	1,31902·40	69
20	0,38232·58	8,88579·16	8,67362·09	1,33814·03	70
21	0,40144·21	8,87274·21	8,67272·20	1,35725·66	71
22	0,42055·84	8,86061·13	8,67186·36	1,37637·29	72
23	0,43967·47	8,84931·18	8,67104·37	1,39548·92	73
24	0,45879·10	8,83876·74	8,67026·06	1,41460·55	74
25	0,47790·73	8,82891·13	8,66951·25	1,43372·18	75
26	0,49702·36	8,81968·46	8,66879·78	1,45283·81	76
27	0,51613·98	8,81103·49	8,66811·51	1,47195·44	77
28	0,53525·61	8,80291·60	8,66746·27	1,49107·07	78
29	0,55437·24	8,79528·63	8,66683·93	1,51018·69	79
30	0,57348·87	8,78810·85	8,66624·36	1,52930·32	80
31	0,59260·50	8,78134·92	8,66567·43	1,54841·95	81
32	0,61172·13	8,77497·81	8,66513·02	1,56753·58	82
33	0,63083·76	8,76896·74	8,66461·02	1,58665·21	83
34	0,64995·39	8,76329·26	8,66411·32	1,60576·84	84
35	0,66907·02	8,75793·06	8,66363·81	1,62488·47	85
36	0,68818·65	8,75286·08	8,66318·40	1,64400·10	86
37	0,70730·27	8,74806·41	8,66274·98	1,66311·73	87
38	0,72641·90	8,74352·30	8,66233·48	1,68223·36	88
39	0,74553·53	8,73922·14	8,66193·80	1,70134·98	89
40	0,76465·16	8,73514·48	8,66155·86	1,72046·61	90
41	0,78376·79	8,73127·91	8,66119·59	1,73958·24	91
42	0,80288·42	8,72761·18	8,66084·91	1,75869·87	92
43	0,82200·05	8,72413·12	8,66051·74	1,77781·50	93
44	0,84111·68	8,72082·64	8,66020·04	1,79693·13	94
45	0,86023·31	8,71768·73	8,65989·71	1,81604·76	95
46	0,87934·94	8,71470·45	8,65960·71	1,83516·39	96
47	0,89846·57	8,71186·91	8,65932·98	1,85428·02	97
48	0,91758·19	8,70917·31	8,65906·46	1,87339·65	98
49	0,93669·82	8,70660·87	8,65881·10	1,89251·28	99
50	0,95581·45	8,70416·90	8,65856·85	1,91162·90	100
			8,65321·25		Perp.

4 5/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,01963·55	0,01963·55	8,71071·30	1,0014090	51
2	0,03927·09	9,72831·23	8,70859·26	1,0210·45	52
3	0,05890·64	9,56185·38	8,70657·58	1,0406·00	53
4	0,07854·19	9,44647·40	8,70465·67	1,0603·54	54
5	0,09817·74	9,35904·90	8,70283·03	1,0799·09	55
6	0,11781·28	9,28927·90	8,70109·20	1,0995·64	56
7	0,13744·83	9,23166·98	8,69943·69	1,1192·18	57
8	0,15708·38	9,18294·19	8,69786·08	1,1388·73	58
9	0,17671·92	9,14097·98	8,69635·97	1,1584·28	59
10	0,19635·47	9,10433·93	8,69492·98	1,1781·83	60
11	0,21599·02	9,07199·05	8,69356·76	1,1976·37	61
12	0,23562·57	9,04317·27	8,69226·94	1,2173·92	62
13	0,25526·11	9,01730·84	8,69103·24	1,2370·47	63
14	0,27489·66	8,99394·89	8,68985·32	1,2567·01	64
15	0,29453·21	8,97273·83	8,68872·92	1,2763·56	65
16	0,31416·75	8,95338·99	8,68765·77	1,2959·11	66
17	0,33380·30	8,93566·93	8,68663·58	1,3155·66	67
18	0,35343·85	8,91938·21	8,68566·15	1,3352·20	68
19	0,37307·39	8,90436·58	8,68473·23	1,3548·75	69
20	0,39270·94	8,89048·29	8,68384·59	1,3744·30	70
21	0,41234·49	8,87761·60	8,68300·05	1,3941·84	71
22	0,43198·04	8,86566·43	8,68219·39	1,4137·39	72
23	0,45161·58	8,85454·04	8,68142·44	1,4333·94	73
24	0,47125·13	8,84416·79	8,68069·03	1,4530·49	74
25	0,49088·68	8,83448·05	8,67998·95	1,4726·03	75
26	0,51052·22	8,82541·88	8,67932·11	1,4922·58	76
27	0,53015·77	8,81693·11	8,67868·30	1,5119·13	77
28	0,54979·32	8,80897·07	8,67807·40	1,5315·67	78
29	0,56942·87	8,80149·62	8,67749·28	1,5512·22	79
30	0,58906·41	8,79447·03	8,67693·79	1,5708·77	80
31	0,60869·96	8,78785·06	8,67640·83	1,5904·32	81
32	0,62833·51	8,78163·38	8,67590·27	1,6101·86	82
33	0,64797·05	8,77576·56	8,67542·00	1,6297·41	83
34	0,66760·60	8,77022·99	8,67495·90	1,6493·96	84
35	0,68724·15	8,76500·40	8,67451·90	1,6690·50	85
36	0,70687·70	8,76006·73	8,67409·89	1,6886·05	86
37	0,72651·24	8,75540·06	8,67369·77	1,7082·60	87
38	0,74614·79	8,75098·67	8,67331·45	1,7279·14	88
39	0,76578·34	8,74680·94	8,67294·85	1,7475·69	89
40	0,78541·88	8,74285·39	8,67259·91	1,7671·24	90
41	0,80505·43	8,73910·67	8,67226·54	1,7868·79	91
42	0,82468·98	8,73555·52	8,67194·66	1,8064·33	92
43	0,84432·53	8,73218·74	8,67164·22	1,8260·88	93
44	0,86396·07	8,72899·29	8,67135·14	1,8457·43	94
45	0,88359·62	8,72596·13	8,67107·38	1,8653·97	95
46	0,90323·17	8,72308·35	8,67080 84	1,8850·52	96
47	0,92286·71	8,72035·04	8,67055·51	1,9046·07	97
48	0,94250·26	8,71775·43	8,67031·30	1,9242·62	98
49	0,96213·81	8,71528·73	8,67008·17	1,9439·16	99
50	0,98177·36	8,71294·24	8,66986·08	1,96354·71	100
			8,66511·17		Per

4 3/4 Per Cent.

ars	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,02015·40	0,02015·40	8,71946·35	1,02785·56	51
2	0,04030·81	9,72908·42	8,71743·01	1,04800·96	52
3	0,06046·21	9,56287·51	8,71549·78	1,06816·37	53
4	0,08061·61	9,44774·06	8,71366·10	1,08831·77	54
5	0,10077·02	9,36055·72	8,71191·48	1,10847·17	55
6	0,12092·42	9,29102·49	8,71025·44	1,12862·58	56
7	0,14107·82	9,23364·92	8,70867·50	1,14877·98	57
8	0,16123·23	9,18515·09	8,70717·27	1,16893·38	58
9	0,18138·63	9,14341·46	8,70574·32	1,18908·79	59
10	0,20154·03	9,10699·61	8,70438·30	1,20924·19	60
11	0,22169·43	9,07486·53	8,70308·84	1,22939·59	61
12	0,24184·84	9,04626·17	8,70185·62	1,24955·00	62
13	0,26200·24	9,02060·78	8,70068·30	1,26970·40	63
14	0,28215·64	8,99745·47	8,69956·60	1,28985·80	64
15	0,30231·05	8,97644·69	8,69850·24	1,31001·21	65
16	0,32246·45	8,95729·73	8,69748·94	1,33016·61	66
17	0,34261·85	8,93977·19	8,69652·44	1,35032·01	67
18	0,36277·26	8,92367·61	8,69560·54	1,37047·41	68
19	0,38292·66	8,90884·76	8,69472·97	1,39062·82	69
20	0,40308·06	8,89514·88	8,69389·54	1,41078·22	70
21	0,42323·47	8,88246·23	8,69310·05	1,43093·62	71
22	0,44338·87	8,87068·74	8,69234·29	1,45109·03	72
23	0,46354·27	8,85973·67	8,69162·10	1,47124·43	73
24	0,48369·68	8,84953·41	8,69093·28	1,49139·83	74
25	0,50385·08	8,84001·27	8,69027·69	1,51155·24	75
26	0,52400·48	8,83111·39	8,68965·17	1,53170·64	76
27	0,54415·89	8,82278·54	8,68905·56	1,55186·04	77
28	0,56431·29	8,81498·08	8,68848·74	1,57201·45	78
29	0,58446·69	8,80765·87	8,68794·56	1,59216·85	79
30	0,60462·09	8,80078·20	8,68742·90	1,61232·25	80
31	0,62477·50	8,79431·72	8,68693·64	1,63247·66	81
32	0,64492·90	8,78823·40	8,68646·64	1,65263·06	82
33	0,66508·30	8,78250·53	8,68601·87	1,67278·46	83
34	0,68523·71	8,77710·58	8,68559·15	1,69293·87	84
35	0,70539·11	8,77201·31	8,68518·40	1,71309·27	85
36	0,72554·51	8,76720·63	8 68479·55	1,73324·67	86
37	0,74569·92	8,76266·67	8,68442·47	1,75340·07	87
38	0,76585·32	8,75837·68	8,68407·12	1,77355·48	88
39	0,78600·72	8,75432·06	8,68373·39	1,79370·88	89
40	0,80616·13	8,75048·34	8,68341·22	1,81386·28	90
41	0,82631·53	8,74685·14	8,68310·52	1,83401·69	91
42	0,84646·93	8,74341·23	8,68281·24	1,85417·09	92
43	0,86662·34	8,74015·43	8,68253·31	1,87432·49	93
44	0,88677·74	8,73706·68	8,68226·66	1,89447·90	94
45	0,90693·14	8,73413·95	8,68201·23	1,91463·30	95
46	0,92708·55	8,73136·33	8,68176·97	1,93478·70	96
47	0,94723·95	8,72872·94	8,68153·83	1,95494·11	97
48	0,96739·35	8,72622·99	8,68131·74	1,97509·51	98
49	0,98754·76	8,72385·71	8,68110·67	1,99524·91	99
50	1,00770·16	8,72160·37	8,68090·56	2,01540·32	100
			8,67669·36		Perp.

4 7/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0.02067·20	0,02067·20	8,72810·03	1,05427·07	51
2	0.04134·39	9,72985·50	8,72615·06	1,07494·26	52
3	0.06201·59	9,56389·48	8,72429·97	1,09561·46	53
4	0,08268·79	9,44900·52	8,72254·20	1,11628·66	54
5	0,10335·99	9,36206·24	8,72087·27	1,13695 86	55
6	0.12403·18	9.29277·85	8,71928 69	1,15763·05	56
7	0.14470·38	9.23562·38	8,71778 03	1,17830·25	57
8	0.16537·58	9.18735·42	8,71634·85	1,19897·45	58
9	0.18604·78	9.14584·26	8,71498·77	1,21964 65	59
10	0,20671·97	9,10964·48	8,71369·41	1,24031·84	60
11	0.22739·17	9.07773·07	8,71246·42	1,26099·04	61
12	0.24806·37	9.04933·97	8,71120·47	1,28166·24	62
13	0.26873·57	9.02389·47	8,71018·25	1,30233·44	63
14	0.28940·76	9.00094·66	8,70912·46	1,32300·63	64
15	0,31007·96	8.98013·97	8,70811·83	1,34397·83	65
16	0,33075·16	8.96118·75	8,70716·10	1,36435·03	66
17	0,35142·36	8.94385·53	8.70625·00	1,38502·22	67
18	0,37209·55	8.92794·91	8,70538·33	1,40569·42	68
19	0,39276·75	8.91330·63	8,70455·84	1,42636·62	69
20	0,41343·95	8,89978·94	8,70377·33	1,44703·82	70
21	0.43411·15	8.88728·13	8,70302·61	1,46771·01	71
22	0.45478·34	8.87568·10	8,70231·47	1,48838·21	72
23	0.47545·54	8.86490·12	8,70163·75	1,50905·41	73
24	0.49612·74	8.65486·59	8,70099·29	1,52972·61	74
25	0,51679·93	8.84550·83	8,70037·90	1,55039·80	75
26	0,53747·13	8.83676·96	8.69979·44	1,57107·00	76
27	0,55814·33	8.82859·78	8,69923·78	1,59174·20	77
28	0,57881·53	8.82094·66	8.69870·77	1,61241·40	78
29	0.59948 72	8.81377·44	8 69820·29	1,63308·59	79
30	0,62015·92	8,80704·41	8,69772·21	1,65375·79	80
31	0 6408312	8.80072·24	8 69726·41	1,67442·99	81
32	0,66150·32	8.79477·01	8 69682·78	1,69510·19	82
33	0,68217·51	8.78918·68	8 69641·23	1,71577·38	83
34	0,70284·71	8.78392·07	8.69601·64	1,73644·58	84
35	0.72351·91	8.77895·82	8.69563·93	1,75711·78	85
36	0.74419·11	8,77427·86	8,69528 00	1.77778·97	86
37	0.76486·30	8.76986·30	8.69493·77	1,79846·17	87
38	0.78553·50	8.76569·40	8.69461·16	1,81913·37	88
39	0.80620·70	8.76175·59	8.69430·07	1,83980·57	89
40	0.82087·90	8.75803·37	8,69400·47	1,86047·76	90
41	0.84755·09	8.75451·40	8 69372·25	1,88114·96	
42	0.86822 29	8.75118·42	8.69345·36	1,90182·16	
43	0 88889·49	8.74803·29	8,69319·74	1,92249·36	
44	0.90956·68	8.74504·91	8.69295·32	1,94316·55	
45	0,93023·88	8.74222·31	8 69272·05	1,96383·75	
46	0.95091·08	8.73954·54	8.69249·88	1,98450·95	
47	0 9715828	8.73700·75	8.69228·74	2.00518·15	
48	0,99225·47	8.73460·12	8.69208·60	2,02585·34	
49	1.01292·67	8.73231·93	8.69189·40	2.04652·54	
50	1,03359·87	8,73015·45	8.69171·11	2,06719·74	
			8 68·97·46		

5 Per Cent.

z. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
118·93	0,02118·93	8,73662·49	1,08065·43	51
237·86	9,73062·48	8,73475·58	1,10184·36	52
356·79	9,56491·28	8,73298·32	1,12303·28	53
475·72	9,45026·73	8,73130·16	1,14422·21	54
594·65	9,36356·45	8,72970·62	1,16541·14	55
713·58	9,29450·47	8,72819·22	1,18660·07	56
832·51	9,23759·36	8,72675·52	1,20779·00	57
951·44	9,18955·15	8,72539·10	1,22897·93	58
1070·37	9,14826·35	8,72409·57	1,25016·86	59
1189·30	9,11228·51	8,72286·57	1,27135·79	60
1308·23	9,08058 65	8,72169·75	1,29254·72	61
1427·16	9,05240·70	8,72058·78	1,31373·65	62
1546·09	9,02716 92	8,71953·37	1,33492·58	63
1665·02	9,00442·44	8,71853·20	1,35611·51	64
1783·95	8,98381·70	8,71758·02	1,37730·44	65
1902·88	8,96506·01	8,71667·58	1,39849·37	66
2021·81	8,94791·94	8,71581·60	1,41968·30	67
2140·74	8,93220·08	8,71499·88	1,44087·23	68
2259·67	8,91774·18	8,71422·21	1,46206·16	69
2378·60	8,90440·49	8,71348·34	1,48325·09	70
2497·53	8,89207·30	8,71278·11	1,50444·02	71
2616·46	8,88064·50	8,71211·34	1,52562·95	72
2735·39	8,87003·40	8,71147·85	1,54681·88	73
2854·32	8,86016·37	8,71087·46	1,56800·81	74
2973·25	8,85096·74	8,71030 03	1,58919·74	75
3092·18	8,84238 66	8,70975·40	1,61038·67	76
3211·11	8,83436 89	8,70923·43	1,63157·60	77
3330·04	8,82686·83	8,70874·00	1,65276·53	78
3448·97	8,81984·33	8,70826·98	1,67395·46	79
3567·90	8,81325·70	8,70782·24	1,69514·39	80
3686·83	8,80707·56	8,70739·67	1,71633·32	81
3805·76	8 80126·94	8,70699·17	1,73752·25	82
3924·69	8,79581·08	8,70660·63	1,75871·18	83
4043·62	8,79067·52	8,70623·97	1,77990·11	84
4162·55	8,78584·01	8,70589·07	1,80109·04	85
4281·48	8,78128·46	8,70555·86	1,82227·97	86
4400·41	8,77699·01	8,70524·26	1,84346·90	87
4519·34	8,77293·92	8,70494·18	1,86465·83	88
4638·27	8,76911·60	8,70465·56	1,88584·76	89
4757·20	8,76550·58	8,70438·32	1,90703·69	90
4876·13	8,76209·53	8,70412·39	1,92822·62	91
4995·06	8,75887·19	8,70387·70	1,94941·55	92
5113·99	8,75582·41	8,70364·21	1,97060·48	93
5232·92	8,75294·11	8,70341·85	1,99179·41	94
5351·85	8,75021·31	8,70320·56	2,01298·34	95
5470·78	8,74763·09	8,70300·30	2,03417·27	96
5589·71	8,74518·58	8,70281·01	2,05536·20	97
5708·64	8,74286·98	8,70262·64	2,07655·13	98
5827·57	8,74067·57	8,70245·16	2,09774·06	99
5946·50	8,73859·61	8,70228·52	2,11892·99	100
		8,69897·00		Perp.

5 1/8 Per Cent.

Years	Log. r^n.	Log. s^n.	Log. s^n.	Log. r^n.
1	0,02170·60	0,02170·60	8,74503·90	1,10700·64
2	0,04341·20	9,73139·34	8,74324·76	1,12871·25
3	0,06511·80	9,56592·92	8,74155·02	1,15041·85
4	0,08682·40	9,45152·72	8,73994·18	1,17212·45
5	0,10853·00	9,36506·37	8,73841·73	1,19383·05
6	0,13023·61	9,29623·89	8,73697·20	1,21553·65
7	0,15194·21	9,23955·85	8,73560·17	1,23724·25
8	0,17364·81	9,19174·31	8,73430·22	1,25894·85
9	0,19535·41	9,15067·75	8,73306·96	1,28065·45
10	0,21706·01	9,11491·73	8,73190·03	1,30236·05
11	0,23876·61	9,08343·27	8,73079·10	1,32406·65
12	0,26047·21	9,05546·32	8,72973·84	1,34577·25
13	0,28217·81	9,03043·14	8,72873·94	1,36747·86
14	0,30388·41	9,00788·84	8,72779·13	1,38918·46
15	0,32559·01	8,98747·86	8,72689·12	1,41089·06
16	0,34729·61	8,96891·55	8,72603·69	1,43259·66
17	0,36900·21	8,95196·44	8,72522·58	1,45430·26
18	0,39070·82	8,93643·16	8,72445·55	1,47600·86
19	0,41241·42	8,92215·44	8,72372·41	1,49771·46
20	0,43412·02	8,90899·55	8,72302·95	1,51942·06
21	0,45582·62	8,89683·75	8,72236·98	1,54112·66
22	0,47753·22	8,88557·98	8,72174·32	1,56283·26
23	0,49923·82	8,87513·51	8,72114·79	1,58453·86
24	0,52094·42	8,86542·75	8,72058·25	1,60624·47
25	0,54265·02	8,85639·03	8,72004·53	1,62795·07
26	0,56435·62	8,84796·48	8,71953·49	1,64965·67
27	0,58606·22	8,84009 88	8,71904·99	1,67136·27
28	0,60776 82	8,83274·64	8,71858 91	1,69306·87
29	0,62947·43	8,82586·60	8,71815·11	1,71477·47
30	0,65118·03	8,81942·08	8,71773·50	1,73648·07
31	0,67288·63	8,81337·71	8,71733·95	1,75818·67
32	0,69459·23	8 80770·53	8,71696·36	1,77989·27
33	0,71629·83	8 80237·78	8,71660·65	1,80159·87
34	0,73800·43	8.797,36·98	8,71626 69	1,82330·47
35	0,75971·03	8.79265·92	8,71594·41	1,84501·07
36	0,78141·63	8,78822·51	8,71563·73	1,86671·68
37	0,80312·23	8.78404·87	8,71534·56	1,88842·28
38	0,82482·83	8,78011·28	8,71506 83	1,91012·88
39	0,84653·43	8,77640·17	8,71480·48	1,93183·48
40	0,86824·04	8,77290·08	8,71455·43	1,95354·08
41	0,88994·64	8,76959·63	8,71431·60	1,97524·68
42	0,91165·24	8,76647·63	8,71408·96	1,99695·28
43	0,93335·84	8,76352·90	8,71387·43	2,01865·88
44	0,95506·44	8,76074·38	8,71366·95	2,04036·48
45	0,97677·04	8,75811·08	8,71347·48	2,06207·08
46	0,99847·64	8,75562·10	8,71328·97	2,08377·68
47	1,02018·24	8,75326·57	8,71311·37	2,10548·29
48	1,04188·84	8,75103·70	8,71294·64	2,12718 89
49	1,06359·44	8,74892·76	8,71278·73	2,14889·49
50	1,08530·04	8,74693·04	8,71263·60	2,17060·09
			8,71249·20	

5 1/4 Per Cent.

·. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
22·21	0 02222·21	8,75334·45	1,13332·73	51
44·42	9,73216·10	8,75162·77	1,15554·94	52
66·63	9 56694·40	8,75000·28	1,17777·15	53
88·84	9,45278·48	8,74846·45	1,19999·36	54
11·05	9,36655·99	8,74700·80	1,22221·57	55
33·26	9,29796·92	8,74562·87	1,24443·79	56
55·47	9,24151·87	8,74432·23	1,26666·00	57
77·68	9,19392·89	8,74308·46	1,28888·21	58
99·89	9,15308·45	8,74191·19	1,31110·42	59
22·10	9,11754·15	8,74080·07	1,33332·63	60
44·31	9.08626·97	8,73974·74	1,35554·84	61
66·53	9 05850·87	8,73874·92	1,37777·05	62
88·74	9 03368·13	8,73780·28	1,39999·26	63
10·95	9,01133·85	8,73690·55	1,42221·47	64
33·16	8.99112·48	8,73605·47	1,44443·68	65
55·37	8 97275·36	8,73524·79	1,46665·89	66
77·58	8 95599·05	8,73448·27	1,48888·10	67
99·79	8.94064·15	8,73375·69	1,51110·31	68
22·00	8.92654·41	8,73306·85	1,53332·52	69
44·21	8,91356·11	8,73241·53	1,55554·73	70
66·42	8,90157·50	8,73179·58	1,57776·94	71
88·63	8,89048·55	8,73120·78	1,59999·15	72
10·84	8,88020·51	8,73065·00	1,62221·36	73
33·05	8.87065·78	8,73012·07	1,64443·57	74
55·26	8.86177·73	8,72961·84	1,66665·78	75
77·47	8.85350·45	8,72914·16	1,68887·99	76
99·68	8.84578·80	8,72868·91	1,71110·20	77
21·89	8.83858·11	8,72825·97	1,73332·42	78
44·10	8.83184·28	8,72785·20	1,75554·63	79
66·31	8,82553·61	8,72746·50	1,77776·84	80
88·52	8,81962·75	8,72709·77	1,79999·05	81
10·73	8,81408·72	8,72674·90	1,82221·26	82
32·94	8,80888·80	8,72641·79	1,84443·47	83
55·16	8,80400·51	8,72610·36	1,86665·68	84
77·37	8,79941·61	8,72580·51	1,88887·89	85
99·58	8,79510·05	8,72552·17	1,91110·10	86
21·79	8,79103·94	8,72525·27	1,93332·31	87
44·00	8,78721·58	8,72499·72	1,95554·52	88
66·21	8,78361·39	8,72475·46	1,97776·73	89
88·42	8,78021·91	8,72452·42	1,99998·94	90
10·63	8,77701·81	8,72430·55	2,02221·15	91
32·84	8,77399·84	8,72409·77	2,04443·36	92
55·05	8,77114·85	8,72390·04	2,06665·57	93
77·26	8,76845·82	8,72371·31	2,08887·78	94
99·47	8,76591·74	8,72353·51	2,11109·99	95
21·68	8,76351·69	8,72336·61	2,13332·20	96
43·89	8,76124·86	8,72320·55	2,15554·41	97
66·10	8,75910·41	8,72305·31	2,17776·62	98
88·31	8,75707·65	8,72290·84	2,19998·83	99
110·52	8,75515·88	8,72277·07	2,22221·05	100
		8,72015·93		Perp.

5 5/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.
1	0,02376·67	0,02376·67	8.77762·42	1,21210·28
2	0,04753·34	9·73445·76	8.77611·49	1,23586·95
3	0,07130·02	9,56997·88	8.77469·07	1,25963·63
4	0,09506·69	9.45654·42	8.77334·67	1,28340·30
5	0,11883·36	9·37003·04	8.77207·81	1,30716·97
6	0,14260·03	9,30313·73	8.77088·05	1,33093·64
7	0,16636·71	9.24737·08	8.76974·98	1,35470·32
8	0,19013·38	9.20045·16	8.76868·19	1,37846·99
9	0,21390·05	9,16026·45	8.76767·32	1,40223·66
10	0,23766·72	9,12536·53	8,76672·05	1,42600·33
11	0,26143·39	9.09472·42	8.76582·04	1,44977·00
12	0,28520·07	9.06758·06	8.76497·00	1,47353·68
13	0,30896·74	9.04335·76	8.76416·63	1,49730·35
14	0,33273·41	9.02160·62	8.76340·69	1,52107·02
15	0,35650·08	9.00197·10	8.76268·91	1,54483·69
16	0,38026·76	8·98416·55	8.76201·06	1,56860·37
17	0,40403·43	8·96795·54	8.76136·92	1,59237·04
18	0,42780·10	8·95314·68	8.76076·29	1,61613·71
19	0,45156·77	8·93957·74	8.76018·97	1,63990·38
20	0,47533·44	8,92711·00	8,75964·76	1,66367·05
21	0,49910·12	8·91562·74	8.75913·51	1,68743·73
22	0,52286·79	8·90502 91	8.75865·03	1,71120·40
23	0,54663·46	8·89522 82	8.75819·18	1,73497·07
24	0,57040·13	8.88614 87	8.75775 84	1,75873·74
25	0,59416·80	8·8 772·41	8.75734·84	1,78250·41
26	0,61793·48	8·86989·61	8.75696·05	1,80627·09
27	0,64170·15	8.86261·26	8.75659·36	1,83003·76
28	0,66546·82	8·85582 80	8.75624 66	1,85380·43
29	0 68923·49	8·8 950·08	8 75591·83	1,87757·10
30	0,71300·17	8,84359·43	8,75560·77	1,90133·78
31	0,73676·84	8,83807·55	8,75531·38	1,92510·45
32	0,76053·51	8.83291·44	8.75503·57	1,94887·12
33	0,78430·18	8.82808·40	8.75477·27	1,97263·79
34	0,80806·85	8.82356·00	8.75452·38	1,99640·46
35	0,83183·53	8,81931·97	8,75428·84	2,02017·14
36	0,85560·20	8,81534·31	8.75406·54	2,04393·81
37	0,87936·87	8.81161·15	8.75385·46	2,06770·48
38	0,90313·54	8,8c810·80	8.75365·50	2,09147·15
39	0,92690·22	8,80481·69	8.75346·62	2,11523·83
40	0,95066·89	8,80172·38	8,75328·74	2,13900·50
41	0,97443·56	8.79881·56	8.75311·84	2,16277·17
42	0,99820·23	8,79608·02	8.75295·82	2,18653·8
43	1,02196·90	8,79350·62	8.75280·68	2,21030·51
44	1,04573·58	8.79108·33	8.75266·34	2,23407·1
45	1,06950·25	8,78880·18	8.75252·77	2,25783·8
46	1,09326·92	8,78665·27	8.75239·93	2,28160·5
47	1,11703·59	8,78462·79	8.75227·77	2,30537·2
48	1,14080·27	8,78271·96	8.75216·27	2,32913·8
49	1,16456·94	8,78092·06	8.75205·38	2,35290·5
50	1,18833·61	8,77922·42	8,75195·07	2,37667·2
			8.75012·25	

5 3/4 Per Cent.

Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
0,02428·04	0,02428·04	8,78551·08	1,23829·92	51
0,04856·08	9,73522·09	8,78406·54	1,26257·96	52
0,07284·11	9,57098·71	8,78270·30	1,28685·99	53
0,09712·15	9,45779·29	8,78141·86	1,31114·03	54
0,12140·19	9,37251·46	8,78020·75	1,33542·07	55
0,14568·23	9,30495·23	8,77906·53	1,35970·11	56
0,16996·26	9,24931·21	8,77798·80	1,38398·14	57
0,19424·30	9,20261·44	8,77697·18	1,40826·18	58
0,21852·34	9,16264·41	8,77601·30	1,43254·22	59
0,24280·38	9,12795·72	8,77510·82	1,45682·26	60
0,26708·41	9,09752·37	8,77425·45	1,48110·29	61
0,29136·45	9,07058·33	8,77344·86	1,50538·33	62
0,31564·49	9,04655·88	8,77268·80	1,52966·37	63
0,33992·53	9,02500·15	8,77196·99	1,55394·41	64
0,36420·56	9,00555·60	8,77129·19	1,57822·44	65
0,38848·60	8,98793·56	8,77065·18	1,60250·48	66
0,41276·64	8,97190·62	8,77004·74	1,62678·52	67
0,43704·68	8,95727·42	8,76947·66	1,65106·56	68
0,46132·71	8,94387·69	8,76893·75	1,67534·59	69
0,48560·75	8,93157·74	8,76842·84	1,69962·63	70
0,50988·79	8,92025·85	8,76794·75	1,72390·67	71
0,53416·83	8,90981·98	8,76749·32	1,74818·71	72
0,55844·86	8,90017·44	8,76706·41	1,77246·74	73
0,58272·90	8,89124·64	8,76665·86	1,79674·78	74
0,60700·94	8,88296·93	8,76627·57	1,82102·82	75
0,63128·98	8,87528·47	8,76591·37	1,84530·86	76
0,65557·02	8,86814·10	8,76557·18	1,86958·90	77
0,67985·05	8,86149·22	8,76524·86	1,89386·93	78
0,70413·09	8,85529·71	8,76494·34	1,91814·97	79
0,72841·13	8,84951·92	8,76465·49	1,94243·01	80
0,75269·17	8,84412·52	8,76438·23	1,96671·05	81
0,77697·20	8,83908·53	8,76412·47	1,99099·08	82
0,80125·24	8,83437·28	8,76388·11	2,01527·12	83
0,82553·28	8,82996·30	8,76365·10	2,03955·16	84
0,84981·32	8,82583·38	8,76343·35	2,06383·20	85
0,87409·35	8,82196·48	8,76322·79	2,08811·23	86
0,89837·39	8,81833·77	8,76303·36	2,11239·27	87
0,92265·43	8,81493·54	8,76284·68	2,13667·31	88
0,94693·47	8,81174·25	8,76267·63	2,16095·35	89
0,97121·50	8,80874·47	8,76251·21	2,18523·38	90
0,99549·54	8,80592·87	8,76235·70	2,20951·42	91
1,01977·58	8,80328·26	8,76221·04	2,23379·46	92
1,04405·62	8,80079·50	8,76207·18	2,25807·50	93
1,06833·65	8,79845·58	8,76194·08	2,28235·53	94
1,09261·69	8,79625·54	8,76181·68	2,30663·57	95
1,11689·73	8,79418·47	8,76169·97	2,33091·61	96
1,14117·77	8,79223·57	8,76158·89	2,35519·65	97
1,16545·81	8,79040·07	8,76148·42	2,37947·68	98
1,18973·84	8,78867·25	8,76138·53	2,40375·72	99
1,21401·88	8,78704·46	8,76129·17	2,42803·76	100
		8,75966·78		Perp

5 7/8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.
1	0,02479·34	0,02479·34	8,79329·68	1,26446·0
2	0,04958·68	9,73598·32	8,79191·28	1,28925·6
3	0,07438·03	9,57199·39	8,79060·96	1,31405·14
4	0,09917·37	9,45903·94	8,78938·24	1,33884·49
5	0,12396·71	9,37399·59	8,78822·64	1,36363·83
6	0,14876·05	9,30656·36	8,78713·74	1,38843·17
7	0,17355·40	9,25124·87	8,78611·13	1,41322·51
8	0,19834·74	9,20477·14	8,78514·44	1,43801·86
9	0,22314·08	9,16501·70	8,78423·30	1,46281·20
10	0,24793·42	9,13054·11	8,78337·41	1,48760·54
11	0,27272·77	9,10031·41	8,78256·43	1,51239·88
12	0,29752·11	9,07357·53	8,78180·09	1,53719·22
13	0,32231·45	9,04974·79	8,78108·10	1,56198·57
14	0,34710·79	9,02838·32	8,78040·22	1,58677·91
15	0,37190·13	9,00912·55	8,77976·21	1,61157·25
16	0,39669·48	8,99168·88	8,77915·82	1,63636·59
17	0,42148·82	8,97583·85	8,77858·88	1,66115·94
18	0,44628·16	8,96138·11	8,77805·15	1,68595·28
19	0,47107·50	8,94815·41	8,77754·48	1,71074·62
20	0,49586·85	8,93602·05	8,77706·67	1,73553·96
21	0,52066·19	8,92486·34	8,77661·55	1,76033·31
22	0,54545·53	8,91458·22	8,77618·98	1,78512·65
23	0,57024·87	8,90509·02	8,77578·81	1,80991·99
24	0,59504·22	8,89631·13	8,77540·91	1,83471·33
25	0,61983·56	8,88817·95	8,77505·15	1,85950·67
26	0,64462·90	8,88063·62	8,77471·40	1,88430·02
27	0,66942·24	8,87362·99	8,77439·53	1,90909·36
28	0,69421·59	8,86711·46	8,77409·45	1,93388·70
29	0,71900·93	8,86104·93	8,77381·08	1,95868·04
30	0,74380·27	8,85539·73	8,77354·29	1,98347·39
31	0,76859·61	8,85012·57	8,77328·99	2,00826·73
32	0,79338·95	8,84520·46	8,77305·13	2,03306·08
33	0,81818·30	8,84060·73	8,77282·59	2,05785·41
34	0,84297·64	8,83630·93	8,77261·31	2,08264·76
35	0,86776·98	8,83228·85	8,77241·23	2,10744·10
36	0,89256·32	8,82852·47	8,77222·27	2,13223·44
37	0,91735·67	8,82499·95	8,77204·36	2,15702·78
38	0,94215·01	8,82169·59	8,77187·47	2,18182·13
39	0,96694·35	8,81859·85	8,77171·51	2,20661·47
40	0,99173·69	8,81569·33	8,77156·43	2,23140·81
41	1,01653·04	8,81296·69	8,77142·22	2,25620·15
42	1,04132·38	8,81040·74	8,77128·79	2,28099·49
43	1,06611·72	8,80800·37	8,77116·11	2,30578·84
44	1,09091·06	8,80574·57	8,77104·12	2,33058·18
45	1,11570·40	8,80362·35	8,77092·81	2,35537·52
46	1,14049·75	8,80162·87	8,77082·14	2,38016·86
47	1,16529·09	8,79975·29	8,77072·06	2,40496·21
48	1,19008·43	8,79798·87	8,77062·54	2,42975·55
49	1,21487·77	8,79632·87	8,77053·55	2,45454·89
50	1,23967·12	8,79476·69	8,77045·05	2,47934·23
			8,76900·79	

6 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,02530·59	0,02530·59	8,80098·36	1,29059·91	51
2	0,05061·17	9,73674·45	8,79965·87	1,31590·50	52
3	0,07591·76	9,57299·90	8,79841·25	1,34121·09	53
4	0,10122·35	9,46028·35	8,79723·99	1,36651·67	54
5	0,12652·93	9,37547·41	8,79613·68	1,39182·26	55
6	0,15183·52	9,30827·11	8,79509·86	1,41712·85	56
7	0,17714·11	9,25318·05	8,79412·15	1,44243·43	57
8	0,20244·69	9,20692·28	8,79320·16	1,46774·02	58
9	0,22775·28	9,16738·30	8,79233·56	1,49304·61	59
10	0,25305·87	9,13311·71	8,79152·02	1,51835·19	60
11	0,27836·45	9,10309·50	8,79075·24	1,54365·78	61
12	0,30367·04	9,07655·68	8,79002·93	1,56896·36	62
13	0,32897·62	9,05292·52	8,78934·82	1,59426·95	63
14	0,35428·21	9,03175·13	8,78870·67	1,61957·54	64
15	0,37958·80	9,01268·01	8,78810·23	1,64488·12	65
16	0,40489·38	8,99542·52	8,78753·29	1,67018·71	66
17	0,43019·97	8,97975·23	8,78699·65	1,69549·30	67
18	0,45550·56	8,96546·77	8,78649·09	1,72079·88	68
19	0,48081·14	8,95240·91	8,78601·46	1,74610·47	69
20	0,50611·73	8,94043·96	8,78556·56	1,77141·06	70
21	0,53142·32	8,92944·22	8,78514·26	1,79671·64	71
22	0,55672·90	8,91931·64	8,78474·38	1,82202·23	72
23	0,58203·49	8,90997·56	8,78436·80	1,84732·82	73
24	0,60734·08	8,90134·39	8,78401·37	1,87263·40	74
25	0,63264·66	8,89335·51	8,78367·97	1,89793·99	75
26	0,65795·25	8,88595·09	8,78336·49	1,92324·58	76
27	0,68325·84	8,87907·97	8,78306·81	1,94855·16	77
28	0,70856·42	8,87269·55	8,78278·83	1,97385·75	78
29	0,73387·01	8,86675·75	8,78252·45	1,99916·34	79
30	0,75917·60	8,86122·91	8,78227·58	2,02446·92	80
31	0,78448·18	8,85607·74	8,78204·13	2,04977·51	81
32	0,80978·77	8,85127·26	8,78182·01	2,07508·10	82
33	0,83509·36	8,84678·81	8,78161·17	2,10038·68	83
34	0,86039·94	8,84259·95	8,78141·51	2,12569·27	84
35	0,88570·53	8,83868·45	8,78122·97	2,15099·86	85
36	0,91101·12	8,83502·33	8,78105·48	2,17630·44	86
37	0,93631·70	8,83159·74	8,78088·99	2,20161·03	87
38	0,96162·29	8,82839·00	8,78073·45	1,22691·61	88
39	0,98692·87	8,82538·58	8,78058·78	2,25222·20	89
40	1,01223·46	8,82257·04	8,78049·95	2,27752·79	90
41	1,03754·05	8,81993·11	8,78031·91	2,30283·37	91
42	1,06284·63	8,81745·57	8,78019·62	2,32813·96	92
43	1,08815·22	8,81513·34	8,78008·01	2,35344·55	93
44	1,11345·81	8,81295·38	8,77987·08	2,37875·13	94
45	1,13876·39	8,81090·77	8,77986·75	2,40405·72	95
46	1,16406·98	8,80898·60	8,77977·02	2,42936·31	96
47	1,18937·57	8,80718·09	8,77967·84	2,45466·89	97
48	1,21468·15	8,80548·49	8,77959·18	2,47997·48	98
49	1,23998·74	8,80389·09	8,77951·01	2,50528·07	99
50	1,26529·33	8,80239·25	8,77943·31	2,53058·65	100
			8,77815·13		Perp.

6 ¼ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,02632·89	0,02632·89	8,81606·66	1,34277·59	51
2	0,05265·79	9,73826·39	8,81485·28	1,36910·48	52
3	0,07898·68	9,57500·48	8,81371·35	1,39543·38	53
4	0,10531·58	9,46276·53	8,81264·40	1,42176·27	54
5	0,13164·47	9,37842·19	8,81163·97	1,44809·16	55
6	0,15797·36	9,31167·48	8,81069·66	1,47442·06	56
7	0,18430·26	9,25703·02	8,80981·10	1,50074·95	57
8	0,21063·15	9,21120·86	8,80897·90	1,52707·84	58
9	0,23696·04	9,17209·49	8,80819·75	1,55340·74	59
10	0,26328·94	9,13824·52	8,80746·32	1,57973·63	60
11	0,28961·83	9,10862·97	8,80677·32	1,60606·53	61
12	0,31594·73	9,08248·81	8,80612·48	1,63239·42	62
13	0,34227·62	9,05924·36	8,80551·55	1,65872·31	63
14	0,36860·51	9,03844·73	8,80494·27	1,68505·21	64
15	0,39493·41	9,01974·43	8,80440·43	1,71138·10	65
16	0,42126·30	9,00284·80	8,80389·82	1,73771·00	66
17	0,44759·20	8,98752·47	8,80342·25	1,76403·89	67
18	0,47392·09	8,97358·03	8,80297·51	1,79036·78	68
19	0,50024·98	8,96085·31	8,80255·45	1,81669·68	69
20	0,52657·88	8,94920·58	8,80215·91	1,84302·57	70
21	0,55290·77	8,93852·19	8,80178·72	1,86935·46	71
22	0,57923·67	8,92870·11	8,80143·75	1,89568·36	72
23	0,60556·56	8,91965·64	8,80110·86	1,92201·25	73
24	0,63189·45	8,91131·26	8,80079·93	1,94834·15	74
25	0,65822·35	8,90360·33	8,80050·84	1,97467·04	75
26	0,68455·24	8,89647·05	8,80023·47	2,00099·93	76
27	0,71088·13	8,88986·25	8,79997·74	2,02732·83	77
28	0,73721·03	8,88373·38	8,79973·53	2,05365·72	78
29	0,76353·92	8,87804·36	8,79950·76	2,07998·62	79
30	0,78986·82	8,87275·53	8,79929·33	2,10631·51	80
31	0,81619·71	8,86783·63	8,79909·18	2,13264·40	81
32	0,84252·60	8,86325·70	8,79890·22	2,15897·30	82
33	0,86885·50	8,85899·07	8,79872·38	2,18530·19	83
34	0,89518·39	8,85501·33	8,79855·61	2,21163·09	84
35	0,92151·29	8,85130·29	8,79839·82	2,23795·98	85
36	0,94784·18	8,84783·92	8,79824·97	2,26428·87	86
37	0,97417·07	8,84460·46	8,79810·99	2,29061·77	87
38	1,00049·97	8,84158·20	8,79797·84	2,31694·66	88
39	1,02682·86	8,83875·63	8,79785·47	2,34327·55	89
40	1,05315·75	8,83611·35	8,79773·83	2,36960·45	90
41	1,07948·65	8,83364·09	8,79762·88	2,39593·34	91
42	1,10581·54	8,83132·63	8,79752·57	2,42226·24	92
43	1,13214·44	8,82915·93	8,79742·87	2,44859·13	93
44	1,15847·33	8,82712·95	8,79733·75	2,47492·02	94
45	1,18480·22	8,82522·77	8,79725·16	2,50124·92	95
46	1,21113·12	8,82344·54	8,79717·08	2,52757·81	96
47	1,23746·01	8,82177·45	8,79709·48	2,55390·71	97
48	1,26378·91	8,82020·77	8,79702·32	2,58023·60	98
49	1,29011·80	8,81873·84	8,79695·59	2,60656·49	99
50	1,31644·69	8,81736·00	8,79689·25	2,63289·39	100
			8,79588·00		Perp.

6 ½ Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,02734·96	0,02734·96	8,83077·22	1,39483·00	51
2	0,05469·92	9,73977·92	8,82966·09	1,42217·96	52
3	0,08204·88	9,57700·41	8,82862·00	1,44952·92	53
4	0,10939·84	9,46523·82	8,82764·50	1,47687·88	54
5	0,13674·80	9,38135·80	8,82673·15	1,50422·84	55
6	0,16409·76	9,31506·37	8,82587·53	1,53157·80	56
7	0,19144·73	9,26086·14	8,82507·31	1,55892·76	57
8	0,21879·69	9,21547·18	8,82432·11	1,58627·73	58
9	0,24614·65	9,17677·99	8,82361·62	1,61362·69	59
10	0,27349·61	9,14334·18	8,82295·53	1,64097·65	60
11	0,30084·57	9,11412·77	8,82233·58	1,66832·61	61
12	0,32819·53	9,08837·77	8,82175·48	1,69567·57	62
13	0,35554·49	9,06551·46	8,82121·00	1,72302·53	63
14	0,38289·45	9,04509·01	8,82069·90	1,75037·49	64
15	0,41024·41	9,02674·88	8,82021·99	1,77772·45	65
16	0,43759·37	9,01020·48	8,81977·04	1,80507·41	66
17	0,46494·33	8,99522·41	8,81934·88	1,83242·37	67
18	0,49229·29	8,98161·30	8,81895·33	1,85977·33	68
19	0,51964·25	8,96920·96	8,81858·22	1,88712·29	69
20	0,54699·22	8,95787·72	8,81823·41	1,91447·25	70
21	0,57434·18	8,94749·91	8,81790·75	1,94182·22	71
22	0,60169·14	8,93797·50	8,81760·10	1,96917·18	72
23	0,62904·10	8,92921·85	8,81731·35	1,99652·14	73
24	0,65639·06	8,92115·41	8,81704·36	2,02387·10	74
25	0,68374·02	8,91371·58	8,81679·04	2,05122·06	75
26	0,71108·98	8,90684·56	8,81655·27	2,07857·02	76
27	0,73843·94	8,90049·21	8,81632·98	2,10591·98	77
28	0,76578·90	8,89460·99	8,81612·05	2,13326·94	78
29	0,79313·86	8,88915·82	8,81592·41	2,16061·90	79
30	0,82048·82	8,88410·09	8,81573·97	2,18796·86	80
31	0,84783·78	8,87940·52	8,81556·67	2,21531·82	81
32	0,87518·74	8,87504·19	8,81540·43	2,24266·78	82
33	0,90253·71	8,87098·44	8,81525·19	2,27001·74	83
34	0,92988·67	8,86720·87	8,81510·88	2,29736·71	84
35	0,95723·63	8,86369·31	8,81497·44	2,32471·67	85
36	0,98458·59	8,86041·78	8,81484·84	2,35206·63	86
37	1,01193·55	8,85736·47	8,81473·01	2,37941·59	87
38	1,03928·51	8,85451·74	8,81461·89	2,40676·55	88
39	1,06663·47	8,85186·07	8,81451·47	2,43411·51	89
40	1,09398·43	8,84938·09	8,81441·67	2,46146·47	90
41	1,12133·39	8,84706·52	8,81432·48	2,48881·43	91
42	1,14868·35	8,84490·21	8,81423·86	2,51616·39	92
43	1,17603·31	8,84288·08	8,81415·75	2,54351·35	93
44	1,20338·27	8,84099·13	8,81408·16	2,57086·31	94
45	1,23073·24	8,83922·46	8,81401·02	2,59821·27	95
46	1,25808·20	8,83757·23	8,81394·32	2,62556·23	96
47	1,28543·16	8,83602·65	8,81388·02	2,65291·20	97
48	1,31278·12	8,83458·00	8,81382·12	2,68026·16	98
49	1,34013·08	8,83322·62	8,81376·57	2,70761·12	99
50	1,36748·04	8,83195·89	8,81371·36	2,73496·08	100
			8,81291·34		Perp.

6 3/4 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.
1	0,02836·79	0,02836·79	8,84511·27	1,44676·21
2	0,05673·58	9,74129·03	8,84409·58	1,47513·00
3	0,08510·37	9,57899·71	8,84314·55	1,50349·78
4	0,11347·15	9,46770·23	8,84225·70	1,53186·57
5	0,14183·94	9,38428·23	8,84142·64	1,56023·36
6	0,17020·73	9,31843·76	8,84064·98	1,58860·15
7	0,19857·52	9,26467·41	8,83992·35	1,61696·94
8	0,22694·31	9,21971·27	8,83924·43	1,64533·73
9	0,25531·10	9,18143·83	8,83860·89	1,67370·51
10	0,28367·88	9,14840·71	8,83801·46	1,70207·30
11	0,31204·67	9,11958·96	8,83745·86	1,73044·09
12	0,34041·46	9,09422·58	8,83693·84	1,75880·88
13	0,36878·25	9,07173·86	8,83645·16	1,78717·67
14	0,39715·04	9,05167·97	8,83599·62	1,81554·46
15	0,42551·83	9,03369·43	8,83557·00	1,84391·24
16	0,45388·61	9,01749·60	8,83517·10	1,87228·03
17	0,48225·40	9,00285·12	8,83479·77	1,90064·82
18	0,51062·19	8,98956·65	8,83444·82	1,92901·61
19	0,53898·98	8,97747·98	8,83412·11	1,95738·40
20	0,56735·77	8,96645·47	8,83381·49	1,98575·19
21	0,59572·56	8,95637·47	8,83352·83	2,01411·97
22	0,62409·34	8,94713·96	8,83325·99	2,04248·76
23	0,65246·13	8,93866·31	8,83300·87	2,07085·55
24	0,68082·92	8,93086·99	8,83277·36	2,09922·34
25	0,70919·71	8,92369·42	8,83255·33	2,12759·13
26	0,73756·50	8,91707·81	8,83234·70	2,15595·92
27	0,76593·29	8,91097·05	8,83215·40	2,18432·70
28	0,79430·07	8,90532·59	8,83197·32	2,21269·49
29	0,82266·86	8,90010·40	8,83180·40	2,24106·28
30	0,85103·65	8,89526·87	8,83164·55	2,26943·07
31	0,87940·44	8,89078·74	8,83149·70	2,29779·86
32	0,90777·23	8,88663·09	8,83135·80	2,32616·65
33	0,93614·02	8,88277·31	8,83122·78	2,35453·43
34	0,96450·80	8,87919·00	8,83110·58	2,38290·22
35	0,99287·59	8,87586·01	8,83099·17	2,41127·01
36	1,02124·38	8,87276·38	8,83088·48	2,43963·80
37	1,04961·17	8,86988·30	8,83078·47	2,46800·59
38	1,07797·96	8,86720·17	8,83069·09	2,49637·38
39	1,10634·75	8,86470·49	8,83060·30	2,52474·16
40	1,13471·53	8,86237·90	8,83052·08	2,55310·95
41	1,16308·32	8,86021·12	8,83044·37	2,58147·74
42	1,19145·11	8,85819·04	8,83037·15	2,60984·53
43	1,21981·90	8,85630·58	8,83030·39	2,63821·32
44	1,24818·69	8,85454·78	8,83024·06	2,66658·11
45	1,27655·48	8,85290·74	8,83018·13	2,69494·90
46	1,30492·27	8,85137·63	8,83012·57	2,72331·68
47	1,33329·05	8,84994·69	8,83007·37	2,75168·47
48	1,36165·84	8,84861·21	8,83002·50	2,78005·26
49	1,39002·63	8,84736·54	8,82997·94	2,80842·05
50	1,41839·42	8,84620·09	8,82993·66	2,83678·84
			8,82930·38	

7 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,02938·38	0,02938·38	8,85910·02	1,49857·27	51
2	0,05876·76	9,74279·72	8,85817·02	1,52795·64	52
3	0,08815·13	9,58098·38	8,85730·29	1,55734·02	53
4	0,11753·51	9,47015·78	8,85649·39	1,58672·40	54
5	0,14691·89	9,38719·52	8,85573·91	1,61610·78	55
6	0,17630·27	9,32179·69	8,85503·50	1,64549·16	56
7	0,20568·64	9,26846·85	8,85437·79	1,67487·53	57
8	0,23507·02	9,22393·12	8,85376·47	1,70425·91	58
9	0,26445·40	9,18607·01	8,85319·24	1,73364·29	59
10	0,29383·78	9,15344·14	8,85265·82	1,76302·67	60
11	0,32322·16	9,12501·56	8,85215·96	1,79241·04	61
12	0,35260·53	9,10003·26	8,85169·41	1,82179·42	62
13	0,38198·91	9,07791·58	8,85125·95	1,85117·80	63
14	0,41137·29	9,05821·70	8,85085·37	1,88056·18	64
15	0,44075·67	9,04058·10	8,85047·48	1,90994·56	65
16	0,47014·04	9,02472·23	8,85012·11	1,93932·93	66
17	0,49952·42	9,01040·68	8,84979·07	1,96871·31	67
18	0,52890·80	8,99744·14	8,84948·21	1,99809·69	68
19	0,55829·18	8,98566·45	8,84919·39	2,02748·07	69
20	0,58767·56	8,97493·95	8,84892·48	2,05686·44	70
21	0,61705·93	8,96514·99	8,84867·34	2,08624·82	71
22	0,64644·31	8,95619·62	8,84843·87	2,11563·20	72
23	0,67582·69	8,94799·18	8,84821·93	2,14501·58	73
24	0,70521·07	8,94046·18	8,84801·44	2,17439·95	74
25	0,73459·44	8,93354·06	8,84782·30	2,20378·33	75
26	0,76397·82	8,92717·03	8,84764·42	2,23316·71	76
27	0,79336·20	8,92130·00	8,84747·72	2,26255·09	77
28	0,82274·58	8,91588·47	8,84732·11	2,29193·47	78
29	0,85212·96	8,91088·39	8,84717·54	2,32131·84	79
30	0,88151·33	8,90626·18	8,84703·91	2,35070·22	80
31	0,91089·71	8,90198·61	8,84691·19	2,38008·60	81
32	0,94028·09	8,89802·78	8,84679·30	2,40946·98	82
33	0,96966·47	8,89436·08	8,84668·19	2,43885·35	83
34	0,99904·84	8,89096·14	8,84657·81	2,46823·73	84
35	1,02843·22	8,88780·83	8,84648·11	2,49762·11	85
36	1,05781·60	8,88488·21	8,84639·05	2,52700·49	86
37	1,08719·98	8,88216·49	8,84630·58	2,55638·87	87
38	1,11658·36	8,87964·09	8,84622·67	2,58577·24	88
39	1,14596·73	8,87729·51	8,84615·28	2,61515·62	89
40	1,17535·11	8,87511·42	8,84608·37	2,64454·00	90
41	1,20473·49	8,87308·58	8,84601·92	2,67392·38	91
42	1,23411·87	8,87119·87	8,84595·88	2,70330·75	92
43	1,26350·24	8,86944·23	8,84590·25	2,73269·13	93
44	1,29288·62	8,86780·72	8,84584·98	2,76207·51	94
45	1,32227·00	8,86628·48	8,84580·06	2,79145·89	95
46	1,35165·38	8,86486·67	8,84575·46	2,82084·27	96
47	1,38103·76	8,86354·55	8,84571·15	2,85022·64	97
48	1,41042·13	8,86231·45	8,84567·14	2,87961·02	98
49	1,43980·51	8,86116·71	8,84563·40	2,90899·40	99
50	1,46918·89	8,86009·74	8,84559·88	2,93837·78	100
			8,84500·80		Perp.

Yrs.			Log. r.	Yrs.
			1.9618317	51
			1.9833401	52
			1.9946486	53
			1.9990571	54
			1.9997655	55
			1.9987740	56
			1.9972825	57
			1.9952909	58
			1.9933994	59
			1.9834579	60
			1.9815163	61
			1.9795748	62
			1.9776332	63
			2.0222417	64
			2.0242502	65
			2.0262586	66
			2.0282671	67
			2.0302756	68
			2.0322840	69
			2.0342925	70
			2.0363010	71
			2.0383094	72
			2.0403179	73
			2.0423264	74
			2.0443348	75
			2.0463433	76
			2.0483517	77
			2.0503602	78
			2.0523687	79
			2.0543771	80
			2.5608856	81
			2.5628941	82
			2.5649025	83
			2.5669110	84
			2.5689195	85
			2.5709279	86
			2.5729364	87
			2.5749449	88
			2.5769533	89
			2.5829618	90
			2.5849702	91
			2.5869787	92
			2.5889872	93
			2.5909956	94
			2.5930041	95
			2.5950126	96
			2.5970210	97
			2.5990295	98
			3.0010379	99
			3.0030464	100
			3.14084·64	Perp.

8 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,03342·38	0,03342·38	8,91174·95	1,70461·15	51
2	0,06684·75	9,74878·43	8,91110·22	1,73803·53	52
3	0,10027·13	9,58886·92	8,91050·36	1,77145·90	53
4	0,13369·50	9,47989·30	8,90995·00	1,80488·28	54
5	0,16711·88	9,39873·22	8,90943·81	1,83830·66	55
6	0,20054·25	9,33508·74	8,90896·47	1,87173·03	56
7	0,23396·63	9,28346·49	8,90852·68	1,90515·41	57
8	0,26739·00	9,24058·61	8,90812·18	1,93857·78	58
9	0,30081·38	9,20433·63	8,90774·70	1,97200·16	59
10	0,33423·76	9,17327·22	8,90740·04	2,00542·53	60
11	0,36766·13	9,14636·48	8,90707·96	2,03884·91	61
12	0,40108·51	9,12285·46	8,90678·27	2,07227·28	62
13	0,43450·88	9,10216·54	8,90650·81	2,10569·66	63
14	0,46793·26	9,08384·95	8,90625·41	2,13912·04	64
15	0,50135·63	9,06755·27	8,90601·89	2,17254·41	65
16	0,53478·01	9,05298·95	8,90580·13	2,20596·79	66
17	0,56820·38	9,03992·72	8,90559·98	2,23939·16	67
18	0,60162·76	9,02817·29	8,90541·34	2,27281·54	68
19	0,63505·14	9,01756·60	8,90524·08	2,30623·91	69
20	0,66847·51	9,00797·05	8,90508·12	2,33966·29	70
21	0,70189·89	8,99927·09	8,90493·34	2,37308·66	71
22	0,73532·26	8,99136·82	8,90479·66	2,40651·04	72
23	0,76874·64	8,98417·69	8,90466·99	2,43993·42	73
24	0,80217·01	8,97762·28	8,90455·27	2,47335·79	74
25	0,83559·39	8,97164·12	8,90444·41	2,50678·17	75
26	0,86901·76	8,96617·52	8,90434·37	2,54020·54	76
27	0,90244·14	8,96117·47	8,90425·07	2,57362·92	77
28	0,93586·52	8,95659·53	8,90416·46	2,60705·29	78
29	0,96928·89	8,95239·79	8,90408·49	2,64047·67	79
30	1,00271·27	8,94854·71	8,90401·12	2,67390·04	80
31	1,03613·64	8,94501·18	8,90394·29	2,70732·42	81
32	1,06956·02	8,94176·38	8,90387·96	2,74074·79	82
33	1,10298·39	8,93877·80	8,90382·11	2,77417·17	83
34	1,13640·77	8,93603·14	8,90376·68	2,80759·55	84
35	1,16983·14	8,93350·38	8,90371·68	2,84101·92	85
36	1,20325·52	8,93117·65	8,90367·03	2,87444·30	86
37	1,23667·90	8,92903·25	8,90362·72	2,90786·67	87
38	1,27010·27	8,92705·68	8,90358·74	2,94129·05	88
39	1,30352·65	8,82523·54	8,90355·06	2,97471·42	89
40	1,33695·02	8,92355·57	8,95351·64	3,00813·80	90
41	1,37037·40	8,92200·62	8,90348·49	3,04156·17	91
42	1,40379·77	8,92057·63	8,90345·56	3,07498·55	92
43	1,43722·15	8,91925·67	8,90342·85	3,10840·93	93
44	1,47064·52	8,91803·83	8,90340·34	3,14183·30	94
45	1,50406·90	8,91691·32	8,90338·02	3,17525·68	95
46	1,53749·28	8,91587·40	8,90335·86	3,20868·05	96
47	1,57091·65	8,91491·41	8,90333·87	3,24210·43	97
48	1,60434·03	8,91402·71	8,90332·04	3,27552·80	98
49	1,63776·40	8,91320·74	8,90330·33	3,30895·18	99
50	1,67118·78	8,91244·98	8,90328·74	3,34237·55	100
			8,90200·00		Perm.

9 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,03742·65	0,03742·65	8,95963·42	1,90875·14	51
2	0,07485·30	9,75470·67	8,95918·65	1,94617·79	52
3	0,11227·95	9,59665·73	8,95877·61	1,98360·44	53
4	0,14970·60	9,48949·26	8,95640·00	2,02103·09	54
5	0,18723·25	9,41008·93	8,95805·52	2,05845·74	55
6	0,22455·90	9,34814·86	8,95773·91	2,09588·39	56
7	0,26198·55	9,29817·72	8,95744·93	2,13331·04	57
8	0,29941·20	9,25689·66	8,95718·36	2,17073·69	58
9	0,33683·85	9,22219·29	8,95694·01	2,20816·34	59
10	0,37426·50	9,19262·35	8,95671·67	2,24558·99	60
11	0,41169·15	9,16715·98	8,95651·19	2,28301·64	61
12	0,44911·80	9,14504·29	8,95632·40	2,32044·29	62
13	0,48654·45	9,12569·77	8,95615·18	2,35786·94	63
14	0,52397·10	9,10867·72	8,95599·38	2,39529·59	64
15	0,56139·75	9,09362·78	8,95584·89	2,43272·24	65
16	0,59882·40	9,08026·53	8,95571·61	2,47014·89	66
17	0,63625·05	9,06835·74	8,95559·43	2,50757·54	67
18	0,67367·70	9,05771·28	8,95548·24	2,54500·19	68
19	0,71110·35	9,04817·14	8,95537·99	2,58242·84	69
20	0,74853·00	9,03959·84	8,95528·59	2,61985·49	70
21	0,78595·65	9,03187·94	8,95519·96	2,65728·14	71
22	0,82338·30	9,02491·64	8,95512·06	2,69470·79	72
23	0,86080·95	9,01862·51	8,95504·80	2,73213·43	73
24	0,89823·60	9,01293·22	8,95498·14	2,76956·08	74
25	0,93566·24	9,00777·44	8,95492·04	2,80698·73	75
26	0,97308·89	9,00319·57	8,95486·44	2,84441·38	76
27	1,01051·54	8,99884·72	8,95481·29	2,88184·03	77
28	1,04794·19	8,99498·57	8,95476·59	2,91926·68	78
29	1,08536·84	8,99147·30	8,95472·26	2,95669·33	79
30	1,12279·49	8,98827·51	8,95468·29	2,99411·98	80
31	1,16022·14	8,98536·18	8,95464·65	3,03154·63	81
32	1,19764·79	8,98270·62	8,95461·32	3,06897·28	82
33	1,23507·44	8,98028·40	8,95458·26	3,10639·93	83
34	1,27250·09	8,97807·36	8,95455·45	3,14382·58	84
35	1,30992·74	8,97605·57	8,95452·87	3,18125·23	85
36	1,34735·39	8,97421·25	8,95450·50	3,21867·88	86
37	1,38478·04	8,97252·83	8,95448·34	3,25610·53	87
38	1,42220·69	8,97098·90	8,95446·35	3,29353·18	88
39	1,45963·34	8,96958·15	8,95444·52	3,33095·83	89
40	1,49705·99	8,96829·43	8,95442·85	3,36838·48	90
41	1,53448·64	8,96711·67	8,95441·31	3,40581·13	91
42	1,57191·29	8,96603·91	8,95439·91	3,44323·78	92
43	1,60933·94	8,96505·28	8,95438·61	3,48066·43	93
44	1,64676·59	8,96415·00	8,95437·42	3,51809·08	94
45	1,68419·24	8,96332·33	8,95436·34	3,55551·73	95
46	1,72161·89	8,96256·63	8,95435·34	3,59294·38	96
47	1,75904·54	8,96187·29	8,95434·43	3,63037·03	97
48	1,79647·19	8,96123·78	8,95433·58	3,66779·68	98
49	1,83389·84	8,96065·59	8,95432·81	3,70522·33	99
50	1,87132·49	8,96012·28	8,95432·11	3,74264·98	100
			8,95424·25		Perp.

10 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,04139·27	0,04139·27	9,00337·63	2,11102·69	51
2	0,08278·54	9,76056·60	9,00306·83	2,15241·96	52
3	0,12417·81	9,60435·01	9,00278·85	2,19381·23	53
4	0,16557·07	9,49895·92	9,00253·42	2,23520·50	54
5	0,20696·34	9,42127·07	9,00230·32	2,27659·77	55
6	0,24835·61	9,36098·59	9,00209·33	2,31799·04	56
7	0,28974·88	9,31261·21	9,00190·26	2,35938·31	57
8	0,33114·15	9,27287·16	9,00172·93	2,40077·57	58
9	0,37253·42	9,23965·11	9,00157·18	2,44216·84	59
10	0,41392·69	9,21150·87	9,00142·87	2,48356·11	60
11	0,45531·95	9,18741·68	9,00129·86	2,52495·38	61
12	0,49671·22	9,16661·75	9,00118·04	2,56634·65	62
13	0,53810·49	9,14853·64	9,00107·30	2,60773·92	63
14	0,57949·76	9,13272·77	9,00097·53	2,64913·19	64
15	0,62089·03	9,11883·91	9,00088·65	2,69052·45	65
16	0,66228·30	9,10658·73	9,00080·59	2,73191·72	66
17	0,70367·57	9,09574·15	9,00073·26	2,77330·99	67
18	0,74506·83	9,08611·14	9,00066·59	2,81470·26	68
19	0,78646·10	9,07753·82	9,00060·53	2,85609·53	69
20	0,82785·37	9,06988·86	9,00055·03	2,89748·80	70
21	0,86924·64	9,06304·95	9,00050·03	2,93888·06	71
22	0,91063·91	9,05692·41	9,00045·47	2,98027·33	72
23	0,95203·18	9,05142·97	9,00041·34	3,02166·60	73
24	0,99342·44	9,04649·44	9,00037·57	3,06305·87	74
25	1,03481·71	9,04205·57	9,00034·16	3,10445·14	75
26	1,07620·98	9,03805·97	9,00031·05	3,14584·41	76
27	1,11760·25	9,03445·86	9,00028·23	3,18723·68	77
28	1,15899·52	9,03121·05	9,00025·66	3,22862·94	78
29	1,20038·79	9,02827·87	9,00023·33	3,27002·21	79
30	1,24178·06	9,02563·04	9,00021·21	3,31141·48	80
31	1,28317·32	9,02323·69	9,00019·28	3,35280·75	81
32	1,32456·59	9,02007·23	9,00017·53	3,39420·02	82
33	1,36595·86	9,01911·38	9,00015·93	3,43559·29	83
34	1,40735·13	9,01734·10	9,00014·48	3,47698·56	84
35	1,44874·40	9,01573·56	9,00013·17	3,51837·82	85
36	1,49013·67	9,01428·13	9,00011·97	3,55977·09	86
37	1,53152·94	9,01296·35	9,00010·88	3,60116·36	87
38	1,57292·20	9,01176·88	9,00009·89	3,64255·63	88
39	1,61431·47	9,01068·57	9,00009·00	3,68394·90	89
40	1,65570·74	9,00964·32	9,00008·17	3,72534·17	90
41	1,69710·01	9,00881·21	9,00007·43	3,76673·43	91
42	1,73849·28	9,00800·36	9,00006·76	3,80812·70	92
43	1,77988·55	9,00726·99	9,00006·14	3,84951·97	93
44	1,82127·81	9,00660·40	9,00005·58	3,89091·24	94
45	1,86267·08	9,00599·94	9,00005·08	3,93230·51	95
46	1,90406·35	9,00545·06	9,00004·61	3,97369·78	96
47	1,94545·62	9,00495·22	9,00004·19	4,01509·05	97
48	1,98684·89	9,00449·97	9,00003·81	4,05648·31	98
49	2,02824·16	9,00408·87	9,00003·47	4,09787·58	99
50	2,06963·43	9,00371·54	9,00003·15	4,13926·85	100
			9,00000·00		Perp.

12 Per Cent.

Years	Log. r^n.	Log. a^n.	Log. a^n.	Log. r^n.	Years
1	0,04921·80	0,04921·80	9,08052·50	2,51011·92	51
2	0,09843·60	9,77210·02	9,08038·09	2,55933·72	52
3	0,14765·41	9,61945·76	9,08025·22	2,60855·52	53
4	0,19687·21	9,51750·53	9,08013·73	2,65777·32	54
5	0,24609·01	9,44312·17	9,08003·48	2,70699·12	55
6	0,29530·81	9,38600·95	9,07994·32	2,75620·93	56
7	0,34452·62	9,34067·76	9,07986·15	2,80542·73	57
8	0,39374·42	9,30384·99	9,07978·86	2,85464·53	58
9	0,44296·22	9,27341·55	9,07972·35	2,90386·33	59
10	0,49218·02	9,24793·44	9,07966·53	2,95308·14	60
11	0,54139·82	9,22638·18	9,07961·35	3,00229·94	61
12	0,59061·63	9,20800·26	9,07956·71	3,05151·74	62
13	0,63983·43	9,19222·50	9,07952·59	3,10073·54	63
14	0,68905·23	9,17860·65	9,07948·88	3,14995·35	64
15	0,73827·03	9,16679·78	9,07945·59	3,19917·15	65
16	0,78748·84	9,15651·89	9,07942·64	3,24838·95	66
17	0,83670·64	9,14754·25	9,07940·02	3,29760·75	67
18	0,88592·44	9,13968·18	9,07937·67	3,34682·55	68
19	0,93514·24	9,13278·15	9,07935·58	3,39604·36	69
20	0,98436·05	9,12671·17	9,07933·71	3,44526·16	70
21	1,03357·85	9,12136·31	9,07932·04	3,49447·96	71
22	1,08279·65	9,11664·26	9,07930·55	3,54369·76	72
23	1,13201·45	9,11247·08	9,07929·31	3,59291·57	73
24	1,18123·25	9,10877·96	9,07928·03	3,64213·37	74
25	1,23045·06	9,10551·01	9,07926·97	3,69135·17	75
26	1,27966·86	9,10261·16	9,07925·96	3,74056·97	76
27	1,32888·66	9,10003·99	9,07925·17	3,78978·77	77
28	1,37810·46	9,09775·65	9,07924·42	3,83900·58	78
29	1,42732·27	9,09572·78	9,07923·75	3,88822·38	79
30	1,47654·07	9,09392·45	9,07923·14	3,93744·18	80
31	1,52575·87	9,09232·07	9,07922·60	3,98665·98	81
32	1,57497·67	9,09089·38	9,07922·12	4,03587·79	82
33	1,62419·47	9,08962·46	9,07921·70	4,08509·59	83
34	1,67341·28	9,08849·27	9,07921·31	4,13431·39	84
35	1,72263·08	9,08748·55	9,07920·97	4,18353·19	85
36	1,77184·88	9,08658·81	9,07920·67	4,23274·99	86
37	1,82106·68	9,08578·84	9,07920·39	4,28196·80	87
38	1,87028·49	9,08507·57	9,07920·15	4,33118·60	88
39	1,91950·29	9,08444·03	9,07919·93	4,38040·40	89
40	1,96872·09	9,08387·38	9,07919·74	4,42962·20	90
41	2,01793·89	9,08336·86	9,07919·57	4,47884·01	91
42	2,06715·70	9,08291·80	9,07919·41	4,52805·81	92
43	2,11637·50	9,08251·61	9,07919·27	4,57727·61	93
44	2,16559·30	9,08215·76	9,07919·15	4,62649·41	94
45	2,21481·10	9,08183·77	9,07919·04	4,67571·22	95
46	2,26402·90	9,08155·23	9,07918·94	4,72493·02	96
47	2,31324·71	9,08129·76	9,07918·86	4,77414·82	97
48	2,36246·51	9,08107·04	9,07918·78	4,82336·62	98
49	2,41168·31	9,08086·76	9,07918·71	4,87258·42	99
50	2,46090·11	9,08068·66	9,07918·65	4,92180·23	100
			9,07918·12		Perp.

TABLE II.

SHEWING

The Logarithms of the Increasing Value of 1£, at the end of every month, from 1 to 12 months.

$$\text{Log}\,(1+i)^{\frac{n}{12}}.$$

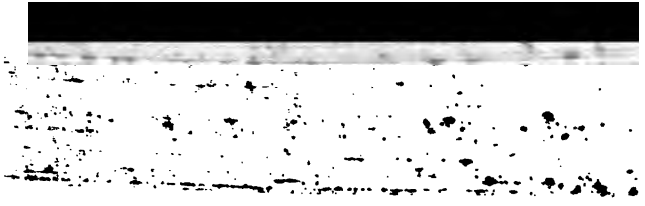

Table II. Shewing the Logarithms of £1 increased by its interest at the end of every month.

Rate per Cent.	1 month.	2 months.	3 months.	4 months.	Rate per Cent.
1/2	0,00018·05	0,00036·10	0,00054·15	0,00072·20	1/2
1	0,00036·01	0,00072·02	0,00108·03	0,00144·05	1
1 1/8	0,00053·88	0,00107·77	0,00161·65	0,00215·53	1 1/8
1 1/4	0,00058·34	0,00116·68	0,00175·01	0,00233·35	1 1/4
1 3/8	0,00062·79	0,00125·57	0,00188·36	0,00251·15	1 3/8
1 1/2	0,00067·23	0,00134·46	0,00201·69	0,00268·92	1 1/2
2	0,00071·67	0,00143·34	0,00215·00	0,00286·67	2
2 1/8	0,00076·10	0,00152·20	0,00228·30	0,00304·40	2 1/8
2 1/4	0,00080·53	0,00161·06	0,00241·58	0,00322·11	2 1/4
2 3/8	0,00084·95	0,00169·90	0,00254·85	0,00339·80	2 3/8
2 1/2	0,00089·37	0,00178·73	0,00268·10	0,00357·46	2 1/2
2 5/8	0,00093·78	0,00187·55	0,00281·33	0,00375·11	2 5/8
2 3/4	0,00098·18	0,00196·36	0,00294·55	0,00392·73	2 3/4
2 7/8	0,00102·58	0,00205·16	0,00307·75	0,00410·33	2 7/8
3	0,00106·98	0,00213·95	0,00320·93	0,00427·91	3
3 1/8	0,00111·37	0,00222·73	0,00334·10	0,00445·47	3 1/8
3 1/4	0,00115·75	0,00231·50	0,00347·25	0,00463·00	3 1/4
3 3/8	0,00120·13	0,00240·26	0,00360·39	0,00480·52	3 3/8
3 1/2	0,00124·50	0,00249·01	0,00373·51	0,00498·01	3 1/2
3 5/8	0,00128·87	0,00257·74	0,00386·61	0,00515·48	3 5/8
3 3/4	0,00133·23	0,00266·47	0,00399·70	0,00532·94	3 3/4
3 7/8	0,00137·59	0,00275·18	0,00412·78	0,00550·37	3 7/8
4	0,00141·94	0,00283·89	0,00425·83	0,00567·78	4
4 1/8	0,00146·29	0,00292·58	0,00438·88	0,00585·17	4 1/8
4 1/4	0,00150·63	0,00301·27	0,00451·90	0,00602·54	4 1/4
4 3/8	0,00154·97	0,00309·94	0,00464·91	0,00619·88	4 3/8
4 1/2	0,00159·30	0,00318·60	0,00477·91	0,00637·21	4 1/2
4 5/8	0,00163·63	0,00327·26	0,00490·89	0,00654·52	4 5/8
4 3/4	0,00167·95	0,00335·90	0,00503·85	0,00671·80	4 3/4
4 7/8	0,00172·27	0,00344·53	0,00516·80	0,00689·07	4 7/8
5	0,00176·58	0,00353·15	0,00529·73	0,00706·31	5
5 1/8	0,00180·88	0,00361·77	0,00542·65	0,00723·53	5 1/8
5 1/4	0,00185·18	0,00370·37	0,00555·55	0,00740·74	5 1/4
5 3/8	0,00189·48	0,00378·96	0,00568·44	0,00757·92	5 3/8
5 1/2	0,00193·77	0,00387·54	0,00581·31	0,00775·08	5 1/2
5 5/8	0,00198·06	0,00396·11	0,00594·17	0,00792·22	5 5/8
5 3/4	0,00202·34	0,00404·67	0,00607·01	0,00809·35	5 3/4
5 7/8	0,00206·61	0,00413·22	0,00619·84	0,00826·45	5 7/8
6	0,00210·88	0,00421·76	0,00632·65	0,00843·53	6
6 1/4	0,00219·41	0,00438·82	0,00658·22	0,00877·63	6 1/4
6 1/2	0,00227·91	0,00455·83	0,00683·74	0,00911·65	6 1/2
6 3/4	0,00236·40	0,00472·80	0,00709·20	0,00945·60	6 3/4
7	0,00244·86	0,00489·73	0,00734·59	0,00979·46	7
7 1/2	0,00261·74	0,00523·47	0,00785·21	0,01046·95	7 1/2
8	0,00278·53	0,00557·06	0,00835·59	0,01114·12	8
9	0,00311·89	0,00623·77	0,00935·66	0,01247·55	9
10	0,00344·94	0,00689·88	0,01034·82	0,01379·76	10
12	0,00410·15	0,00820·30	0,01230·45	0,01640·60	12

Table II. Shewing the Logarithms of £1 increased by its interest at the end of every month.

Rate per Cent.	5 months.	6 months.	7 months.	8 months.	Rate per Cent.
1/8	0,00090·25	0,00108·30	0,00126·35	0,00144·40	1/8
1	0,00180·06	0,00216·07	0,00252·08	0,00288·09	1
1 1/8	0,00269·42	0,00323·30	0,00377·19	0,00431·07	1 1/8
1 2/8	0,00291·69	0,00350·03	0,00408·37	0,00466·70	1 2/8
1 3/4	0,00313·93	0,00376·72	0,00439·51	0,00502·29	1 3/4
1 7/8	0,00336·15	0,00403·38	0,00470·61	0,00537·84	1 7/8
2	0,00358·34	0,00430·01	0,00501·68	0,00573·34	2
2 1/8	0,00380·50	0,00456·60	0,00532·70	0,00608·80	2 1/8
2 1/4	0,00402·64	0,00483·17	0,00563·69	0,00644·22	2 1/4
2 2/8	0,00424·75	0,00509·70	0,00594·65	0,00679·59	2 2/8
2 1/2	0,00446·83	0,00536·19	0,00625·56	0,00714·92	2 1/2
2 5/8	0,00468·88	0,00562·66	0,00656·43	0,00750·21	2 5/8
2 3/4	0,00490·91	0,00589·09	0,00687·27	0,00785·46	2 3/4
2 7/8	0,00512·91	0,00615·49	0,00718·07	0,00820·66	2 7/8
3	0,00534·88	0,00641·86	0,00748·84	0,00855·82	3
3 1/8	0,00556·83	0,00668·20	0,00779·56	0,00890·93	3 1/8
3 1/4	0,00578·75	0,00694·50	0,00810·25	0,00926·00	3 1/4
3 2/8	0,00600·65	0,00720·78	0,00840·91	0,00961·03	3 2/8
3 1/2	0,00622·51	0,00747·02	0,00871·52	0,00996·02	3 1/2
3 5/8	0,00644·36	0,00773·23	0,00902·10	0,01030·97	3 5/8
3 3/4	0,00666·17	0,00799·41	0,00932·64	0,01065·87	3 3/4
3 7/8	0,00687·96	0,00825·55	0,00963·14	0,01100·74	3 7/8
4	0,00709·72	0,00851·67	0,00993·61	0,01135·56	4
4 1/8	0,00731·46	0,00877·75	0,01024·04	0,01170·33	4 1/8
4 1/4	0,00753·17	0,00903·80	0,01054·44	0,01205·07	4 1/4
4 2/8	0,00774·85	0,00929·82	0,01084·80	0,01239·77	4 2/8
4 1/2	0,00796·51	0,00955·81	0,01115·12	0,01274·42	4 1/2
4 5/8	0,00818·14	0,00981·77	0,01145·40	0,01309·03	4 5/8
4 3/4	0,00839·75	0,01007·70	0,01175·65	0,01343·60	4 3/4
4 7/8	0,00861·33	0,01033·60	0,01205·87	0,01378·13	4 7/8
5	0,00882·89	0,01059·46	0,01236·04	0,01412·62	5
5 1/8	0,00904·42	0,01085·30	0,01266·18	0,01447·07	5 1/8
5 1/4	0,00925·92	0,01111·11	0,01296·29	0,01481·47	5 1/4
5 2/8	0,00947·40	0,01136·88	0,01326·36	0,01515·84	5 2/8
5 1/2	0,00968·85	0,01162·62	0,01356·39	0,01550·16	5 1/2
5 5/8	0,00990·28	0,01188·34	0,01386·39	0,01584·45	5 5/8
5 3/4	0,01011·68	0,01214·02	0,01416·36	0,01618·69	5 3/4
5 7/8	0,01033·06	0,01239·67	0,01446·28	0,01652·89	5 7/8
6	0,01054·41	0,01265·29	0,01476·18	0,01687·06	6
6 1/4	0,01097·04	0,01316·45	0,01535·85	0,01755·26	6 1/4
6 1/2	0,01139·57	0,01367·48	0,01595·39	0,01823·31	6 1/2
6 3/4	0,01182·00	0,01418·39	0,01654·79	0,01891·19	6 3/4
7	0,01224·32	0,01469·19	0,01714·05	0,01958·92	7
7 1/2	0,01308·69	0,01570·42	0,01832·16	0,02093·90	7 1/2
8	0,01392·66	0,01671·19	0,01949·72	0,02228·25	8
9	0,01559·44	0,01871·32	0,02183·21	0,02495·10	9
10	0,01724·70	0,02069·63	0,02414·57	0,02759·51	10
12	0,02050·75	0,02460·90	0,02871·05	0,03281·20	12

Table II. Shewing the Logarithms of £1 increased by its interest at the end of every month.

Rate per Cent.	9 months.	10 months.	11 months.	12 months.	Rate per Cent.
1/2	0,00162·45	0,00180·51	0,00198·56	0,00216·61	1/2
1	0,00324·10	0,00360·11	0,00396·13	0,00432·14	1
1 1/8	0,00484·95	0,00538·84	0,00592·72	0,00646·60	1 1/8
1 1/4	0,00525·04	0,00583·38	0,00641·72	0,00700·06	1 1/4
1 3/8	0,00565·08	0,00627·87	0,00690·65	0,00753·44	1 3/8
1 1/2	0,00605·07	0,00672·30	0,00739·53	0,00806·76	1 1/2
1 5/8	0,00645·01	0,00716·68	0,00788·35	0,00860·02	1 5/8
1 3/4	0,00684·91	0,00761·01	0,00837·11	0,00913·21	1 3/4
1 7/8	0,00724·75	0,00805·28	0,00885·80	0,00966·33	1 7/8
2	0,00764·54	0,00849·49	0,00934·44	0,01019·39	2
2 1/8	0,00804·29	0,00893·66	0,00983·02	0,01072·39	2 1/8
2 1/4	0,00843·99	0,00937·76	0,01031·54	0,01125·32	2 1/4
2 3/8	0,00883·64	0,00981·82	0,01080·00	0,01178·18	2 3/8
2 1/2	0,00923·24	0,01025·82	0,01128·40	0,01230 98	2 1/2
2 5/8	0,00962·79	0,01069·77	0,01176·75	0,01283·72	2 5/8
2 3/4	0,01002·30	0,01113·66	0,01225·03	0,01336·40	2 3/4
2 7/8	0,01041·75	0,01157·51	0,01273·26	0,01389·01	2 7/8
3	0,01081·16	0,01201·29	0,01321·42	0,01441·55	3
3 1/8	0,01120·53	0,01245·03	0,01369·53	0,01494·03	3 1/8
3 1/4	0,01159·84	0,01288·71	0,01417·58	0,01546·45	3 1/4
3 3/8	0,01199·11	0,01332·34	0,01465·58	0,01598·81	3 3/8
3 1/2	0,01238·33	0,01375·92	0,01513·51	0,01651·10	3 1/2
3 5/8	0,01277·50	0,01419·44	0,01561·39	0,01703·33	3 5/8
3 3/4	0,01316·63	0,01462·92	0,01609·21	0,01755·50	3 3/4
3 7/8	0,01355·70	0,01506·34	0,01656·97	0,01807·61	3 7/8
4	0,01394·74	0,01549·71	0,01704·68	0,01859·65	4
4 1/8	0,01433·72	0,01593·02	0,01752·33	0,01911·63	4 1/8
4 1/4	0,01472·66	0,01636·29	0,01799·92	0,01963·55	4 1/4
4 3/8	0,01511·55	0,01679·50	0,01847·45	0,02015·40	4 3/8
4 1/2	0,01550·40	0,01722·66	0,01894·93	0,02067·20	4 1/2
4 5/8	0,01589·20	0,01765·77	0,01942·35	0,02118·93	4 5/8
4 3/4	0,01627·95	0,01808·83	0,01989·72	0,02170·60	4 3/4
4 7/8	0,01666·66	0,01851·84	0,02037·03	0,02222·21	4 7/8
5	0,01705·32	0,01894·80	0,02084·28	0,02273·76	5
5 1/8	0,01743·93	0,01937·70	0,02131·48	0,02325·25	5 1/8
5 1/4	0,01782·50	0,01980·56	0,02178·62	0,02376·67	5 1/4
5 3/8	0,01821·03	0,02023·36	0,02225·70	0,02428·04	5 3/8
5 1/2	0,01859·51	0,02066·12	0,02272·73	0,02479·34	5 1/2
5 5/8	0,01897·94	0,02108·82	0,02319·70	0,02530·59	5 5/8
5 3/4	0,01974·67	0,02194·08	0,02413·49	0,02632·89	5 3/4
5 7/8	0,02051·22	0,02279·13	0,02507·05	0,02734·96	5 7/8
6	0,02127·59	0,02363·99	0,02600·39	0,02836·79	6
6 1/4	0,02203·78	0,02448·65	0,02693·51	0,02938·38	6 1/4
6 1/2	0,02355·63	0,02617·37	0,02879·11	0,03140·85	6 1/2
6 3/4	0,02506·78	0,02785·31	0,03063·84	0,03342·38	6 3/4
7	0,02806·99	0,03118·87	0,03430·76	0,03742·65	7
7 1/2	0,03104·45	0,03449·39	0,03794·33	0,04139·27	7 1/2
8	0,03691·35	0,04101·50	0,04511·65	0,04921·80	8

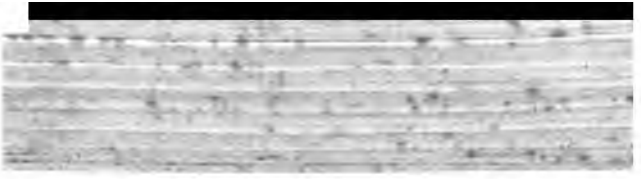

TABLE III.

SHEWING

For every rate contained in the two preceding tables the logarithms, to 10 and 7 decimals,
of i, i being the interest of £1 per annum or the rate;
of r, r being £1 increased by interest for one year;
and the logarithm of $\log^2 r$.

For every rate between 0 and 10 per Cent., proceeding by 10ths, the logarithms of i and r.

For every fractionary rate between 0 and 10 per Cent., proceeding by 12ths, the logarithms of i and r.

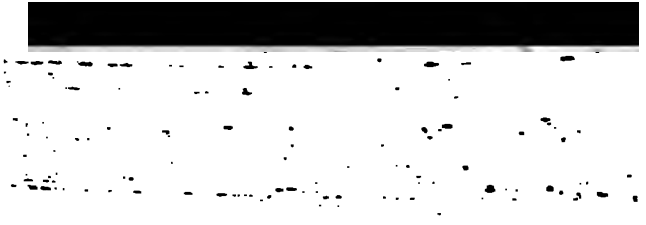

Table III. This table shews the Logarithms of (t), (r), and $(\text{Log}^2.r)$, t being the rate of interest per cent. and r £1 increased by its interest for one year.

Rate per Cent.	Log. t.	Log. r.	$\text{Log}^2.$ r.	Rate per Cent.
1/2	7,69897·00043	0,00216·60618	7,33567·08	1/2
1	8,00000·00000	0,00432·13738	7,63562·18	1
1 1/8	8,17609·12591	0,00646·60422	7,81063·85	1 1/8
1 1/4	8,21085·33653	0,00700·05586	7,84513·27	1 1/4
1 3/8	8,24303·80487	0,00753·44179	7,87704·97	1 3/8
1 1/2	8,27300·12721	0,00806·76217	7,90674·55	1 1/2
2	8,30102·99957	0,00860·01718	7,93450·71	2
2 1/8	8,32735·89344	0,00913·20695	7,96056·92	2 1/8
2 1/4	8,35218·25181	0,00966·33167	7,98512·62	2 1/4
2 3/8	8,37566·36140	0,01019·39148	8,00834·10	2 3/8
2 1/2	8,39794·00087	0,01072·38654	8,03035·14	2 1/2
2 5/8	8,41912·93077	0,01125·31701	8,05127·49	2 5/8
2 3/4	8,43933·26938	0,01178·18305	8,07121·28	2 3/4
2 7/8	8,45863·78490	0,01230·98482	8,09025·27	2 7/8
3	8,47712·12547	0,01283·72247	8,10847·11	3
3 1/8	8,49485·00217	0,01336·39616	8,12593·52	3 1/8
3 1/4	8,51188·33610	0,01389·00603	8,14270·41	3 1/4
3 3/8	8,52827·37772	0,01441·55226	8,15883·04	3 3/8
3 1/2	8,54406·80444	0,01494·03498	8,17436·08	3 1/2
3 5/8	8,55930·80109	0,01546·45436	8,18933·71	3 5/8
3 3/4	8,57403·12677	0,01598·81054	8,20379·70	3 3/4
3 7/8	8,58827·17068	0,01651·10368	8,21777·43	3 7/8
4	8,60205·99913	0,01703·33393	8,23129·98	4
4 1/8	8,61542·39529	0,01755·50144	8,24440·12	4 1/8
4 1/4	8,62838·89301	0,01807·60636	8,25710·39	4 1/4
4 3/8	8,64097·80574	0,01859·64885	8,26943·10	4 3/8
4 1/2	8,65321·25138	0,01911·62904	8,28140·36	4 1/2
4 5/8	8,66511·17371	0,01963·54710	8,29304·13	4 5/8
4 3/4	8,67669·36096	0,02015·40316	8,30436·19	4 3/4
4 7/8	8,68797·46200	0,02067·19738	8,31538·19	4 7/8
5	8,69897·00043	0,02118·92991	8,32611·66	5
5 1/8	8,70969·38697	0,02170·60088	8,33658·00	5 1/8
5 1/4	8,72015·93034	0,02222·21045	8,34678·52	5 1/4
5 3/8	8,73037·84686	0,02273·75876	8,35674·44	5 3/8
5 1/2	8,74036·26895	0,02325·24596	8,36646·89	5 1/2
5 5/8	8,75012·25268	0,02376·67220	8,37596·93	5 5/8
5 3/4	8,75966·78447	0,02428·03760	8,38525·54	5 3/4
5 7/8	8,76900·78709	0,02479·34233	8,39433·65	5 7/8
6	8,77815·12504	0,02530·58653	8,40322·12	6
6 1/4	8,79588·00173	0,02632·89387	8,42043·34	6 1/4
6 1/2	8,81291·33566	0,02734·96078	8,43695·11	6 1/2
6 3/4	8,82930·37728	0,02836·78837	8,45282·69	6 3/4
7	8,84509·80400	0,02938·37777	8,46810·76	7
7 1/2	8,87506·12634	0,03140·84643	8,49704·67	7 1/2
8	8,90308·99870	0,03342·37555	8,52405·52	8
9	8,95424·25094	0,03742·64979	8,57317·92	9
10	9,00000·00000	0,04139·26852	8,61692·36	10
12	9,07918·12460	0,04921·80227	8,69212·42	12

Table III. This table shews the Logarithms of t and r, t being the rate of interest per cent. and r being £1 increased by its interest for one year.

Rate per Cent.	Log. r.	Log. t.	Log. t.	Log. r.	Rate per Cent.
1/10	0,00043·40775	7,00000·00	8,70757·02	0,02160·27160	5 1/10
2/10	0,00086·77215	7,30103·00	8,71600·33	0,02201·57398	5 2/10
3/10	0,00130·09330	7,47712·13	8,72427·59	0,02242·83712	5 3/10
4/10	0,00173·37128	7,60206·00	8,73239·38	0,02284·06109	5 4/10
5/10	0,00216·60618	7,69897·00	8,74036·27	0,02325·24596	5 5/10
6/10	0,00259·79807	7,77815·13	8,74818·80	0,02366·39182	5 6/10
7/10	0,00302·94706	7,84509·80	8,75587·49	0,02407·49873	5 7/10
8/10	0,00346·05321	7,90309·00	8,76342·80	0,02448·56677	5 8/10
9/10	0,00389·11662	7,95424·25	8,77085·20	0,02489·59601	5 9/10
1	0,00432·13738	8,00000·00	8,77815·13	0,02530·58653	6
1 1/10	0,00475·11556	8,04139·27	8,78532·98	0,02571·53839	6 1/10
1 2/10	0,00518·05125	8,07918·12	8,79239·17	0,02612·45167	6 2/10
1 3/10	0,00560·94454	8,11394·34	8,79934·05	0,02653·32645	6 3/10
1 4/10	0,00603·79550	8,14612·80	8,80618·00	0,02694·16280	6 4/10
1 5/10	0,00646·60422	8,17609·13	8,81291·34	0,02734·96078	6 5/10
1 6/10	0,00689·37079	8,20412·00	8,81954·39	0,02775·72047	6 6/10
1 7/10	0,00732·09529	8,23044·89	8,82607·48	0,02816·44194	6 7/10
1 8/10	0,00774·77780	8,25527·25	8,83250·89	0,02857·12527	6 8/10
1 9/10	0,00817·41840	8,27875·36	8,83884·91	0,02897·77052	6 9/10
2	0,00860·01718	8,30103·00	8,84509·80	0,02938·37777	7
2 1/10	0,00902·57421	8,32221·93	8,85125·83	0,02978·94708	7 1/10
2 2/10	0,00945·08958	8,34242·27	8,85733·25	0,03019·47854	7 2/10
2 3/10	0,00987·56337	8,36172·78	8,86332·29	0,03059·97220	7 3/10
2 4/10	0,01029·99566	8,38021·12	8,86923·17	0,03100·42814	7 4/10
2 5/10	0,01072·38654	8,39794·00	8,87506·13	0,03140·84643	7 5/10
2 6/10	0,01114·73608	8,41497·33	8,88081·36	0,03181·22713	7 6/10
2 7/10	0,01157·04436	8,43136·38	8,88649·07	0,03221·57033	7 7/10
2 8/10	0,01199·31147	8,44715·80	8,89209·46	0,03261·87609	7 8/10
2 9/10	0,01241·53748	8,46239·80	8,89762·71	0,03302·14447	7 9/10
3	0,01283·72247	8,47712·13	8,90309·00	0,03342·37555	8
3 1/10	0,01325·86653	8,49136·17	8,90848·50	0,03382·56940	8 1/10
3 2/10	0,01367·96973	8,50515·00	8,91381·39	0,03422·72608	8 2/10
3 3/10	0,01410·03215	8,51851·39	8,91907·81	0,03462·84566	8 3/10
3 4/10	0,01452·05388	8,53147·89	8,92427·93	0,03502·92822	8 4/10
3 5/10	0,01494·03498	8,54406·80	8,92941·89	0,03542·97382	8 5/10
3 6/10	0,01535·97554	8,55630·25	8,93449·85	0,03582·98253	8 6/10
3 7/10	0,01577·87564	8,56820·17	8,93951·93	0,03622·95441	8 7/10
3 8/10	0,01619·73535	8,57978·36	8,94448·27	0,03662·88954	8 8/10
3 9/10	0,01661·55476	8,59106.46	8,94939·00	0,03702·78798	8 9/10
4	0,01703·33393	8,60206·00	8,95424·25	0,03742·64979	9
4 1/10	0,01745·07295	8,61278·39	8,95904·14	0,03782·47506	9 1/10
4 2/10	0,01786·77190	8,62324·93	8,96378·78	0,03822·26384	9 2/10
4 3/10	0,01828·43084	8,63346·85	8,96848·29	0,03862·01619	9 3/10
4 4/10	0,01870·04987	8,64345·27	8,97312·79	0,03901·73220	9 4/10
4 5/10	0,01911·62904	8,65321·25	8,97772·36	0,03941·41192	9 5/10
4 6/10	0,01953·16845	8,66275·78	8,98227·12	0,03981·05541	9 6/10
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5	0,02118·02991	8,69897·00	9,00000·00	0,04136·26852	10

Table III. This table shews the Logarithms of t and r, t being the rate of interest per cent. and r being £1 increased by its interest for one year.

Rate per Cent.	Log. r.	Log. t.	Log. t.	Log. r.	Rate per Cent.
1/12	0,00036·17614	6,92081·88	8,70614·86	0,02153·38405	5 1/12
1/6	0,00072·32216	7,22184·87	8,71321·04	0,02187·81089	5 1/6
1/3	0,00144·52409	7,52287·87	8,72699·87	0,02256·58279	5 1/3
5/12	0,00180·58009	7,61978·88	8,73373·21	0,02290·92795	5 5/12
7/12	0,00252·60240	7,76591·68	8,74689·36	0,02359·53688	5 7/12
2/3	0,00288·56882	7,82390·87	8,75332·77	0,02393·80075	5 2/3
5/6	0,00360·41243	7,92081·88	8,76591·68	0,02462·24749	5 5/6
11/12	0,00396·28971	7,96221·14	8,77207·71	0,02496·43045	5 11/12
1 1/12	0,00467·95548	8,03476·21	8,78414·16	0,02564·71576	6 1/12
1 1/6	0,00503·74407	8,06694·68	8,79005·05	0,02598·81820	6 1/6
1 1/3	0,00575·23289	8,12493·87	8,80163·23	0,02666·94283	6 1/3
1 5/12	0,00610·93322	8,15126·77	8,80730·95	0,02700·96512	6 5/12
1 7/12	0,00682·24596	8,19957·24	8,81844·58	0,02768·92984	6 7/12
1 2/3	0,00717·85846	8,22184·87	8,82390·87	0,02802·87236	6 2/3
1 5/6	0,00788·99599	8,26324·14	8,83463·26	0,02870·67791	6 5/6
1 11/12	0,00824·52110	8,28254·66	8,83989·68	0,02904·54103	6 11/12
2 1/12	0,00895·48427	8,31875·88	8,85023·77	0,02972·18816	7 1/12
2 1/6	0,00930·92241	8,33579·21	8,85531·72	0,03005·97225	7 1/6
2 1/3	0,01001·71208	8,36797·68	8,86530·14	0,03073·46170	7 1/3
2 5/12	0,01037·06368	8,38321·68	8,87020·88	0,03107·16713	7 5/12
2 7/12	0,01107·68069	8,41218·04	8,87986·01	0,03174·49902	7 7/12
2 2/3	0,01142·94618	8,42596·87	8,88460·66	0,03208·12676	7 2/3
2 5/6	0,01213·39136	8,45229·77	8,89394·66	0,03275·30303	7 5/6
2 11/12	0,01248·57115	8,46488·68	8,89854·24	0,03308·85224	7 11/12
3 1/12	0,01318·84536	8,48902·05	8,90759·05	0,03375·87300	8 1/12
3 1/6	0,01353·93986	8,50060·24	8,91204·48	0,03409·34464	8 1/6
3 1/3	0,01424·04391	8,52287·87	8,92081·88	0,03476·21063	8 1/3
3 5/12	0,01459·05355	8,53360·26	8,92514·01	0,03509·60505	8 5/12
3 7/12	0,01528·98826	8,55428·72	8,93365·60	0,03576·31697	8 7/12
3 2/3	0,01563·91343	8,56427·14	8,93785·21	0,03609·03453	8 2/3
3 5/6	0,01633·67963	8,58357·66	8,94612·46	0,03676·19309	8 5/6
3 11/12	0,01668·52074	8,59291·66	8,95020·25	0,03709·43415	8 11/12
4 1/12	0,01738·11923	8,61101·48	9,95824·53	0,03775·84005	9 1/12
4 1/6	0,01772·87670	8,61978·88	9,96221·14	0,03809·00496	9 1/6
4 1/3	0,01842·30828	8,63682·21	8,97003·68	0,03875·25890	9 1/3
4 5/12	0,01876·98249	8,64509·46	8,97389·72	0,03908·34800	9 5/12
4 7/12	0,01946·24798	8,66118·14	8,98151·66	0,03974·45068	9 7/12
4 2/3	0,01980·83934	8,66900·68	8,98527·67	0,04007·46432	9 2/3
4 5/6	0,02049·93951	8,68424·67	8,99270·08	0,04073·41642	9 5/6
4 11/12	0,92084·44841	8,69167·08	8,99636·57	0,04106·35495	9 11/12

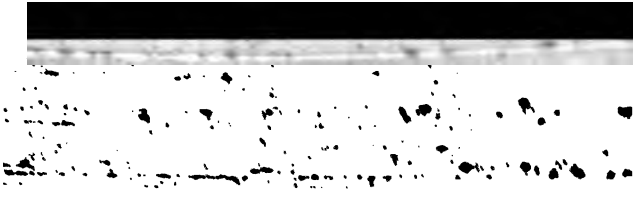


TABLE IV.

SHEWING

he Logarithms to 7 places which are to be *subtracted* from the logarithm of the annuity, when the *interest* is paid either 2, 4, 6, or 12 times *more often* than the *amortizement;* or which are to be *added* to the logarithm of the annuity when the *interest* is paid either 2, or 4, times *less often* than the *amortizement.*

$$P_2 = \log \frac{1}{\frac{2}{t}(r^{\frac{1}{2}} - 1)} \qquad P_4 = \log \frac{1}{\frac{4}{t}(r^{\frac{1}{4}} - 1)}$$

$$P_6 = \log \frac{1}{\frac{6}{t}(r^{\frac{1}{6}} - 1)} \qquad P_{12} = \log \frac{1}{\frac{12}{t}(r^{\frac{1}{12}} - 1)}$$

$$Q_2 = \log \frac{\left(1 + \frac{t}{2}\right)^2 - 1}{t} \qquad Q_4 = \log \frac{\left(1 + \frac{t}{4}\right)^4 - 1}{t}.$$

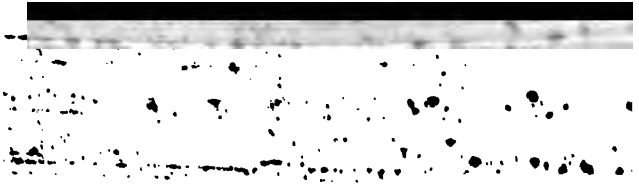
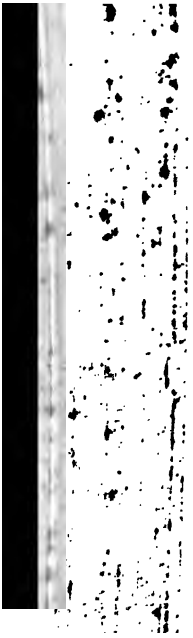


Table IV. Logarithms to 7 places for finding the value of annuity when the periods for paying the annuity and for paying the interest are not according to each other.

Rate per Cent.	P_2.	P_4.	P_6.	P_{12}.	Q_2.	Q_4.	Rate of Inter.
	−0,000·	−0,000·	−0,000·	−0,000·	+0,000·	+0,000·	
1/4	·2712	·4067	·4519	·4971	·2713	·4071	1/4
1/2	·5419	·8127	·9030	·9932	·5425	·8142	1/2
3/4	·8120	1·2178	1·3531	1·4883	·8135	1·2213	3/4
1	1·0817	1·6222	1·8023	1·9824	1·0844	1·6283	1
1 1/4	1·3509	2·0258	2·2506	2·4755	1·3551	2·0352	1 1/4
1 1/2	1·6195	2·4285	2·6981	2·9676	1·6256	2·4421	1 1/2
1 5/8	1·7537	2·6296	2·9215	3·2133	1·7607	2·6456	1 5/8
1 3/4	1·8877	2·8305	3·1446	3·4587	1·8959	2·8490	1 3/4
1 7/8	2·0216	3·0312	3·3676	3·7039	2·0310	3·0524	1 7/8
2	2·1554	3·2317	3·5903	3·9488	2·1661	3·2558	2
2 1/8	2·2890	3·4320	3·8128	4·1935	2·3011	3·4592	2 1/8
2 1/4	2·4225	3·6321	4·0351	4·4380	2·4361	3·6626	2 1/4
2 3/8	2·5560	3·8321	4·2572	4·6822	2·5710	3·8660	2 3/8
2 1/2	2·6892	4·0318	4·4790	4·9261	2·7059	4·0694	2 1/2
2 5/8	2·8224	4·2313	4·7006	5·1698	2·8407	4·2727	2 5/8
2 3/4	2·9554	4·4307	4·9220	5·4132	2·9756	4·4761	2 3/4
2 7/8	3·0884	4·6298	5·1432	5·6564	3·1103	4·6794	2 7/8
3	3·2212	4·8288	5·3642	5·8994	3·2451	4·8827	3
3 1/8	3·3538	5·0276	5·5849	6·1421	3·3797	4·0860	3 1/8
3 1/4	3·4864	5·2261	5·8055	6·3846	3·5144	5·2893	3 1/4
3 3/8	3·6188	5·4245	6·0258	6·6269	3·6490	5·4926	3 3/8
3 1/2	3·7511	5·6227	6·2460	6·8689	3·7835	5·6959	3 1/2
3 5/8	3·8833	5·8208	6·4658	7·1107	3·9181	5·8992	3 5/8
3 3/4	4·0154	6·0186	6·6856	7·3523	4·0525	6·1024	3 3/4
3 7/8	4·1474	6·2162	6·9051	7·5936	4·1870	6·3057	3 7/8
4	4·2792	6·4136	7·1243	7·8346	4·3214	6·5089	4
4 1/8	4·4109	6·6108	7·3433	8·0754	4·4557	6·7122	4 1/8
4 1/4	4·5425	6·8079	7·5622	8·3160	4·5900	6·9154	4 1/4
4 3/8	4·6740	7·0048	7·7808	8·5563	4·7243	7·1186	4 3/8
4 1/2	4·8054	7·2015	7·9992	8·7965	4·8585	7·3218	4 1/2
4 5/8	4·9366	7·3980	8·2174	9·0364	4·9927	7·5250	4 5/8
4 3/4	5·0677	7·5943	8·4354	9·2759	5·1269	7·7281	4 3/4
4 7/8	5·1987	7·7904	8·6532	9·5153	5·2610	7·9313	4 7/8
5	5·3296	7·9864	8·8708	9·7545	5·3950	8·1344	5
5 1/8	5·4604	8·1821	9·0881	9·9935	5·5291	8·3376	5 1/8
5 1/4	5·5911	8·3777	9·3052	10·2322	5·6630	8·5407	5 1/4
5 3/8	5·7216	8·5731	9·5222	10·4707	5·7970	8·7438	5 3/8
5 1/2	5·8520	8·7683	9·7390	10·7089	5·9309	8·9469	5 1/2
5 5/8	5·9823	8·9633	9·9555	10·9469	6·0647	9·1500	5 5/8
5 3/4	6·1125	9·1582	10·1718	11·1849	6·1985	9·3531	5 3/4
5 7/8	6·2426	9·3528	10·3879	11·4222	6·3323	9·5562	5 7/8
6	6·3725	9·5473	10·6038	11·6595	6·4660	9·7592	6
6 1/4	6·6321	9·9357	11·0350	12·1335	6·7334	10·1653	6 1/4
6 1/2	6·8912	10·3234	11·4654	12·6065	7·0006	10·5714	6 1/2
6 3/4	7·1499	10·7103	11·8950	13·0786	7·2676	10·9774	6 3/4
7	7·4081	11·0966	12·3237	13·5498	7·5344	11·3833	7
7 1/2	7·9231	11·8669	13·1784	14·4895	8·0676	12·1951	7 1/2
8	8·4363	12·6344	14·0307	15·4256	8·6002	13·0067	8
9	9·4574	14·1609	15·7250	17·2873	9·6633	14·6293	9
10	10·4714	15·6764	17·4068	19·1349	10·7239	16·2513	10
12	12·4788	18·6746	20·7334	22·7890	12·8372	19·4930	12



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